MTH 231 Calculus I

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- math tutoring 1S-214
- student w/ disabilities

Text: Calculus, w/ early transcendentals, Rogawski, Adams.
HW: webworks.

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§1.2 Linear and quadratic functions

recall: input set (domain) $\xmapsto{\text{function}}$ output set (range)

examples: $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$ ($\mathbb{R}$ = set of real numbers) or $f(x) = x^2$

example values: $0 \mapsto 0$, $-1 \mapsto 1$, $1 \mapsto 1$, $2 \mapsto 4$ etc.

notation: $f : \mathbb{R} \to \mathbb{R}$

name ↗ ↗ inputs / domain ↑ outputs / range.

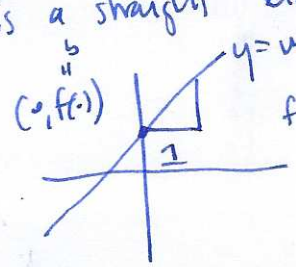
<u>Examples</u>

- $+ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $(a,b) \mapsto a+b$
- evaluation at 0: $\{\text{functions } f:\mathbb{R}\to\mathbb{R}\} \to \mathbb{R}$, $f \mapsto f(0)$

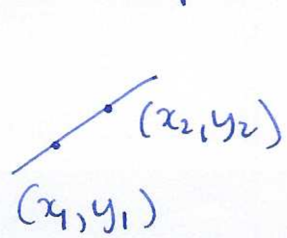
<u>key property</u> each input x gets mapped to an output $f(x)$, not multiple values...

A <u>linear function</u> $f: \mathbb{R} \to \mathbb{R}$ has the form $f(x) = mx + b$ (m, b "constants", i.e. don't depend on x) The graph of a linear function is a straight line.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

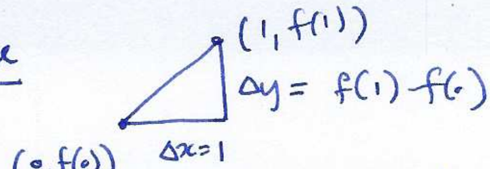


$f(1) - f(0) = m$
= slope



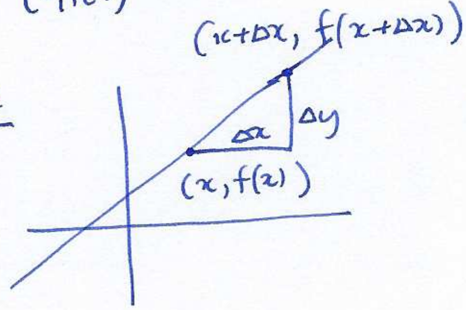
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

<u>special case</u>



$$\frac{\Delta y}{\Delta x} = \frac{f(1) - f(0)}{1 - 0} = \frac{m + b - b}{1} = m$$

<u>general case</u>

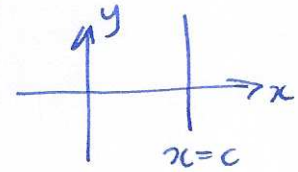


$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$$

$$= \frac{m(x+\Delta x) + b - (mx+b)}{\Delta x} = \frac{m\Delta x}{\Delta x} = m$$

<u>useful fact</u>: a straight line has constant slope everywhere.

<u>observations</u>
- $|m|$ large $\iff$ steep slope
- $m = 0$ horizontal line
- $m > 0$ increasing (from left to right)
- $m < 0$ decreasing
- vertical lines not graphs of functions



equation vs function

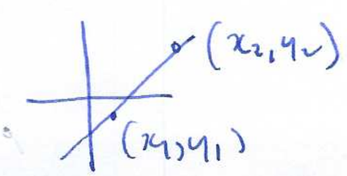
relation between variables
$x = c$
$x^2 + y^2 = 1$

a map from one set to another
e.g. $f: \mathbb{R} \to \mathbb{R}$
$x \mapsto f(x)$

the <u>graph</u> of $f: \mathbb{R} \to \mathbb{R}$ is $y = f(x)$ (an equation!)
but not every equation comes from a function.
to deal with <u>any</u> straight line, use the <u>general / symmetric linear equation</u> $ax + by = c$, at least one of a, b not zero),

e.g $x = c$, set $b = 0$, $a = 1$.

<u>useful technique</u>: find equation of line through two points



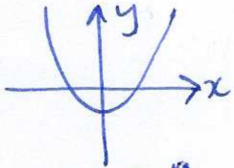
- find slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
- line $y - y_1 = m(x - x_1)$

point slope formula for a line.

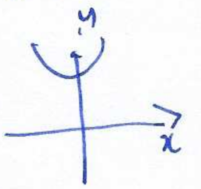
quadratic functions

given by $f(x) = ax^2 + bx + c$ (a, b, c constants, do not depend on x)

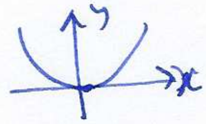
graphs are parabolas



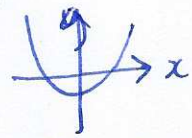
at most two distinct real solutions to $f(x) = 0$, given by $f(x) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



$b^2 - 4ac < 0$



$b^2 - 4ac = 0$



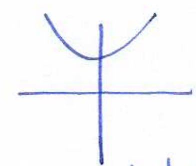
$b^2 - 4ac > 0$

useful techniques:

- factorization: $ax^2 + bx + c = a(x - r_1)(x - r_2)$, then r_1, r_2 are solutions | roots.
- complete the square: any quadratic can be written as $a(x+b)^2 + c$ example: $x^2 + 2x + 3$

$(x+1)^2 + 2$

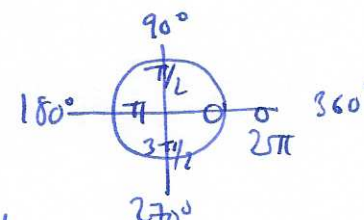
$x^2 + 2x + 1 + 2$



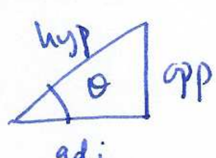
no solutions!

§1.4 Trig functions

- angles vs. radians, radians win.



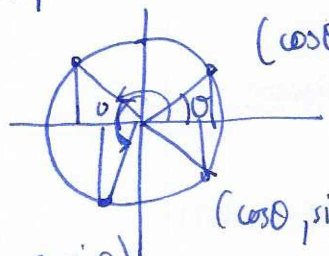
- angle in radians = distance travelled around unit circle.
- right angled triangles



$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

can extend these functions to be defined for all $\theta \in \mathbb{R}$.



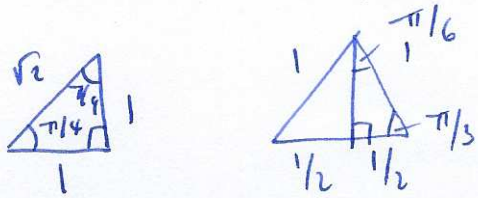
useful facts: $\sin(-\theta) = -\sin(\theta)$ (odd function)

$\cos(-\theta) = \cos\theta$ (even function)

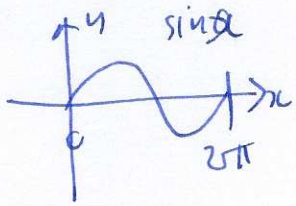
special values:

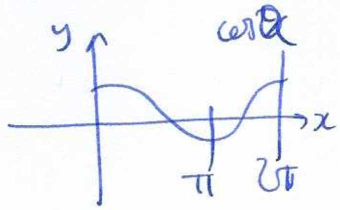
θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/\sqrt{2}$	0

this comes from special triangles



- graphs of $\sin(\theta)$, $\cos(\theta)$ are periodic with period 2π.





<u>More trig functions</u>

$$\tan(x) = \frac{\text{opp}}{\text{adj}} = \frac{\sin(x)}{\cos(x)}$$

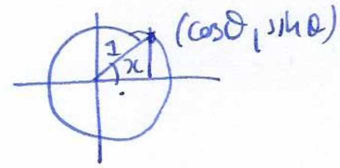
$$\frac{1}{\sin x} = \operatorname{cosec}(x)$$

$$\frac{1}{\cos x} = \sec(x)$$

$$\cot(x) = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

<u>Trig identities</u>

<u>Pythagorean identity</u> $\sin^2 x + \cos^2 x = 1$



$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \longleftrightarrow \tan^2 x + 1 = \sec^2 x$$

$$\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \longleftrightarrow 1 + \cot^2 x = \operatorname{cosec}^2 x$$

<u>Double angle</u>

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

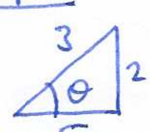
<u>Addition</u>

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

<u>special case</u> (shift) : $\sin(x + \frac{\pi}{2}) = \cos(x)$.

<u>Example</u> suppose $\sin\theta = \frac{2}{3}$, find $\cos\theta$, $\tan\theta$, $\sin 2\theta$



$\cos\theta = \frac{\sqrt{5}}{3}$ $\tan\theta = \frac{2}{\sqrt{5}}$ $\sin 2\theta = 2\sin\theta\cos\theta = 2\frac{2}{3}\frac{\sqrt{5}}{3} = \frac{4\sqrt{5}}{9}$.