Math 231 Calculus 1 Spring 22 Midterm 2b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of the following functions.

(a) $f(x) = x^3 e^x$.

$3x^2 e^x + x^3 e^x$

(b) $f(x) = \frac{\ln(x)}{\sin(x)}$.

$$\frac{\sin(x) \cdot \frac{1}{x} - \ln(x) \cdot \cos(x)}{\cancel{\cos(x)}(\sin(x))^2}$$

(2) (10 points) Find the derivative of the function $f(x) = \tan^{-1}\left(2 - \sqrt{x}\right)$.

$$f'(x) = \frac{1}{1-(2-\sqrt{x})^2}\left(-\frac{1}{2}x^{-1/2}\right).$$

(3) (10 points) Find the second derivative of the function $f(x) = \sqrt{3-4x}$.

$(3-4x)^{1/2}$

$f'(x) = \frac{1}{2}(3-4x)^{-1/2}(-4)$

$f''(x) = -\frac{1}{4}(3-4x)^{-3/2} \cdot 16 = -4(3-4x)^{-3/2}$

(4) (10 points) Use implicit differentiation to find the tangent line to the curve given by the equation $xy^2 + 3y^3 = 1$ at the point $(-2, 1)$.

$$y^2 + x\,2yy' + 9y^2y' = 0$$

$x = -2, y = 1$:

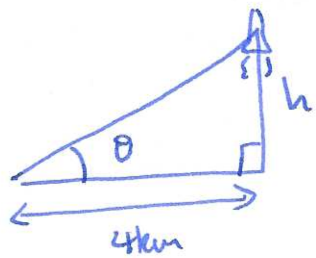
$$1 + -4y' + 9y' = 0 \qquad y' = \frac{1}{5}$$

tangent line: $\quad y - 1 = \frac{1}{5}(x+2)$.

(5) Find the following limit: $\lim_{x\to 0} \dfrac{e^{2x} - 1 - 2\tan(x)}{\sin(3x^2)}$

$$\overset{l'H}{=} \lim_{x\to 0} \frac{2e^{2x} - 2\sec^2(x)}{\cos(3x^2)\,6x} \overset{l'H}{=} \frac{4e^{2x} - 4\sec(x)\cdot\sec(x)\tan(x)}{-\sin(3x^2)\cdot 36x^2 + \cos(3x^2)\cdot 6} = \frac{4}{6} = \frac{2}{3}$$

(6) (10 points) A rocket is launched from ground level vertically upwards from a distance of 4km away. When you see it an angle of $\pi/4$ radians, the angle is increasing at a rate of 0.1 radians/sec. How fast is the rocket going up?



$$\frac{h}{4} = \tan\theta$$

$$\frac{1}{4}\frac{dh}{dt} = \sec^2\theta \frac{d\theta}{dt}$$

$$\frac{dh}{dt} = 4\sec^2\left(\frac{\pi}{4}\right)0.1 = \frac{8}{10} = \frac{4}{5} \text{ km/s.}$$

(7) (10 points) Use linear approximation to estimate $\sqrt{79}$. What is the percentage error in your approximation?

$81 = 9^2$

$79 = 81 - 2$

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f(79) \approx f(81) + f'(81) \cdot (-2)$

$= 9 + \frac{1}{18} \cdot -2 = 8\frac{8}{9} = \frac{80}{9}$

percentage error : $\frac{\left| \sqrt{79} - 8\frac{8}{9} \right|}{\sqrt{79}} \times 100 \approx 0.00781\ \%$

(8) Find the critical points for the function $f(x) = 12x - x^3$ and use the second derivative test to classify them.

$f'(x) = 12 - 3x^2 = 3(4 - x^2)$

critical pts $f'(x) = 0$

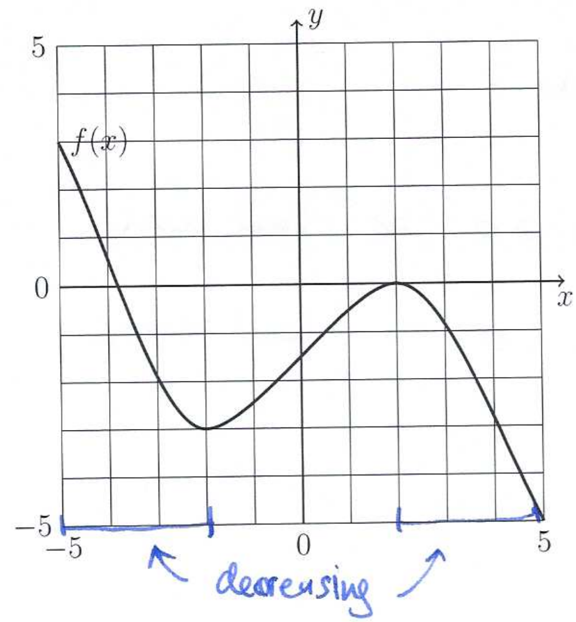
$\Rightarrow \quad x = \pm 2$

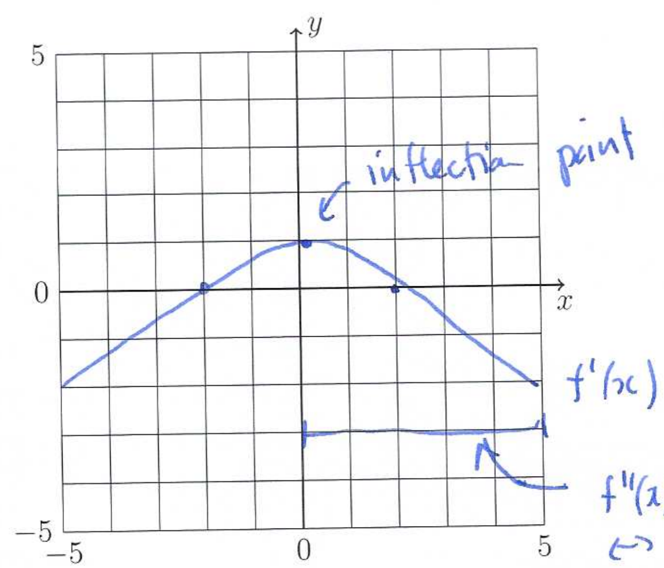
$f''(x) = -6x$

$f''(-2) = 12 > 0 \Rightarrow$ local min

$f''(2) = -12 < 0 \Rightarrow$ local max

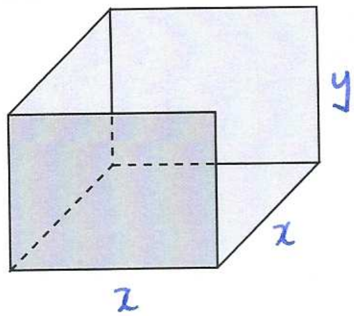
(9) (10 points) The graph of the function $f(x)$ is shown below. On the top set of axes mark where $f(x)$ is decreasing. On the lower set of axes sketch $f'(x)$, and then use this to find where $f(x)$ is concave down.





$f''(x) < 0$
$\Leftrightarrow f'(x)$ decreasing
$\Leftrightarrow f(x)$ concave down

(10) A container consists of a square base and three vertical sides, but without a top side. If the total volume of the container is 2m^3, what is the smallest possible area of the container?



$V = x^2 y = 2$

$A = x^2 + 3xy$

$y = \frac{2}{x^2} \Rightarrow A = x^2 + 3x\frac{2}{x^2} = x^2 + \frac{6}{x}$

$\frac{dA}{dx} = 2x - \frac{6}{x^2}$ critical pts $\frac{dA}{dx} = 0$: $x^3 = 3$ $x = \sqrt[3]{3}$.

$y = \frac{2}{\sqrt[3]{9}}$

$A = \sqrt[3]{9} + \frac{3\sqrt[3]{3} \cdot 2}{\sqrt[3]{9}} = \sqrt[3]{9} + \frac{6}{\sqrt[3]{3}}$