<u>last time</u> Δ n-simplices $[v_0, v_1, \dots, v_n]$ ← ordered!

Δ-complex: • Δ_i ← collection of simplices (may have different dimensions)

• F_j ← face identifications (same dimension in each pair, different pairs may have different dimension)

<u>geometric realization</u> topological space $X = \bigsqcup \Delta_i / \sim$ ← equivalence generated by linear maps between face pairings.

<u>Examples</u> 

$\Delta = \{ [v_0, v_1, v_2, v_3], [w_0, w_1, w_2], [u_0, u_1] \}$

pairings $\{ [v_2, v_3] \sim [w_0, w_1], \ [w_2] \sim [u_1] \}$

• $S^1 = \Delta = \{ [v_0, v_1] \}$, $F = \{ [v_0] \sim [v_1] \}$ 

$= \Delta = \{ [v_0, v_1], [w_0, w_1] \}$, $F = \{ [v_0] \sim [w_0], [v_1] \sim [w_1] \}$ 

• S^2 $\Delta = \{ [v_0, v_1, v_2], [w_0, w_1, w_2] \}$ $F = \{ [v_0, v_1] \sim [w_0, w_1], [v_1, v_2] \sim [w_1, w_2], [v_0, v_2] \sim [w_0, w_2] \}$.

• S^n: 2 n-simplices $[v_0, \dots, v_n]$, $[w_0, \dots, w_n]$ identify $[v_0, \dots \hat{v}_i, \dots v_n] \sim [w_0, \dots \hat{w}_i, \dots w_n]$

• dim 0 : collection of points $\{ (v), (w), \dots \}$

• dim 1 : any graph is a Δ-complex 

• dim 2 : <u>Thm</u> classification of ~~compact~~ closed orientable surfaces:

 ...

S^2 $T^2 = S^1 \times S^1$

non-orientable: connect sum with $\mathbb{RP}^2$

 K^2 ...

$\mathbb{RP}^2$ Klein bottle

Fact: every surface can be realized by a Δ-complex

· glue triangles by pairing edges ⇝ gives surface

Thm (alg. invariant) X Δ-complex which is a surface. Then defn $\chi(X) = V - E + F$ and if $X \cong Y$ then $\chi(X) = \chi(Y)$

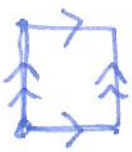
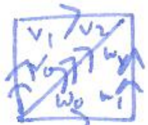
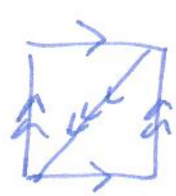
Euler characteristic — homeomorphic

 … $\mathbb{R}P^2$ K …

χ 2 0 −2 … −1 0 1 …

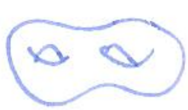
Fact complete invariant for closed surfaces.

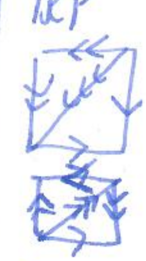
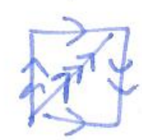
Example S^2:  $V - E + F$, $3 - 3 + 2 = 2$  $V - E + F$, $4 - 6 + 4 = 2$.

$T^2 = S^1 \times S^1$  S^1, $S^1\times S^1 / \sim$   warning:  doesn't work!

$\Delta = \{[v_0,v_1,v_2],[w_0,w_1,w_2]\}$ $F = \{[w_0,w_1]\sim[v_1,v_2], [v_0,v_1]\sim[w_1,w_2], [w_0,w_2]\sim[v_0,v_2]\}$

$\chi = V - E + F = 1 - 3 + 2 = 0$

S_2:  =  octagon ∼ 6 triangles.

$\mathbb{R}P^2$  K^2 

Δ-complex ⇝ chain complex: $\dots \to C_2 \to C_1 \to C_0 \to 0$

C_n = free abelian group whose generators are n-simplices in quotient X

e.g. T^2 get $0 \to \mathbb{Z}^2 \to \mathbb{Z}^3 \to \mathbb{Z} \to 0$ Q: what are the maps?

Motivation/intuition 1-manifolds:  $\partial = \emptyset$

boundary $\partial = \cdot\ \cdot$

2-manifolds  note $\partial\partial = 0$.

$\partial = S^1$ … $\partial = S^1$ $\partial = \emptyset$