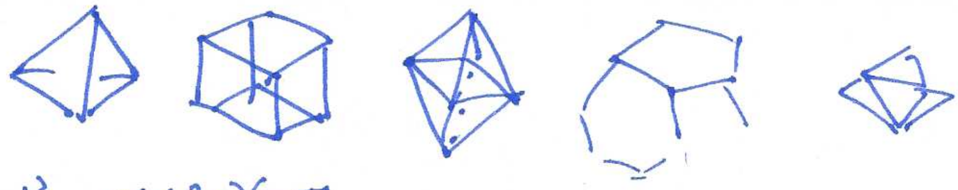
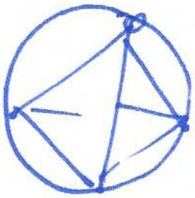
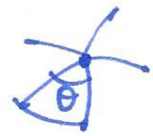
Euclidean / regular solids



<u>recall</u> area of spherical triangle is $\alpha+\beta+\gamma-\pi$.

 $\Leftarrow$ project onto S^2 $\Leftarrow$ area of S^2 is 4π

Tilings by equilateral triangles:  n triangles meet at a vertex

angle $\theta = \frac{2\pi}{n}$, area of triangle is $\frac{2\pi}{n}\times 3-\pi = \pi\left(\frac{6}{n}-1\right)$

$n=3$: area is π, so $\frac{4\pi}{\pi} = 4$ triangles.

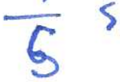
$n=4$: area is $\frac{\pi}{2}$ so $4\pi/\frac{\pi}{2} = 8$ triangles.

$n=5$: area is $\frac{\pi}{5}$ so $4\pi/\frac{\pi}{5} = 20$ triangles.

<u>Observation</u>: symmetry groups of regular solids are another collection of finite groups.

<u>Fact</u>:   have same symmetry group - in fact are dual to each other.

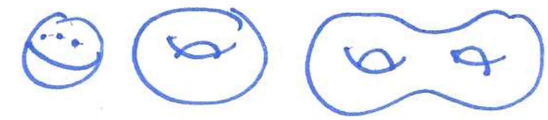
Classification of surfaces

Q: what is a surface? A: locally two-dimensional , e.g. any union of triangles 

<u>compact</u>: finitely many triangles.

<u>closed</u>: no boundary

<u>orientable</u>: no Möbius bands.

Thm closed orientable surfaces:  ...

genus 0 1 2 ...

χ 2 0 -2

closed non-orientable surfaces $\mathbb{R}P^2$, K^2, $K^2 \# \mathbb{R}P^2$ ...

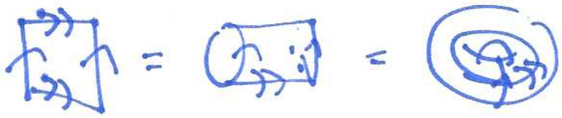
1 0 -1 ...

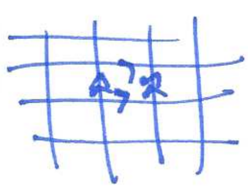
compact surfaces w/ boundary: closed surfaces with discs removed.

e.g.: S^2  ...

T^2  ...

How to build T^2 from triangles/square:

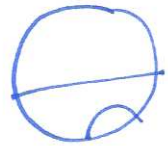
 equivalently: take tiling of $\mathbb{R}^2$:

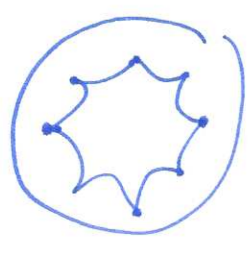


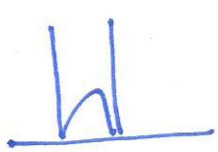
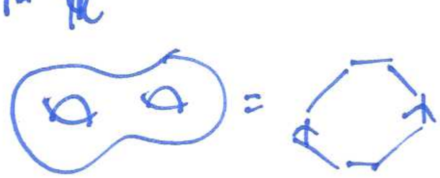
$z \mapsto z+1$
$z \mapsto z+i$

tilings $\leftrightarrow$ surfaces (orbifolds)

Q: what about higher genus surfaces? A: $\mathbb{H}^2$.

Models for $\mathbb{H}^2$  angles same as in $\mathbb{R}^2$ 



ideal triangles/polygons

look for regular octagon w/ angle $\frac{2\pi}{8}$