Math 231 Calculus 1 Spring 22 Midterm 2a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of the following functions.

(a) $f(x) = x^2 e^x$.

$2xe^x + x^2 e^x$

(b) $f(x) = \dfrac{\cos(x)}{\ln(x)}$.

$$\frac{\ln(x)(-\sin x) - \cos x \cdot \frac{1}{x}}{\ln(x)^2}$$

(2) (10 points) Find the derivative of the function $f(x) = \tan^{-1}(3 - \sqrt{x})$.

$$\frac{1}{1+(3-\sqrt{x})^2} \cdot \left(-\frac{1}{2}x^{-1/2}\right)$$

(3) (10 points) Find the second derivative of the function $f(x) = \sqrt{4 - 3x}$.

$(4-3x)^{1/2}$

$f'(x) = \frac{1}{2}(4-3x)^{-1/2} \cdot (-3) = -\frac{3}{2}(4-3x)^{-1/2}$

$f''(x) = \frac{3}{4}(4-3x)^{-3/2} \cdot (-3) = -\frac{9}{4}(4-3x)^{-3/2}$

(4) (10 points) Use implicit differentiation to find the tangent line to the curve given by the equation $x^2y + 2y^3 = 6$ at the point $(-2, 1)$.

$$2xy + x^2y' + 6y^2 \cdot y' = 0$$

$x = -2,\ y = 1$:

$$-4 + 4y' + 6y' = 0 \qquad y' = \frac{4}{10} = \frac{2}{5}$$

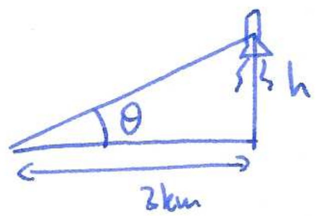
tangent line: $y - 1 = \frac{2}{5}(x+2)$

(5) Find the following limit: $\lim_{x\to 0} \dfrac{e^{3x} - 1 - 3\tan(x)}{\sin(2x^2)}$

$$\overset{l'H}{\Rightarrow} = \lim_{x\to 0} \frac{3e^{3x} - 3\sec^2(x)}{\cos(\cancel{4x}\ 2x^2)\cdot 4x}$$

$$\overset{l'H}{=} \lim_{x\to 0} \frac{9e^{3x} - 6\sec(x)\sec(x)\tan(x)}{-\sin(2x^2)\cdot 16x^2 + 4\cos(2x^2)} = \frac{9}{4}$$

(6) (10 points) A rocket is launched from ground level vertically upwards from a distance of 3km away. When you see it an angle of $\pi/6$ radians, the angle is increasing at a rate of 0.2 radians/sec. How fast is the rocket going up?



$$\frac{h}{3} = \tan\theta$$

$$\frac{1}{3}\frac{dh}{dt} = \sec^2\theta \,\frac{d\theta}{dt}$$

$$\frac{dh}{dt} = 3 \cdot \sec^2\left(\frac{\pi}{6}\right) 0.2 = \frac{3}{5}\cdot\frac{4}{3} = \frac{4}{5} \text{ km/sec}$$

(7) (10 points) Use linear approximation to estimate $\sqrt{47}$. What is the percentage error in your approximation?

$47 = 49-2$
$= 7^2-2$.

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f(47) \approx f(49) + f'(49).(-2)$

$7 + \frac{1}{2.7}.(-2) = 6\frac{6}{7} = \frac{48}{7}$

percentage error : $\frac{\left|\sqrt{47} - 6\frac{6}{7}\right|}{\sqrt{47}} \times 100 \approx 0.0217\%$

(8) Find the critical points for the function $f(x) = 27x - x^3$ and use the second derivative test to classify them.

$f'(x) = 27 - 3x^2 = 3(9 - x^2)$

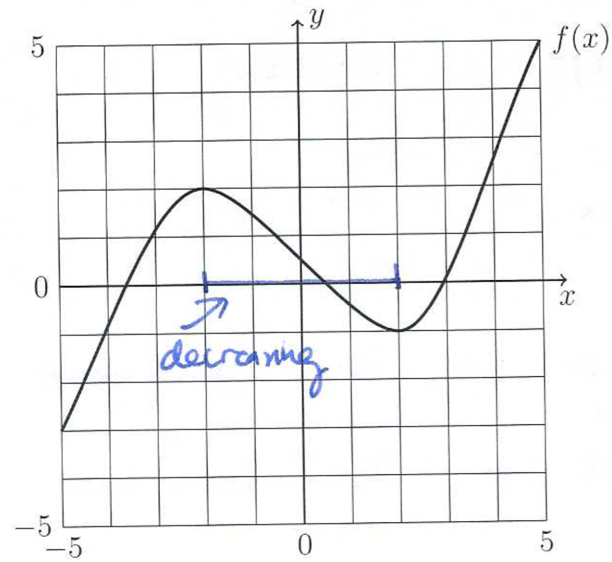
critical pts, solve $f'(x) = 0$: $x = \pm 3$.

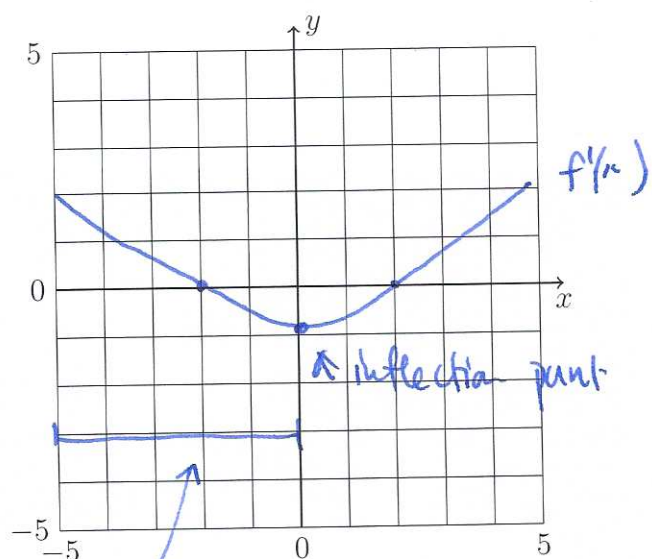
$f''(x) = -6x$

$f''(3) = -18 < 0 \Rightarrow$ local max

$f''(-3) = 18 > 0 \Rightarrow$ local min

(9) (10 points) The graph of the function $f(x)$ is shown below. On the top set of axes mark where $f(x)$ is decreasing. On the lower set of axes sketch $f'(x)$, and then use this to find where $f(x)$ is concave down.

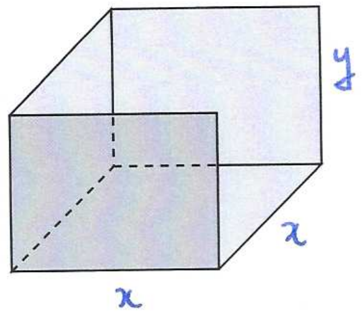




$f(x)$ concave down $\leftrightarrow$ $f''(x) < 0$

$\leftrightarrow$ $f'(x)$ decreasing

(10) A container consists of a square base and three vertical sides, but without a top side. If the total area of the container is 1m^2, what is the largest possible volume of the container?



$V = x^2 y$

$A = x^2 + 3xy = 1$

$y = \frac{1-x^2}{3x}$

$V = \frac{x^2(1-x^2)}{3x} = \frac{1}{3}x(1-x^2) = \frac{1}{3}(x - x^3)$

$\frac{dV}{dx} = \frac{1}{3}(1-3x^2)$ critical pt $\frac{dV}{dx} = 0$: $x^2 = \frac{1}{3}$ $x = \frac{1}{\sqrt{3}}$.

$y = \frac{1-1/3}{3/\sqrt{3}} = \frac{2/3}{3/\sqrt{3}} = \frac{2\sqrt{3}}{9}$.

$V = \frac{1}{3} \cdot \frac{2\sqrt{3}}{9}$