Math 231 Calculus 1 Spring 22 Midterm 3a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Find $\lim_{x\to 0} \dfrac{\ln(3x+1)}{\sin(2x)}$.

$$\overset{l'H}{=} \lim_{x\to 0} \frac{\frac{1}{3x+1}\cdot 3}{\cos(2x)\cdot 2} = \frac{3}{2}$$

(2) Find $\lim_{x\to 0} \dfrac{e^{3x^2}-1}{\cos(5x)-1}$.

$$\overset{l'H}{=} \lim_{x\to 0} \frac{e^{3x^2}\cdot 6x}{-\sin(5x)\cdot 5} \overset{l'H}{=} \lim_{x\to 0} \frac{e^{3x^2}\cdot 36x^2 + e^{3x^2}\cdot 6}{-25\cos(5x)} = -\frac{6}{25}$$

(3) Consider the function $f(x) = x + \frac{9}{x}$.

(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them

a) $f'(x) = 1 - \frac{9}{x^2}$ solve $f'(x) = 0$: $1 - \frac{9}{x^2} = 0$ $x^2 = 9$ $x = \pm 3$

b) $f''(x) = \frac{18}{x^3}$

$x = -3$, $f''(x) < 0 \Rightarrow$ local max

$x = +3$ $f''(x) > 0 \Rightarrow$ local min

(4) Consider the function $f(x) = e^{-x}(x^2+1)$.
(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all the points of inflection.
(c) Determine the intervals where $f(x)$ is concave up and concave down.

a) no vertical asymptotes $\lim_{x\to\infty} \frac{x^2+1}{e^x} \overset{l'H}{=} \lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x} = 0$

right horizontal asymptote at $y=0$ $\lim_{x\to-\infty} e^{-x}(x^2+1) = \lim_{x\to\infty} e^{x}(x^2+1) = \infty$

no left horizontal asymptote.

b) $f'(x) = -e^{-x}(x^2+1) + e^{-x}(2x) - e^{-x}(-x^2+2x-1)$

$f''(x) = -e^{-x}(-x^2+2x-1) + e^{-x}(-2x+2) = e^{-x}(x^2-4x+3)$
$(x-3)(x-1)$

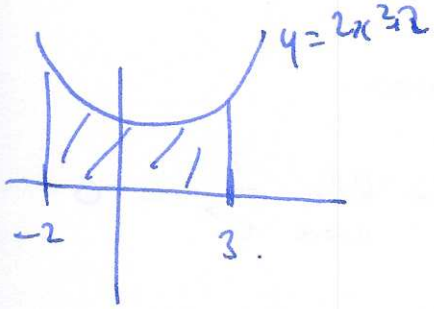
inflection points $x = 1, 3$.

		1		3	
e^{-x}	+		+		+
$(x-3)$	−		−		+
$(x-1)$	−		+		+
$f''(x)$	+		−		+

concave up $(-\infty, 1) \cup (3, \infty)$

concave down $(1, 3)$

(5) Find the area under the graph $y = 2x^2 + 2$ between $x = -2$ and $x = 3$.



$$\int_{-2}^{3} 2x^2 + 2\,dx = \left[\frac{2}{3}x^3 + 2x\right]_{-2}^{3}$$

$$= 18 + 6 - \left(\frac{-16}{3} - 4\right) = 28 + 5 + \frac{1}{3} = \frac{100}{3}$$

(6) Find the indefinite integral $\int \frac{2}{x} - 3e^x + 4\sin(x)\ dx$.

$2\ln(x) - 3e^x - 4\cos(x) + C$

(7) Find the indefinite integral $\int x\sqrt{2+3x^2}\,dx$.

$u = 2+3x^2$

$\frac{du}{dx} = 6x$

$\int x\, u^{1/2} \frac{dx}{du}\, du = \int x u^{1/2} \frac{1}{6x}\, du$

$= \frac{1}{6}\int u^{1/2}\, du = \frac{1}{6}\frac{2}{3} u^{3/2} + C = \frac{1}{9}(2+3x^2)^{3/2} + C$

(8) Find the definite integral $\int_0^1 e^x \sin(3e^x)dx$.

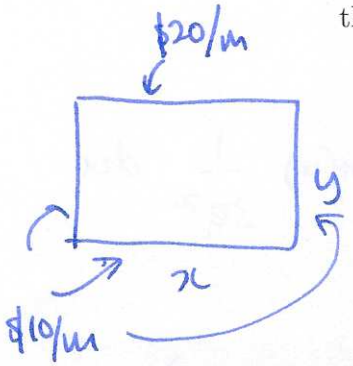
$u = 3e^x$

$\frac{du}{dx} = 3e^x$

$$\int_3^{3e} e^x \sin(u) \frac{dx}{du} du = \int_3^{3e} e^x \sin(u) \frac{1}{3e^x} du$$

$$= \frac{1}{3}\int_3^{3e} \sin(u)\, du = \frac{1}{3}\left[-\cos(u)\right]_3^{3e} = \frac{1}{3}\left(-\cos(3e) + \cos(3)\right)$$

(9) You wish to enclose a rectangular garden with a brick wall on one side, costing \$20/m and wooden fencing on the other three sides, costing \$10/m. If the total area of the garden is 200m^2, what are the dimensions which minimize the cost?



$A = xy = 200 \Rightarrow y = \frac{200}{x}$

$C = 20x + 10x + 20y = 30x + 20y$

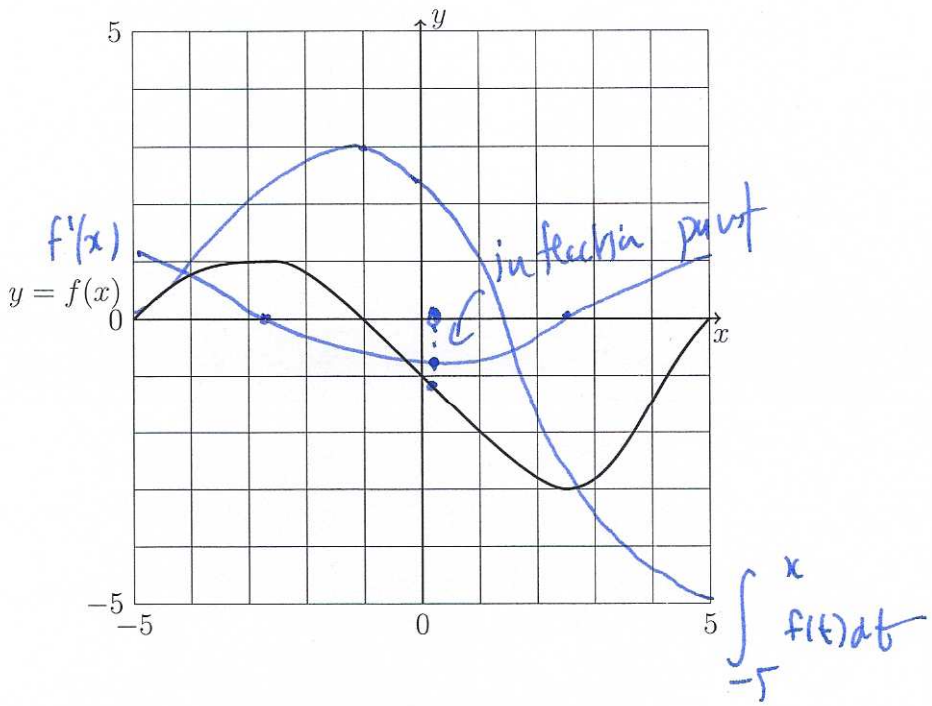
$C(x) = 30x + 20 \cdot \frac{200}{x} = 30x + \frac{4000}{x}$

$C'(x) = 30 - \frac{4000}{x^2}$ Solve $C'(x) = 0$: $x^2 = \frac{4000}{30}$

$x = \sqrt{\frac{4000}{30}} = \frac{20\sqrt{10}}{\sqrt{30}}.$

$y = \frac{200\sqrt{30}}{\sqrt{4000}}.$

(10) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection of $f(x)$.

(c) Sketch the graph of $\displaystyle\int_{-5}^{x} f(t)\,dt$.