need r in terms of h: $\frac{r}{h} = \frac{4}{10}$, i.e. $r = \frac{2}{5}h$

so $V = \frac{1}{3}\pi h \left(\frac{2}{5}h\right)^2 = \frac{4}{75}\pi h^3$

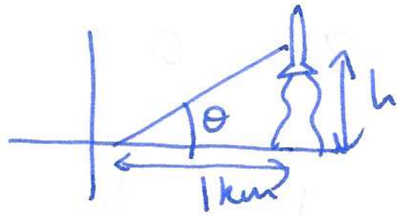
$\frac{dV}{dt} = \frac{12}{75}\pi h^2 \frac{dh}{dt}$ so when $h = 5$: $10 = \frac{12}{75}\pi (5)^2 \frac{dh}{dt}$ $\quad \frac{dh}{dt} = \frac{10}{4\pi}$ ft/min

advice: ① give things names

② write down relations between them and use implicit differentiation

③ plug in numbers as necessary.

Example



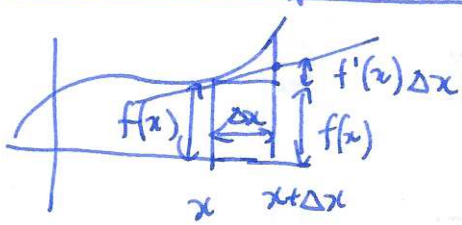
if angle is $\theta = \frac{\pi}{3}$ and rate of change is $\frac{d\theta}{dt} = \frac{1}{2}$ radians/sec

how fast is the rocket going?

$\frac{h}{1} = \tan\theta \quad \frac{dh}{dt} = \sec^2\theta \frac{d\theta}{dt} \quad \frac{dh}{dt} = \sec^2\left(\frac{\pi}{3}\right)\cdot\frac{1}{2}$

≈ 0.1 km/sec

§4.1 Linear approximation



If $f(x)$ is differentiable at x, and Δx is small,

then $f(x+\Delta x) \approx f(x) + f'(x)\Delta x$

so change in f is $\Delta f \approx f(x+\Delta x) - f(x)$

$\approx f'(x)\Delta x$

Example estimate $\sqrt{103}$

$f(x) = \sqrt{x} = x^{1/2}$ $\qquad f(100) = 10$ $\qquad$ so $\Delta f \approx f'(x)\Delta x$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\qquad f'(100) = \frac{1}{20}$ $\qquad \frac{1}{20}\cdot 3.$

so $\sqrt{103} \approx 10 + \frac{3}{20} = 10.15$

Example you make an 18" pizza. If the diameter is accurate to ± 0.4 in how much pizza do you gain/lose?

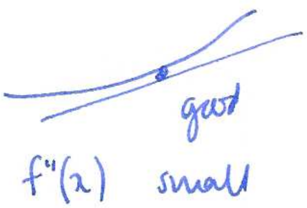
$A = \pi r^2 \quad 2r = D, \quad A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}, \quad A'(D) = \frac{2\pi D}{4} = \frac{\pi D}{2}$

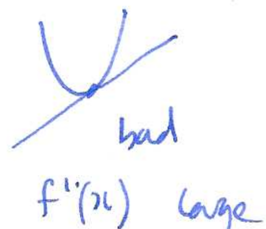
$\Delta A \approx A'(18)\Delta D = \frac{1}{2}\pi 18 . 0.4 \approx 11 \text{ in}^2$

Q: is this good or bad? absolute error $= 11$

percentage error $= \left| \frac{\text{absolute error}}{\text{actual value}} \right| \times 100 = \frac{11}{\pi . 18^2/4} \times 100 \sim 4\%$

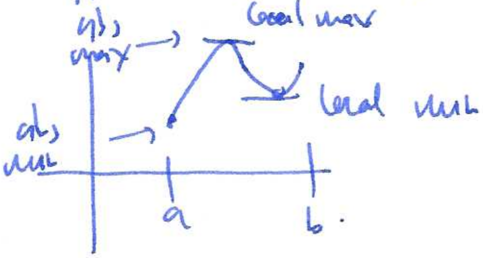
<u>observation</u>: when is the linear approximation a good approximation?





<u>§4.2 Extreme values</u>

suppose $f(x)$ is defined on a closed interval $[a,b]$



<u>Defn</u> $f(c)$ is the <u>absolute max</u> if $f(c) \geq f(x)$ for all $x \in [a,b]$

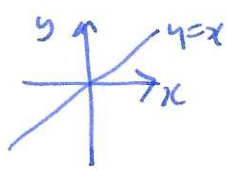
$f(c)$ is the <u>absolute min</u> if $f(c) \leq f(x)$ for all $x \in [a,b]$.

<u>note</u>: Q: <u>where</u> is the local/abs max/min ← want x-value

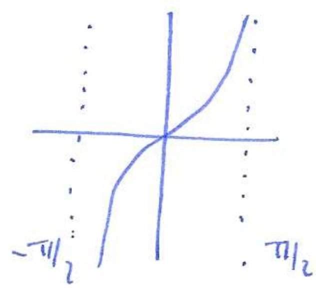
Q: <u>what</u> is the local/abs max/min ← want y-value

<u>warning</u> some functions do not have any max or min

<u>Examples</u> $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x$



$f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$, $x \mapsto \tan(x)$



<u>Thm</u> If $f(x)$ is continuous on a <u>closed and bounded interval</u> then $f(x)$ has both an absolute max and absolute min.

<u>Defn</u> $f(x)$ has a local max at $x=c$ if there is a small interval containing c s.t. $f(c)$ is an abs max on this interval.

$f(x)$ has a local min at $x=c$ if there is a small interval containing c s.t. $f(c)$ is an abs min on this interval.

<u>Defn</u> we say that $x=c$ is a critical point if $f'(c)=0$ (or undefined)