# Math 301 Introduction to Proof Fall 22 Sample Midterm 1

(1) Find three distinct elements for the truth sets of the following statements:
  (a) $x^2 - y^2 = 1$, where the universe is $\mathbb{R} \times \mathbb{R}$.
  (b) $A$ is a subset of $\mathbb{Z}$ which is closed under subtraction.
  (c) $A$ is an element of $\mathcal{P}(\mathbb{Z})$.
  (d) $A$ is a subset of $\mathcal{P}(\mathbb{Z})$.
  (e) The numbers $a, b$ and $c$ are not the lengths of the sides of any triangle, where the universe is $\mathbb{R}_{>0} \times \mathbb{R}_{>0} \times \mathbb{R}_{>0}$.

(2) Consider the statement: If $n$ is a cube of an integer, then $n$ is a square number.

  Which, if any, of the following substitutions give a counter example.
     a) $n = 64$    b) $n = 8$    c) $n = 16$

(3) Write out a careful proof of the fact that the square of an odd number is odd.

(4) What is $\mathcal{P}(\{\varnothing\})$? What is $\mathcal{P}(\mathcal{P}(\{\varnothing\}))$?

(5) If $A \cap B \subseteq A \cap C$ does this imply that $B \subseteq C$?

(6) State which of the following statements, are true, vacuously true, or false.
  (a) If $A \subseteq A \cap B$, then $A \subseteq B$.
  (b) If $\mathcal{P}(A) = \varnothing$, then $A = \varnothing$.
  (c) If $A \in B$ and $B \in C$, then $A \in C$.

(7) Suppose $A$ and $B$ are finite sets with $|A| = a$, $|B| = b$ and $|A \cap B| = c$. Find
  (a) $|A \setminus B|$
  (b) $|A \times (A \cup B)|$
  (c) $|\mathcal{P}(B \setminus A)|$

(8) There is an island where all people either always lie, or always tell the truth. You meet three people, $A$, $B$ and $C$.
    $A$ says: At least one of us is a truth teller.
    $B$ says: $A$ is lying.
    $C$ says: $B$ is lying.
  What can you deduce?

(9) There is an island where all people either always lie, or always tell the truth. You meet $D$ who says "I am lying." What can you deduce?