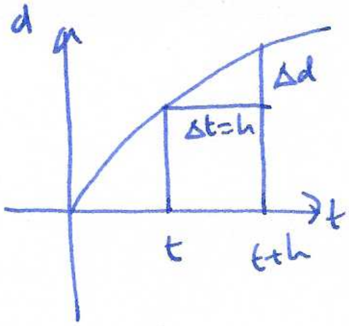
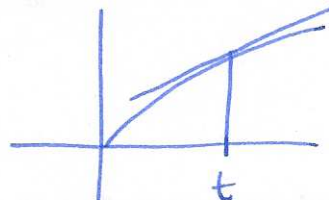


$d = f(t)$ average speed on an interval $[t, t+h]$

is $\dfrac{\Delta d}{\Delta t} = \dfrac{f(t+h) - f(t)}{t+h-t} = \dfrac{f(t+h)-f(t)}{h}$



Q: what is the speed at time t?

(sometimes called the instantaneous speed / rate of change)

A: speed is the slope of the tangent line at t

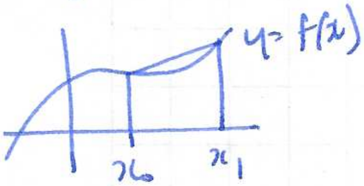
idea/hope: as the length of the interval $[t, t+h]$ gets small, the average speed gets close to the slope of the tangent line.

this works for "nice" functions

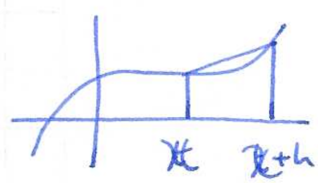
observation: this works for any function $y = f(x)$, not just speed.

summary: average rate of change over an interval $[x_0, x_1]$ is

$\dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$



often:

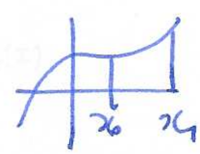


$\dfrac{f(x+h) - f(x)}{h}$

§2.2 Limits

aim: want to find slopes of tangent lines

know: average rate of change $\dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$



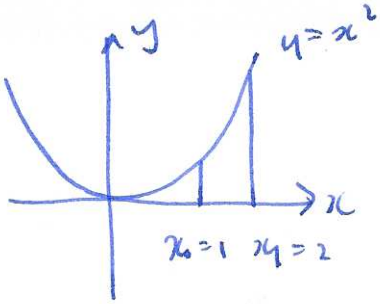
Q: why not set $x_0 = x_1$?

A: doesn't work, get $\dfrac{f(x_1) - f(x_1)}{x_1 - x_1} = \dfrac{0}{0}$ undefined.

Observations

① if we draw careful pictures, the average slope gets close to the slope of the tangent line as the length of the interval gets small

② seems to work for sample calculations:



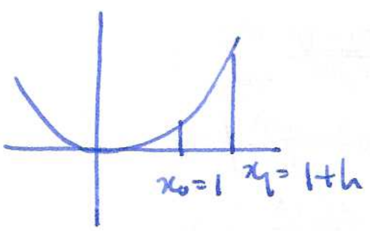
$x_1 = 2$: $\dfrac{f(2)-f(1)}{2-1} = \dfrac{4-1}{1} = 3$

$x_1 = 1.5$: $\dfrac{f(3/2)-f(1)}{3/2 - 1} = \dfrac{9/4 - 1}{1/2} = \dfrac{5}{2} = 2.5$

$x_1 = 1.1$ $\dfrac{1.21 - 1}{0.1} = 2.1$

$x_1 = 1.01$ $\dfrac{1.0201 - 1}{0.01} = 2.01$

3 seems to work algebraically

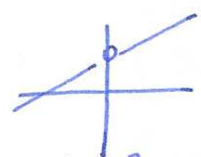


average rate of change from $x_0 = 1$ to $x_1 = 1+h$

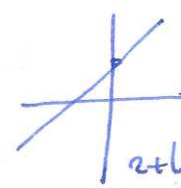
$= \dfrac{f(1+h)-f(1)}{1+h-1} = \dfrac{(1+h)^2 - 1^2}{h} = \dfrac{1+2h+h^2-1}{h}$

$= \dfrac{2h+h^2}{h} = 2+h$

($h \neq 0$!)



$\dfrac{2h+h^2}{h}$



$2+h$.

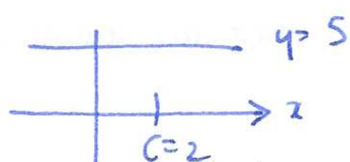
<u>Def</u>

Defⁿ: Let f be a function defined on an interval containing c, but not necessarily at c. We say " the limit of $f(x)$ as x approaches c is equal to L " if $|f(x) - L|$ becomes arbitrarily small as x gets closer to c

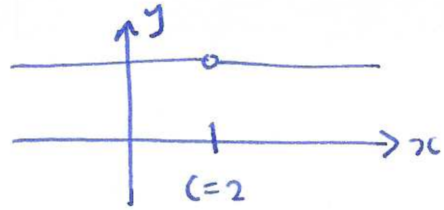
notation : $\lim_{x \to c} f(x) = L$ or $f(x) \to L$ as $x \to c$

we also say : " $f(x)$ converges to L as x tends to c "

Examples a) $f(x) = 5$, $c = 2$



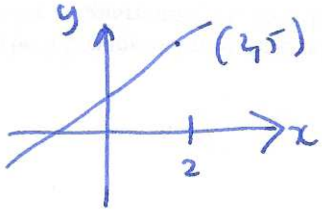
b) $f(x) = \dfrac{5(x-2)}{(x-2)}$, $c=2$



want to show : $|f(x)-5|$ close to 0 if x close to 2

$|f(x)-5| = |5-5| = 0$ for all $x \neq 2$, so this is true.

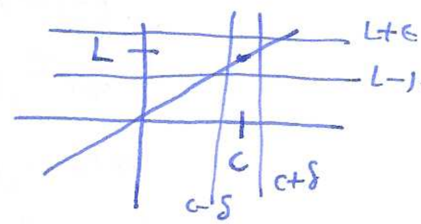
c) $\lim_{x \to 2} 2x+1 = 5$



want to show : $|f(x)-5|$ close to zero when x close to 2

$|f(x)-5| = |2x+1-5| = |2x-4| = 2|x-2|$. x close to 2 $\Leftrightarrow$ $|x-2|$ small.

<u>Precise defn</u> Let $f(x)$ be defined on an interval containing c, but not nec. at c. We say $\lim_{x \to c} f(x) = L$ if for all $\epsilon > 0$ there is a $\delta > 0$ s.t. if $|c-x| < \delta$ then $|f(x) - L| < \epsilon$.



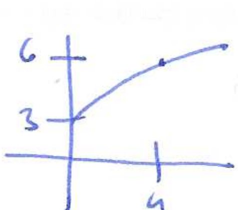
<u>useful facts</u>

<u>Thm</u> for any constants k, c :
$$\lim_{x \to c} k = k$$
$$\lim_{x \to c} x = c$$

<u>investigating limits</u> : try
- drawing a picture
- calculating close values
- algebra

<u>Example</u> $\lim_{x \to 9} \dfrac{x-9}{\sqrt{x}-3}$ problem : can't plug in $x=9$, get $\frac{0}{0}$ undefined

- draw picture



looks like $f(9) = 6$