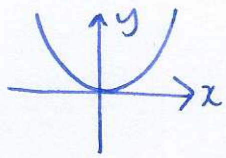
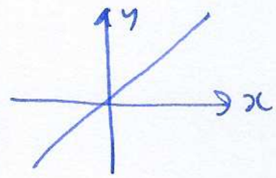
if $f'(x) < 0$ for all $x \in (a,b)$ then f is decreasing on (a,b)

Example 1. $f(x) = x^2$ $\qquad f'(x)$ $\quad y = 2x$

increasing on $(0,\infty)$

decreasing on $(-\infty,0)$



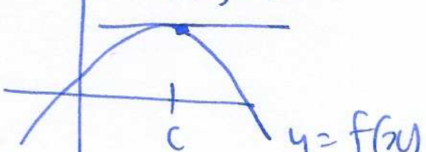


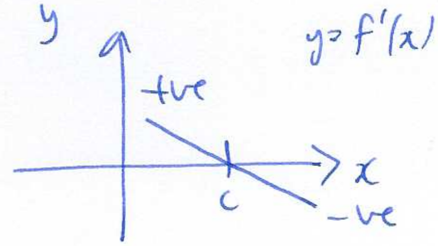
2. $f(x) = x^2 - 2x - 3$ $\qquad f'(x) > 0$ where $2x - 2 > 0$

$f'(x) = 2x - 2$ $\qquad\qquad\qquad\qquad x > 1$

First derivative test

Local max: $f'(c) = 0$

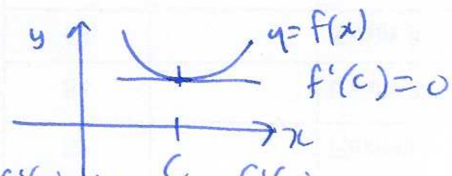


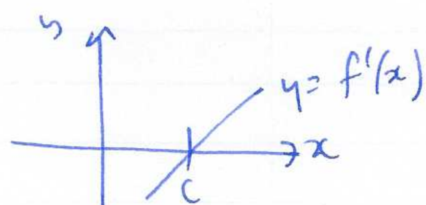


$f'(x) > 0$ $\qquad f'(x) < 0$

- if $f'(x)$ goes from positive to negative $\Rightarrow$ local max

Local min:





$f'(x) < 0$ $\qquad f'(x) > 0$

- if $f'(x)$ goes from negative to positive, $\Rightarrow$ local min.

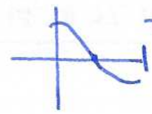
Thm **First derivative test** If $f(x)$ is differentiable and $f'(c) = 0$ then if $f'(x)$ changes from +ve to -ve at c $\Rightarrow$ c local max

-ve to +ve $\Rightarrow$ c local min

Example classify critical points of $f(x) = \cos^2 x + \sin x$ on $[0,\pi]$

find critical points: $f'(x) = -2\cos(x)\sin x + \cos(x)$

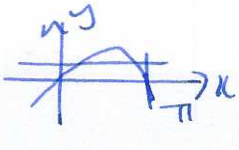
solve $f'(x) = 0$: $\cos(x)(1 - 2\sin x) = 0$ $\qquad$ • $\cos(x) = 0$



$x = \frac{\pi}{2}$

• $1 - 2\sin(x) = 0$

$\Leftrightarrow \sin(x) = \frac{1}{2}$



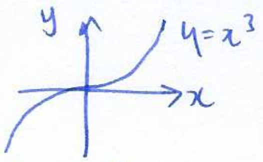
$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad$ so critical points are $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

find sign of $f'(x)$:

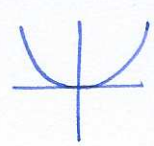
	0		$\pi/6$		$\pi/2$		$5\pi/6$		π
$\cos(x)$		$+$		$+$		$-$		$-$	
$1-2\sin(x)$		$+$		$-$		$-$		$+$	
$f'(x)$		$+$		$-$		$+$		$-$	
			local max		local min		local max		

Example critical point not max or min

$f(x) = x^3$



$f'(x) =$



$y=3x^2$

$f'(x)=0 \Rightarrow x=0$

		0	
$f'(x)$	$+$		$+$

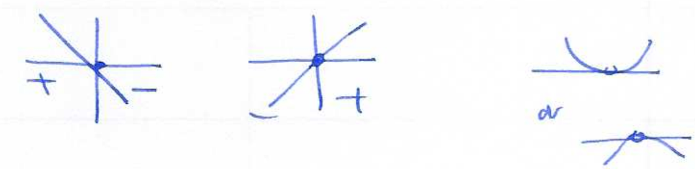
$\Rightarrow$ $x=0$ not local max or min

recall critical point $f'(x)=0$

$f(x)$

local max   local min   neither

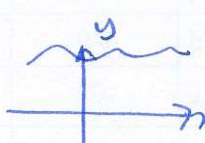
$f'(x)$



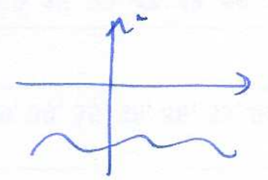
$f''(x)$ $-$ve $+$ve 0

§4.4 Second derivative test

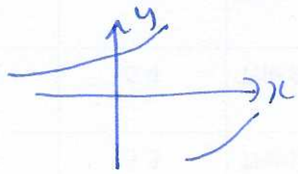
recall $f(x)>0$
f positive



$f(x)$ $f(x)<0$
f negative



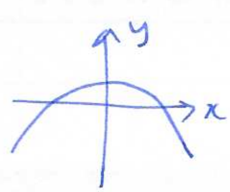
$f'(x)>0$
f increasing



$f'(x)<0$
f decreasing

$f''(x)>0$
concave up

$f''(x)<0$
concave down



mnemonic: up ∪ ☺   ∩ down.