MTH 338 Linear Algebra

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- math tutoring 1S-214
- students w/ disabilities

Text: Linear Algebra, Hefferon (free).
HW: webworks.

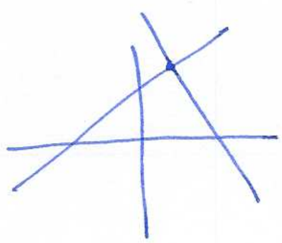
Introduction

aims : • solve linear equations } algebraicly / geometrically
• understand linear functions. }

Example 2 equations in 2 unknowns:

$$1x + 2y = 3 \quad ①$$
$$4x + 5y = 6 \quad ②$$

expect unique solution:



two lines usually intersect in a unique point.

Elimination :

$$4x + 5y = 6 \quad ②$$
$$1x + 2y = 3 \quad ①$$

② − 4①:

$$\begin{array}{r} 4x + 5y = 6 \\ -4x - 8y = -12 \\ \hline -3y = -6 \end{array}$$

one equation in one unknown!

$y = 2$.

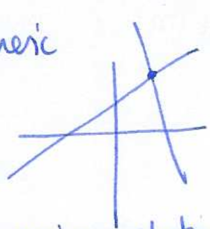
substitute in ①. $x + 4 = 3$
$x = 1$

Note: this works for any number of equations in any number of variables.

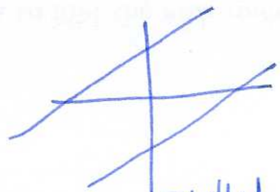
Problems : not always a unique (or any) solution.

Q: what happens geometrically for 2 equations in 2 unknowns?

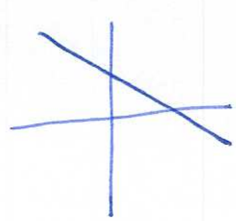
generic



unique solution



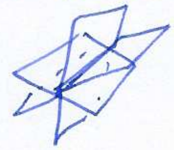
parallel
no solutions



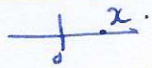
same line
→ infinitely many solutions

efficient calculations: use matrix notation. $\begin{array}{l} 4x+5y=6 \\ 1x+2y=3 \end{array} \leftrightarrow \left[\begin{array}{cc|c} 4 & 5 & 6 \\ 1 & 2 & 3 \end{array}\right]$.

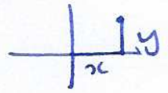
Higher dimensions: 3 equations in 3 unknowns $\leftrightarrow$ 3 planes in $\mathbb{R}^3$.

generic  unique solution. other cases: ...  infinitely many solutions.

Higher dimensional geometry $\leftrightarrow$ vectors. (ww 1).



$\mathbb{R}$ real line x

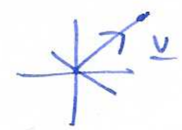
$\mathbb{R}^2$ plane (x,y). 

$\mathbb{R}^3$ 3d space  (x,y,z).

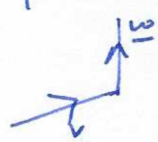
$\mathbb{R}^n$ n-dim space $(x_1, x_2, \dots, x_n)$

row vector; col vector.

Defn (algebraic) a vector is a list of n-numbers $\langle x_1, x_2, \dots, x_n \rangle \sim \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

(geometric) a vector has length and direction 

key properties addition, scalar multiplication.

addition geometric.  in coordinates $\underline{v} = \langle v_1, v_2, \dots v_n \rangle$

$\underline{w} = \langle w_1, w_2, \dots, w_n \rangle$

$\underline{v}+\underline{w} = \langle v_1+w_1, v_2+w_2, \dots, v_n+w_n \rangle$.

properties $\underline{v}+\underline{w} = \underline{w}+\underline{v}$ commutes

$\underline{u}+(\underline{v}+\underline{w}) = (\underline{u}+\underline{v})+\underline{w}$ associative

special vector $\underline{0} = \langle 0, 0, \dots, 0 \rangle$ s.t. $\underline{0}+\underline{v} = \underline{v}+\underline{0} = \underline{v}$.

inverse vector $-\underline{v} = \langle -v_1, -v_2, \dots, -v_n \rangle$ s.t. $\underline{v}+(-\underline{v}) = \underline{0}$.

warning: can only add vectors in same dim! $\langle 1,2 \rangle + \langle 3,4,5 \rangle$ X does n't make sense!

scalar multiplication geometric: $4\underline{v}$ = vector in same direction as $\underline{v}$ but 4 times as long

$-\underline{v}$ = vector in opp. direction to $\underline{v}$.

in coordinates: $4\underline{v} = \langle 4v_1, 4v_2, \dots, 4v_n \rangle$.

properties $1\underline{v} = \underline{v}$, $a(b\underline{v}) = (ab)\underline{v}$ $a(\underline{v}+\underline{w}) = a\underline{v}+a\underline{w}$ $(a+b)\underline{v} = a\underline{v}+b\underline{v}$