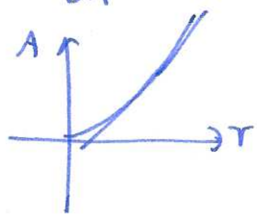
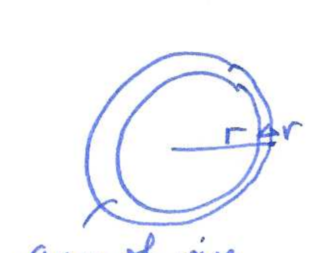
Example area of circle is $\pi r^2$

calculate rate of change of area wrt radius: $\frac{dA}{dr} = 2\pi r$

e.g. $\left.\frac{dA}{dr}\right|_{r=2} = 4\pi \qquad \left.\frac{dA}{dr}\right|_{r=5} = 10\pi$




area of ring $\approx 4\pi\Delta r$

for small $\overset{\Delta r}{h}$, $f'(x_0) \simeq \frac{f(x_0+h) - f(x_0)}{h}$

so $f(x_0+h) \approx f(x_0) + h f'(x_0)$ ← linear approximation formula

Example stopping distance in feet given by $F(s) = 1.1s + 0.05s^2$

($s$ speed in mph) calculate stopping distance when $s = 30$ $F(30) = 78$

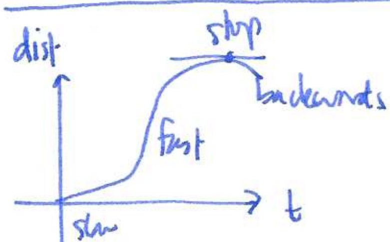
$F'(s) = 1.1 + 0.1s \qquad F'(30) = 1.1 + 0.1 \cdot 30 = 4.1$ ft/mph.

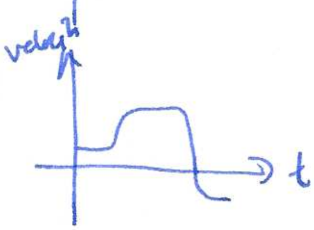
estimate stopping distance at $s = 31$ (using above info)

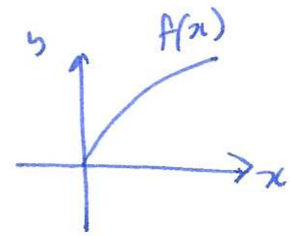
$F(s+h) \approx F(s) + hF'(s)$

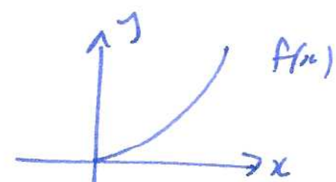
$F(31) \simeq F(30) + 1 \cdot F'(30) = 78 + 4.1 = 82.1$

On the interpretation of graphs



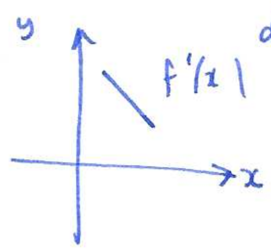



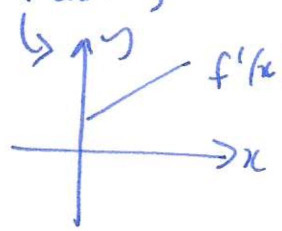



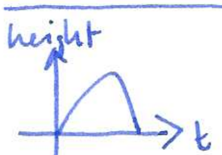

rate of change decreasing

increasing




Motion under gravity




$s_0 = s(0) =$ height at $t = 0$

$v_0 = v(0) =$ velocity at $t = 0$

$s''(t) = v'(t) = a(t) = -g \qquad g = 9.8\,m/s^2$ (constant)

$32\,ft/s^2$

$s'(t) = v(t) = -gt + v_0$

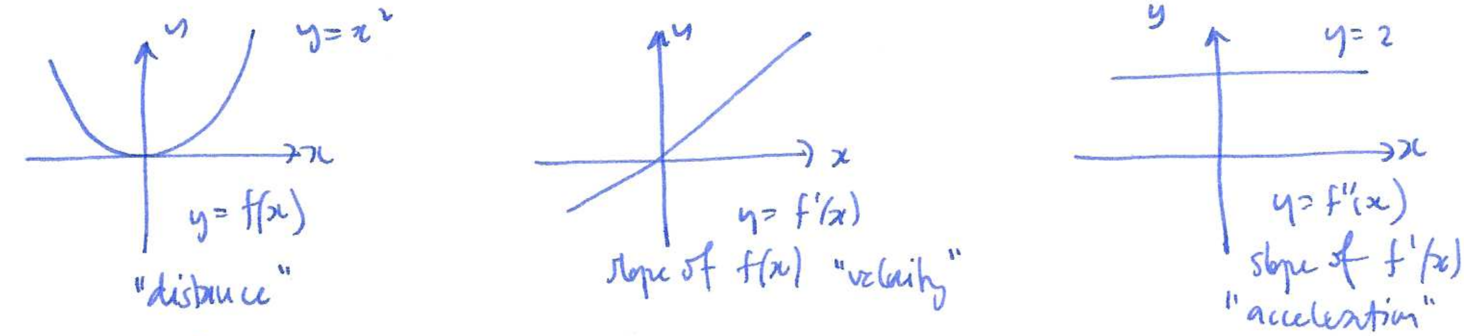
$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$

Example throw a stone upwards at 10 m/s from height 2m, what is max height? $s(t) = 2 + 10t - \frac{1}{2}gt^2$

$v(t) = 10 - gt \qquad v(t) = 0 \Rightarrow t = \frac{10}{g} \approx 1 \qquad s(1) = 2 + 10 - 5 = 7m.$

- how fast is it going when it hits the ground?
- if I can throw a stone 10m high, how fast can I throw it?

## §3.5 Higher derivatives



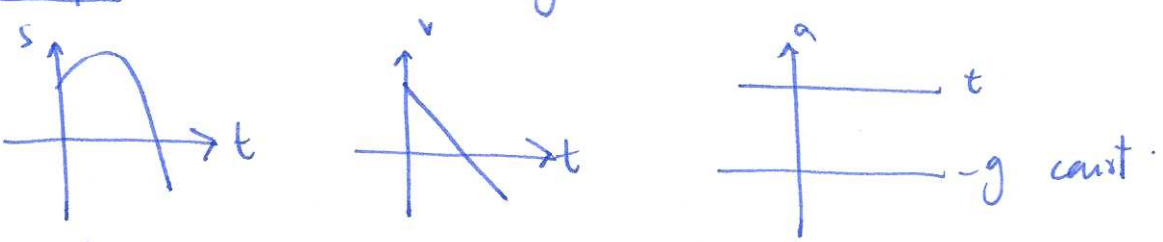

Example $f(x) = xe^x$

$f'(x) = xe^x + e^x$

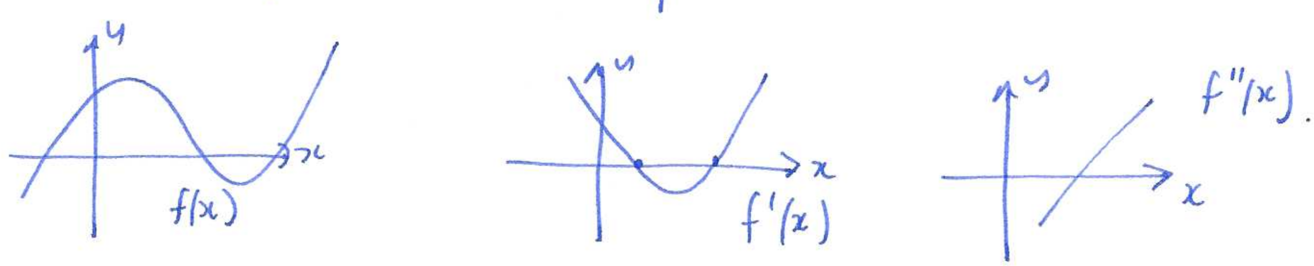
$f''(x) = xe^x + e^x + e^x = xe^x + 2e^x$

$f^{(3)}(x) = xe^x + e^x + 2e^x = xe^x + 3e^x.$ etc.

Example acceleration due to gravity




Example




## §3.6 Trigonometric functions

Thm $\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x.$

Proof (for $\sin x$) $\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

recall $\sin(A+B) = \sin A \cos B + \cos A \sin B$