Math 231 Calculus 1 Spring 22 Midterm 3b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Find $\lim_{x\to 0} \dfrac{\sin(4x)}{\ln(3x+1)}$.

$$\overset{l'H}{=} \lim_{x\to 0} \frac{4\cos(4x)}{\frac{1}{3x+1}\cdot 3} = \frac{4}{3}$$

(2) Find $\lim_{x\to 0} \dfrac{\cos(3x)-1}{e^{2x^2}-1}$.

$$\overset{l'H}{=} \lim_{x\to 0} \frac{-\sin(3x)\cdot 3}{e^{2x^2}\,4x} \overset{l'H}{=} \lim_{x\to 0} \frac{-\cos(3x)\cdot 9}{e^{2x^2}\cdot 16x^2 + e^{2x^2}\cdot 4} = \frac{-9}{4}$$

(3) Consider the function $f(x) = x + \frac{4}{x}$.

(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them

a) $f'(x) = 1 - \frac{4}{x^2}$ critical pts solve $f'(x)=0$: $x^2 = 4$ $x = \pm 2$

b) $f''(x) = \frac{8}{x^3}$ $f''(-2) < 0 \Rightarrow -2$ local max

$f''(2) > 0 \Rightarrow 2$ local min

(4) Consider the function $f(x) = e^{-x}(x^2 - 7)$.
(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all the points of inflection.
(c) Determine the intervals where $f(x)$ is concave up and concave down.

a) no vertical asymptotes

right horizontal asymptote

$\lim_{x\to\infty} \frac{x^2-7}{e^x} \overset{l'H}{=} \lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x} = 0$ $\quad y=0$

$\lim_{x\to-\infty} e^{-x}(x^2-7) = \lim_{x\to\infty} e^{+x}(x^2+7) = \infty$ no left horizontal asymptote

b) $f'(x) = -e^{-x}(x^2-7) + e^{-x}(2x) = e^{-x}(-x^2+2x+7)$

$f''(x) = -e^{-x}(-x^2+2x+7) + e^{-x}(-2x+2) = e^{-x}(x^2-4x-5) = e^{-x}(x-5)(x+1)$

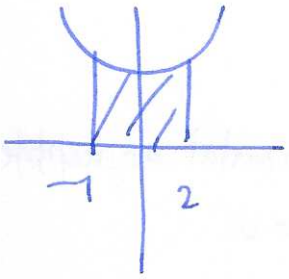
inflection points: $x=-1,\ x=+5$.

	$-\infty$	-1	5	∞
e^{-x}	+	+	+	
$x-5$	−	−	+	
$x+1$	−	+	+	
$f''(x)$	+	−	+	

concave up $(-\infty, -1) \cup (5, \infty)$

concave down $(-1, 5)$

(5) Find the area under the graph $y = 2x^2 + 3$ between $x = -1$ and $x = 2$.



$$\int_{-1}^{2} 2x^2 + 3\,dx = \left[\frac{2}{3}x^3 + 3x\right]_{-1}^{2}$$

$$= \frac{16}{3} + 6 - \left(-\frac{2}{3} - 3\right)$$

$$= 9 + \frac{18}{3} = 15$$

(6) Find the indefinite integral $\int 3\sin(x) - \frac{2}{x} + 4e^x \, dx$.

$-3\cos(x) - 2\ln|x| + 4e^x + c$

(7) Find the indefinite integral $\int x\sqrt{3+2x^2}\,dx$.

$u = 3+2x^2$

$\frac{du}{dx} = 4x$

$$\int x\, u^{1/2} \frac{dx}{du}\, du = \int x u^{1/2} \frac{1}{4x}\, du = \frac{1}{4}\int u^{1/2}\, du$$

$$= \frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{6}(3+2x^2)^{3/2} + C$$

(8) Find the definite integral $\int_0^1 e^x \cos(2e^x)dx$.

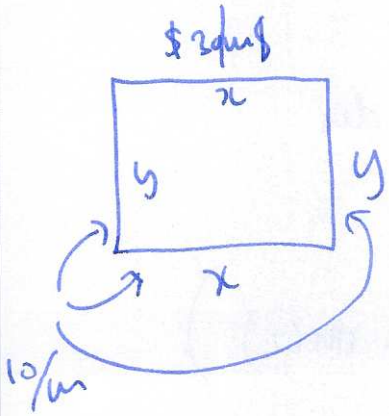
$u = 2e^x$

$\frac{du}{dx} = 2e^x$

$$\int_2^{2e} e^x \cos(u) \frac{dx}{du} du = \int_2^{2e} e^x \cos(u) \frac{1}{2e^x} du$$

$$= \frac{1}{2} \int_2^{2e} \cos(u) du = \left[\frac{1}{2} \sin(u) \right]_2^{2e} = \frac{1}{2}\left(\sin(2e) - \sin(2)\right).$$

(9) You wish to enclose a rectangular garden with a brick wall on one side, costing \$30/m and wooden fencing on the other three sides, costing \$10/m. If the total area of the garden is 100m^2, what are the dimensions which minimize the cost?



$A = xy = 100 \Rightarrow y = \frac{100}{x}$

$C = 30x + 10x + 20y = 40x + 20y$

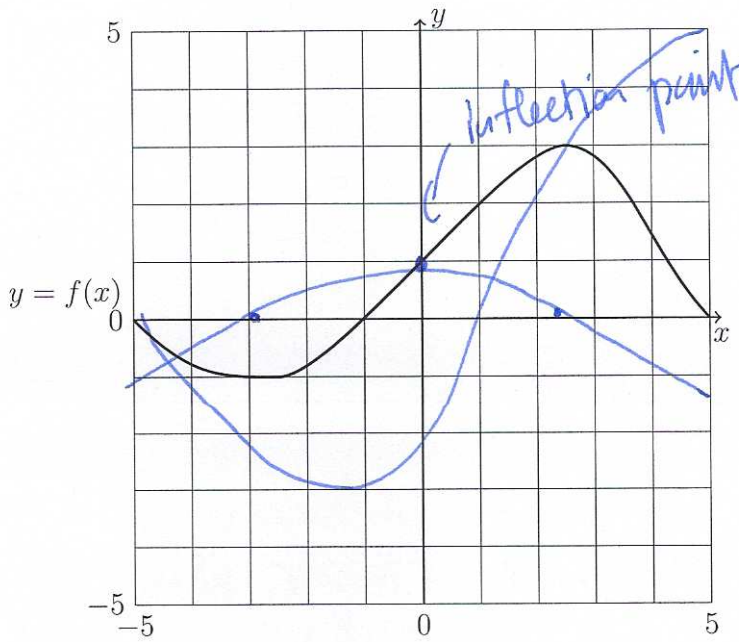
$C(x) = 40x + 20 \cdot \frac{100}{x} = 40x + \frac{2000}{x}$

$C'(x) = 40 - \frac{2000}{x^2}$ critical pt $C'(x) = 0$:

$x^2 = \frac{2000}{40}$ $\quad x = \sqrt{50}$

$y = \frac{100}{\sqrt{50}}$

(10) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection of $f(x)$.

(c) Sketch the graph of $\int_{-5}^{x} f(t)\,dt$.