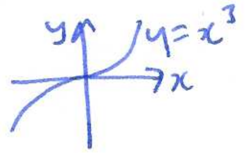
<u>warning</u> $f'(c) = 0 \not\Rightarrow$ c is local max or min

<u>Example</u> $y = x^3$



$f'(x) = 3x^2$ $f'(0) = 0$, but $x = 0$ not local max or min.

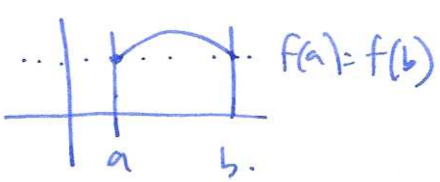
• How to find the absolute max or min of a differentiable function on a closed interval $[a,b]$ ① find critical points, evaluate function there
② check endpoints.

<u>Example</u> ① find abs max/min of $2x^3 - 15x^2 + 24x + 7$ on $[0,3]$

② $x^2 - 8$ on $[1,4]$ ③ $\cos(x)\sin(x)$ on $[0,\pi]$.

<u>Thm</u> (Rolle's Thm) suppose $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b). If $f(a) = f(b)$, then there is $c \in (a,b)$ s.t. $f'(c) = 0$

<u>Proof</u>

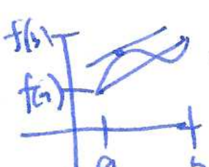


• if there is a local max/min at c, then $f'(c) = 0$

• if no local max/min, then $f(x) = \text{const} = f(a) = f(b)$, so $f'(c) = 0$ for all $c \in (a,b)$ □.

§4.3 First derivative test

<u>Thm</u> (MVT) (Mean Value Theorem) Suppose f is continuous on $[a,b]$ and differentiable on (a,b), then there is a $c \in (a,b)$ s.t. $f'(c) = \frac{f(b) - f(a)}{b-a}$, i.e. there is a point where the slope is equal to the average rate of change



<u>Proof</u> (Rolle's Thm turned sideways) □.

<u>Corollary</u> If $f(x)$ is differentiable, and $f'(x) = 0$, then $f(x) = c$ constant.

<u>Proof</u> suppose there is $a \neq b$ with $f(a) \neq f(b)$, then there is c with $f'(c) = \frac{f(b) - f(a)}{b-a} \neq 0$ □.

<u>Monotonicity</u> suppose f is differentiable on (a,b);

If $f'(x) > 0$ for all $x \in (a,b)$ then f is increasing on (a,b)