

SMT1 Solutions

①

Q1 a) $(1,0), (-1,0), (\sqrt{2},1)$

b) $\mathbb{Z}, \{0\}, 2\mathbb{Z} = \{2n | n \in \mathbb{Z}\}$

c) $\emptyset, \mathbb{Z}, \{1\}$

d) $\emptyset, P(\mathbb{Z}), \{\emptyset\}$

e) $(1,2,4), (1,3,5), (1,4,6)$

Q2 b) counterexample.

Q3 Thm If n is odd then n^2 is odd.

Proof If n is odd, then $n=2k+1$ for some integer k .

Then $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$, so n^2 is odd $\square$

Q4 $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$

$P(P(\{\emptyset\})) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

Q9 This is a
contradicting statement,
can't be either T or F.

Q5 No. Choose $A = \emptyset, B = \{1\}, C = \emptyset$.

Q6 a) T b) VT c) F.

Q7 a) $a-c$ b) $a(a+b-c)$ c) 2^{b-c}

Q8

A	B	C	A's statement T/F	B's statement T/F	C's statement T/F
T	T	T	T	F # B is a Truth teller	
T	T	L	T	F # "	
T	L	T	T	F	T ← only possibility T # C is a liar.
T	L	L	T	F	
L	T	T	T # A is a liar		
L	T	L	T # "		
L	L	T	T # "		
L	L	L	F	T # B is a liar	

THIS IS THE ONLY POSSIBILITY