Math 231 Calculus 1 Fall 21 Sample Midterm 1
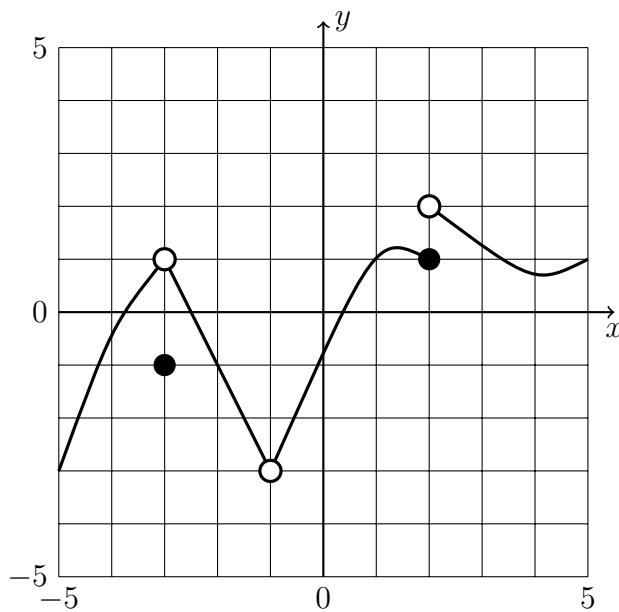
(1) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.

(a) $\lim\limits_{x \to 2-} f(x)$

(b) $\lim\limits_{x \to 2+} f(x)$

(c) $\lim\limits_{x \to 2} f(x)$

(d) $\lim\limits_{x \to -1-} f(x)$

(e) $\lim\limits_{x \to -1+} f(x)$

(f) $\lim\limits_{x \to -3} f(x)$

(2) Evaluate these limits. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

(a) $\displaystyle\lim_{x\to-3}\frac{x^2+x-6}{x+3}$

(b) $\displaystyle\lim_{x\to\infty}\frac{\sqrt[3]{2+4x^3}}{3-2x}$

(c) $\displaystyle\lim_{x\to 0}\frac{\sin 3x}{4x}$

(d) $\displaystyle\lim_{x\to 0}\frac{x}{3-\sqrt{x+9}}$

(3) For what value of c (if any) is the function $f(x)$ continuous at $x=2$? Justify your answer.

$$f(x)=\begin{cases} 3x-\frac{1}{x+2} & x<2 \\ c & x=2 \\ \frac{\cos(\pi x)}{x} & x>2 \end{cases}$$

(4) For a sphere of radius r, its volume is $V=\frac{4}{3}\pi r^3$. What is the average rate of change of volume when the radius increases from $r=4$ to $r=5$?

(5) Show that $e^x=1/x$ has a solution for some $x>0$. You do not need to find this solution.

(6) Consider the function $f(x)$ defined by the following graph.
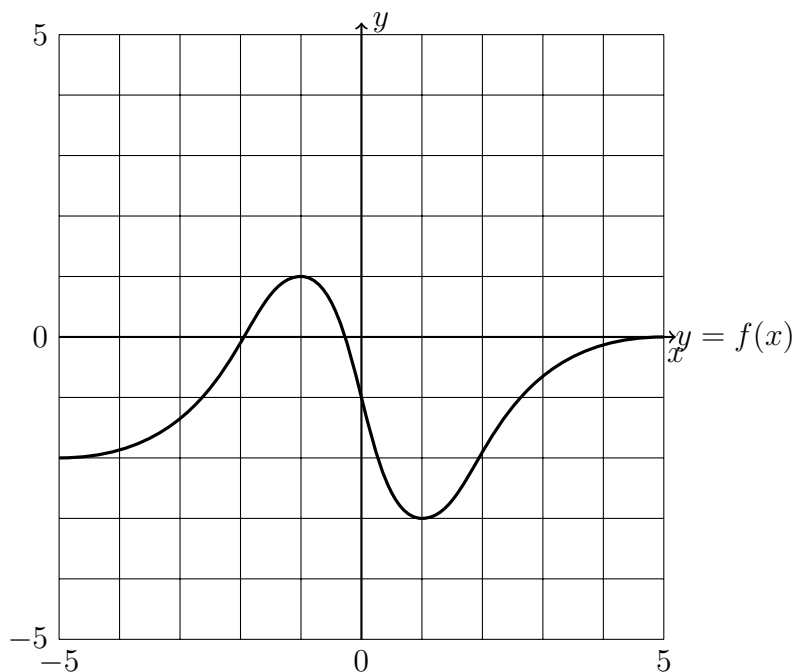
(a) Label all regions where $f'(x) < 0$.
(b) Label all regions where $f'(x) > 0$.
(c) Sketch a graph of $f'(x)$ on the figure.
(d) What is $\lim_{x \to \infty} f(x)$?
(e) What is $\lim_{x \to -\infty} f'(x)$?

(7) Use the limit definition of the derivative to evaluate $f'(3)$, where

(a) $f(x) = \dfrac{1}{x-2}$

(b) $f(x) = \dfrac{1}{\sqrt{x-1}}$

(8) Find the derivatives of the following functions

(a) $3x^2 - 2x^3 - \sqrt{x^3} - 2\sqrt[3]{1/x^2}$

(b) $3x^4 e^x$

(c) $\dfrac{3x-2}{3-2x}$

(d) $\dfrac{\sqrt{3x}-2}{1-\cos(x)}$

(e) $\tan(x)$

(f) $\sin^2(x)$

(g) $x^4 e^{-2x^4}$

(h) $\dfrac{\sqrt{1-2x}}{4-\sin(3x)}$

(i) x^{x^2}

(j) $\sqrt{\csc(\ln(x))}$

(k) $\sin^{-1}(3x-2)$

(9) Find the second derivatives of the functions above (skip (d, j, k) as they takes a long time).

(10) The graphs of the functions f and g are shown below.
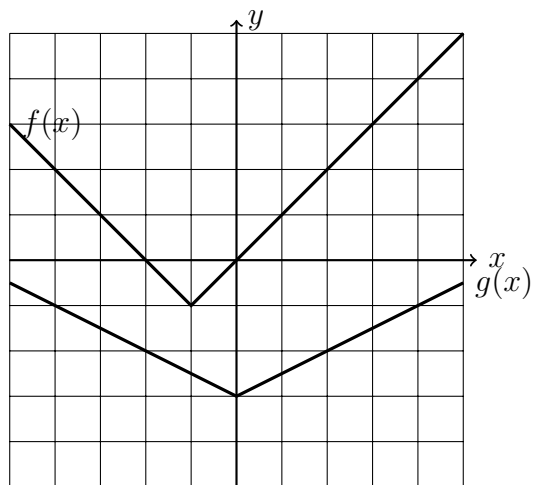
(a) Let $h(x) = f(x)g(x)$ Find $h'(2)$.
(b) Let $h(x) = f(x)/g(x)$. Find $h'(-4)$.
(c) Let $h(x) = f(g(x))$. Find $h'(1)$.