Assignment for week 2

1. Consider the first R example:

   `https://www.math.csi.cuny.edu/~maher/teaching/2023/spring/tda/r/index.html`

   Try running the example with more or fewer points, i.e. change the number 50 in these lines:

   ```
   theta <- runif(50, 0, 2*pi)
   radius <- runif(50, 1, 1.5)
   ```

   (a) What is the smallest number of points you can have, and still get a reasonably long barcode for H_1, say at least four times as long as the second longest one.

   (b) What is the largest number of points you can have before your computer takes more than say 30 seconds to computer the answer?

2. Each of the following matrices defines a map $f\colon G \to H$, where both G and H are isomorphic to $\mathbb{Z}^2$.

   (a) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}$

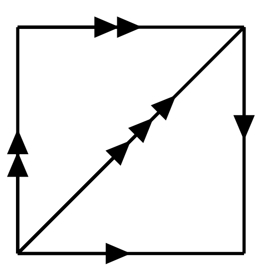
   Show that the two quotients $H/f(G)$ are isomorphic, and that the two matrices are equivalent by row and column operations.

3. Consider the sequence of abelian groups: $0 \xrightarrow{f_3} \mathbb{Z}^2 \xrightarrow{f_2} \mathbb{Z}^3 \xrightarrow{f_1} \mathbb{Z}^2 \xrightarrow{f_0} 0$ where the first non-zero map is $\begin{bmatrix} 0 & 0 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$ and the second one is $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$.

   (a) Show that the composition of any two adjacent maps is zero.

   (b) For each non-zero group, compute $\ker(f_i)/\mathrm{im}(f_{i+1})$.

4. The following Δ-complex is a surface.

   (a) Which one? Hint: compute the Euler characteristic.

   (b) Write down the simplicial homology chain complex.

(c) Compute the simplicial homology groups. Hint: compare with Q2 above.

5. Write down an explicit Δ-complex structure on $S^2 \vee S^1 \vee S^1$ and show that it has the same homology groups as the torus $S^1 \times S^1$. Remark: these spaces have the same homology groups, but they are not just not homeomorphic, they are not even homotopy equivalent, though I don't think we have an easy way of showing that right now.

6. Consider the map $f\colon S^1 \to S^1$ given by $\theta \mapsto 3\theta$ in polar coordinates on the circle. Choose Δ-complex structures on the two circles so you can realize f by a simplicial map, i.e. a map which takes simplices to simplices by linear maps. Use this to compute the induced homomorphisms on homology $f_*\colon H_k(S^1) \to H_k(S^1)$.