Math 231 Calculus 1 Spring 22 Midterm 1a

Name: Solutions

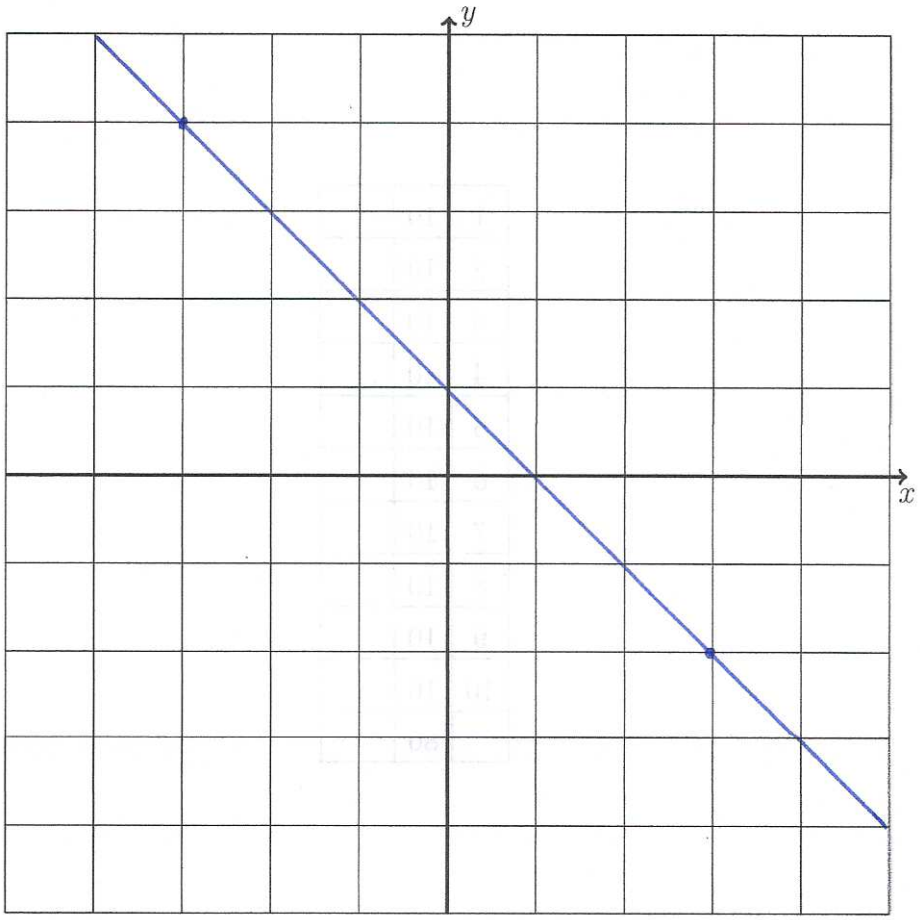
- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(3, -2)$ and $(-3, 4)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.

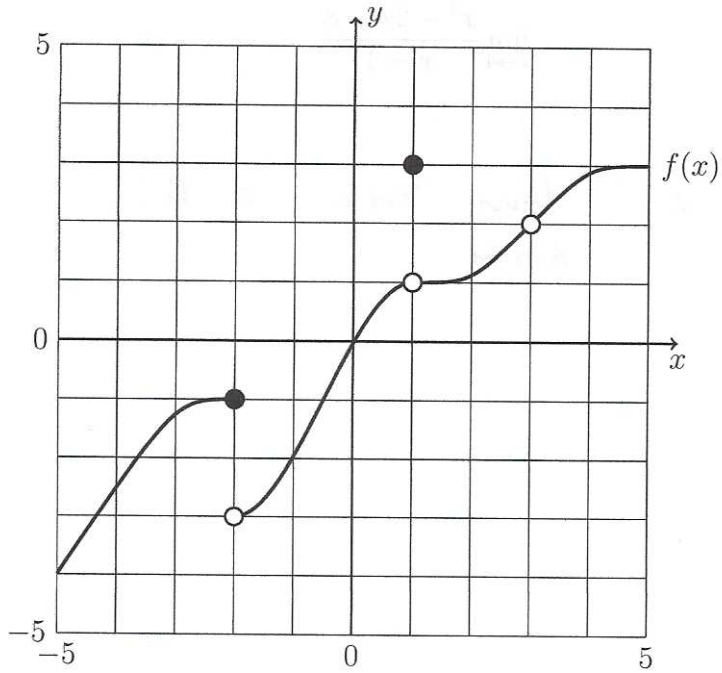
slope $m = \frac{4-(-2)}{-3-3} = \frac{6}{-6} = -1$



$y - (-3) = (-1)(x-4)$

$y = -x + 1$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x\to -2-} f(x)$ -1

(b) $\lim_{x\to -2} f(x)$ DNE

(c) $\lim_{x\to 1+} f(x)$ 1

(d) $\lim_{x\to 1} f(x)$ 1

(e) $\lim_{x\to 3-} f(x)$ 2

(f) $\lim_{x\to 3} f(x)$ 2

(3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 4} \frac{x^2-2x-8}{x-4}$$

$$\lim_{x\to 4} \frac{(x-4)(x+2)}{(x-4)} = \lim_{x\to 4} x+2 = 6$$

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 4} \frac{x-4}{2-\sqrt{x}}$$

$$\lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(2-\sqrt{x})} = \lim_{x\to 4} -(\sqrt{x}+2) = -4$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = x^2 - 3x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$

$$= \lim_{h \to 0} 2x + h - 3 = 2x - 3$$

(6) (10 points) Find the following limit.

$$\lim_{x\to\infty} \frac{\sqrt{2x^2-2}}{3x-3}$$

$$\lim_{x\to\infty} \frac{\sqrt{2-2/x^2}}{3-3/x} = \frac{\sqrt{2}}{3}$$

$2x^{-1/2}$

(7) Find the first and second derivatives of $f(x) = x^4 - \sin(x) + 2/\sqrt{x}$.

$f'(x) = 4x^3 - \cos(x) - x^{-3/2}$

$f''(x) = 12x^2 + \sin(x) + \frac{3}{2}x^{-5/2}$

(8) Find the first and second derivatives of $f(x) = \frac{e^x}{x} - \sqrt[3]{x}$.

$$f'(x) = \frac{xe^x - e^x}{x^2} - \frac{1}{3}x^{-2/3} = \frac{e^x}{x} - \frac{e^x}{x^2} - \frac{1}{3}x^{-2/3}$$

$$f''(x) = \frac{e^x}{x} - \frac{e^x}{x^2} - \frac{x^2e^x - e^x \cdot 2x}{x^4} + \frac{2}{9}x^{-5/3}$$

$$f''(x) = \frac{e^x}{x} - \frac{2e^x}{x^2} + \frac{2e^x}{x^3} + \frac{2}{9}x^{-5/3}$$

(9) Find the first and second derivatives of $f(x) = \cos(x^2)$.

$f'(x) = -\sin(x^2) \cdot 2x$

$f''(x) = -\cos(x^2) \cdot 4x^2 - 2\sin(x^2)$

(10) (10 points) The graph of $f(x)$ is given in the top picture. Sketch the graph of $f'(x)$ in the bottom picture.