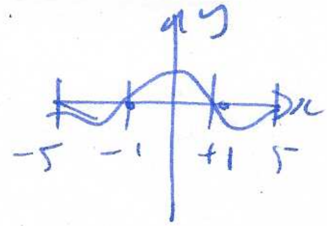
# SMT2 Solutions

**Q1** a) $[-5,-1] \cup [1,5]$ b) $[-1,1]$ c)




d) 1 e) 0 f) $-2, 0, 2$

**Q2** a) $5x^4 e^{-3x^2} + x^5 e^{-3x^2} \cdot (-6x)$

b) $$\frac{(2-\cos(4x))\frac{1}{2}(1-3x)^{-1/2}\cdot(-3) - \sqrt{1-3x}\cdot 4\sin(4x)}{(2-\cos(4x))^2}$$

c) $(e^{\sqrt{x}\ln(x)})' = e^{\sqrt{x}\ln(x)} \cdot \left(\frac{1}{2}x^{-1/2} \cdot \ln(x) + \sqrt{x}\cdot\frac{1}{x}\right)$.

d) $\frac{1}{\sqrt{\sec(x)}} \cdot \frac{1}{2}(\sec(x))^{-1/2} \cdot \sec(x)\tan(x)$.

e) $\frac{1}{1+(4x^{-1/4})^2} \cdot 4 \cdot -\frac{1}{4}x^{-5/4} = \frac{-x^{-5/4}}{1+16x^{-1/2}}$.

f) $\frac{-1}{\sqrt{1-(2x-3)^2}} \cdot 2 = -2(1-(2x-3)^2)^{-1/2}$.

**Q3** a) $20x^3 e^{-3x^2} + 5x^4 \cdot e^{-3x^2} \cdot -6x + -30x^5 e^{-3x^2} - 6x^6 e^{-3x^2} \cdot (-6x)$

c) too long. e) $$\frac{(1+16x^{-1/2})(\frac{5}{4}x^{-9/4}) + x^{-5/4}(-8x^{-3/2})}{(1+16x^{-1/2})^2}.$$

f) $(1-(2x-3)^2)^{-3/2} \cdot 2(2x-3) \cdot 2$.

**Q4** $3x^2 - 2y^2 = 10$  $6x - 4yy' = 0$ at $(-2,-1)$: $\cancel{y}$

$\cancel{-8}$ $-12 + 4y' = 0$  $y' = \frac{1}{3}$.  $y+1 = \frac{1}{3}(x+2)$.

**Q5** $x^2y - xy^3 = \cos(x+y)$  $2xy + x^2y' - y^3 - x3y^2y' = -\sin(x+y)(1+y')$

$$y'\left( x^2 - x3y^2 + \sin(x+y) \right) = -2xy + y^3 - \sin(x+y).$$

$$y' = \frac{y^3 - 2xy - \sin(x+y)}{x^2 - 3xy^2 + \sin(x+y)}.$$

**Q6** $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\quad \frac{dV}{dt} = 5,\ r = 10 :\ 5 = 400\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{\pi}{80}$ cm/sec $\quad A = \cancel{4}\pi r^2$ $\quad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$ $\quad r = 10,\ \frac{dr}{dt} = \frac{\pi}{80}$

$$\frac{dA}{dt} = \frac{8\pi\, 10 . \pi}{80} = \pi^2 \text{ cm}^2/\text{sec}.$$

**Q7** $f(x) = x^{1/3}$ $\quad f'(x) = \frac{1}{3}x^{-2/3}.$ $\quad f(x+a) \approx f(x) + f'(x)a$

$x = 27,\ a = -2$ $\quad \sqrt[3]{25} \approx 3 + \frac{1}{3}\frac{1}{9}(-2) = 3 - \frac{2}{27}.$

abs error : $\left| \sqrt[3]{25} - (3 - 2/27) \right|$ $\quad$ percentage error $= \frac{\text{abs. error}}{\sqrt[3]{25}} \times 100.$

**Q8** $f(x) = e^x(x^2 - 3x - 9)$ $\quad f'(x) = e^x(2x - 3x) + e^x(x^2 - 3x - 9)$

$$= e^x(x^2 - x - 12) = e^x(x-4)(x+3)$$

critical pts: $x = -3,\ x = 4.$

| | | $-3$ | | $4$ | |
|---|---|---|---|---|---|
| $e^x$ | $+$ | | $+$ | | $+$ |
| $(x-4)$ | $-$ | | $-$ | | $+$ |
| $x+3$ | $-$ | | $+$ | | $+$ |
| $f'(x)$ | $+$ | | $-$ | | $+$ |
| | | local max | | local min. | |

**Q9** $f(x) = x^2 - 4x - 2$ $\quad f'(x) = 2x - 4$ $\quad$ critical pt $x = 2$

$f(-2) = 10$ abs max

$f(2) = -6$ abs min

**Q10** a) vertical asymptotes $x = \pm 2$

horizontal asymptotes $\lim_{x\to\infty} \frac{e^x}{4-x^2} = -\infty$ $\lim_{x\to -\infty} \frac{e^x}{4-x^2} = \lim_{x\to +\infty} \frac{e^{-x}}{4-x^2} \overset{l'H}{=} \lim_{x\to+\infty} \frac{-e^{-x}}{2x}$

$= \lim_{x\to\infty} \frac{e^{-x}}{2} = 0$. left horizontal asymptote $y=0$.

b) $f'(x) = \frac{(4-x^2)e^x - e^x(-2x)}{(4-x^2)^2} = -e^x \frac{(x^2-2x-4)}{(4-x^2)^2}$

$f'(x)=0$
$x^2-2x-4=0$
$x = \frac{2\pm\sqrt{4+16}}{2}$
$= 1\pm\sqrt{5}$.

c)

| | $-2$ | $1-\sqrt{5}$ | $2$ | $1+\sqrt{5}$ | |
|---|---|---|---|---|---|
| $-e^{-x}$ | $-$ | $-$ | $-$ | $-$ | $-$ |
| $(x-(1-\sqrt{5}))$ | $-$ | $-$ | $+$ | $+$ | $+$ |
| $(x-(1+\sqrt{5}))$ | $-$ | $-$ | $-$ | $-$ | $+$ |
| ~~$(2-x)$~~ | ~~$+$~~ | ~~$+$~~ | ~~$+$~~ | ~~$-$~~ | ~~$-$~~ |
| ~~$(2+x)$~~ | ~~$-$~~ | ~~$+$~~ | ~~$+$~~ | ~~$+$~~ | ~~$+$~~ |

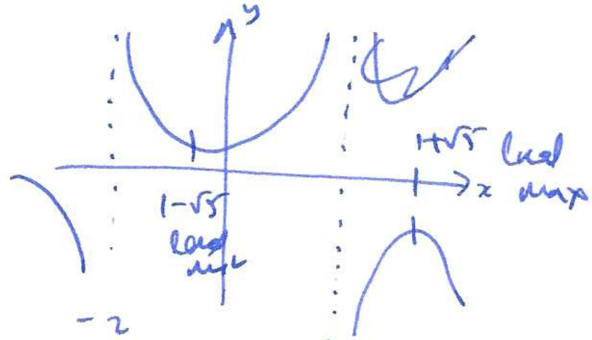
$(4-x^2)^2 > 0$.

| | $-2$ | $1-\sqrt{5}$ | $2$ | $1+\sqrt{5}$ | |
|---|---|---|---|---|---|
| $f'(x)$ | $-$ | $-$ | $+$ | $+$ | $-$ |

d) first derivative test easier

$1-\sqrt{5}$ local min
$1+\sqrt{5}$ local max

e)




**Q11** $f'(x) = \frac{1}{e^{x^2}+1} > 0 \Rightarrow$ increasing

so max value on $[1,3]$ at $x=3$.

**Q12** a) $\lim_{x\to\infty} \frac{3+4x}{\sqrt{2x^2-3}} = \lim_{x\to\infty} \frac{4+3/x}{\sqrt{2-3/x^2}} = \frac{4}{\sqrt{2}}$.

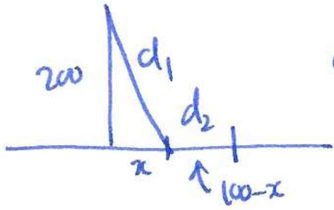
b) $\lim_{x\to 0} \frac{\sin^{-1}(3x)}{\cos^{-1}(2x)} \overset{l'H}{=} \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-(3x)^2}}\cdot 3}{\frac{-1}{\sqrt{1-(2x)^2}}\cdot 2} = \frac{3}{2}$.

c) $\lim_{x\to 0} \frac{\ln(x)}{\text{cosec}(x)} \overset{l'H}{=} \lim_{x\to 0} \frac{1/x}{-\text{cosec}(x)\cot(x)} = \lim_{x\to 0} \frac{-\sin^2(x)}{x\cos(x)} \overset{l'H}{=} \lim_{x\to 0} \frac{-2\sin x\cos(x)}{\cos(x) - x\sin x} = 0$

d) $\lim_{x\to 0} \frac{e^{2x}-1-2\sin x}{2\sin x\,(e^{2x}-1)} \overset{l'H}{=} \lim_{x\to 0} \frac{2e^{2x}-2\cos x}{2\cos x\,(e^{2x}-1) + 2\sin x\, 2e^{2x}} \overset{l'H}{=} \lim_{x\to 0} \frac{4e^{2x}+2\sin x}{-2\sin x\,(e^{2x}-1) + 2\cos x\, 2e^{2x} + 2\cos x\, 2e^{2x} + 8\sin x\, e^{2x}}$

$= 4/8 = \frac{1}{2}$.

Q13




$d_1 = \sqrt{200^2+x^2}$

$d_2 = 100-x$

$t = \sqrt{200^2+x^2} + \frac{100-x}{4}$

$\frac{dt}{dx} = \frac{1}{2}(200^2+x^2)^{-1/2}\cdot 2x - \frac{1}{4} = 0.$

$4x = \sqrt{200^2+x^2}$ $\quad 16x^2 = 200^2+x^2$ $\quad 15x^2 = 200^2$ $\quad x = \frac{200}{\sqrt{15}}$

Q14

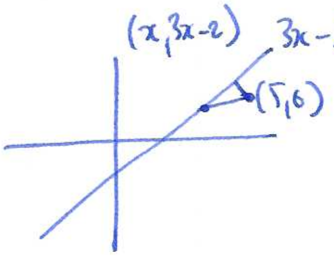
$V = h(40-h)(60-h)$ $\quad \frac{dV}{dh} = \cancel{(40-h)(60-h)+}$

$V = h(2400-100h+h^2)$

$V = 2400h-100h^2+h^3$

$\frac{dV}{dh} = 2400 - 200h + 3h^2$ $\quad h = \frac{200 \pm \sqrt{200^2-4\cdot 3\cdot 2400}}{6}$

Q15



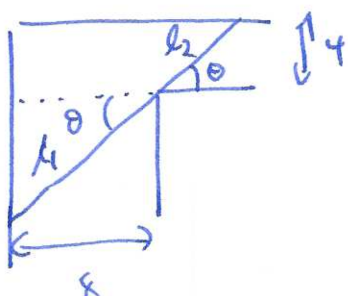

$d^2 = (x-5)^2 + (\overbrace{3x-2-6}^{3x-8})^2$

$= x^2-10x+25+9x^2-48x+64$

$= 10x^2-58x+89$ $\quad \cancel{x = \frac{58\pm\sqrt{58^2-4\cdot 10\cdot 89}}{20}}$

$\frac{d}{dx}(d^2) = 20x-58$ $\quad x = \frac{58}{20} = 2.9.$

Q16




$\frac{4}{l_2} = \frac{\sin}{\cancel{\cos}\theta}$ $\quad \frac{8}{l_1} = \cos\theta$

$l_2 = \frac{4}{\sin\theta}$ $\quad l_1 = \frac{8}{\cos\theta}.$

$l = l_1+l_2 = \frac{4}{\sin\theta} + \frac{8}{\cos\theta}$

$\frac{dl}{d\theta} = -4\sin^{-2}\theta\cdot\cos\theta - 8\cos^{-2}\theta\cdot(-\sin\theta) = \frac{-4\cos\theta}{\sin^2\theta} + \frac{8\sin\theta}{\cos^2\theta}$

$= \frac{8\sin^3\theta-4\cos^3\theta}{\sin^2\theta\cos^2\theta} = 0$ $\quad 8\sin^3\theta = 4\cos^3\theta$

$\tan^3\theta = \frac{1}{2}$ $\quad \theta = \sqrt[3]{\tan^{-1}\left(\frac{1}{2}\right)}.$