Proof · L_1, L_2 extended lines. if $L_1 \parallel L_2$, intersect in $I_{L_1} = I_{L_2}$.
if $L_1 \not\parallel L_2$ then intersect in unique ordinary intersection point. ~~the~~.
· L_1 extended, L_2 ideal : intersect in ideal point of L_1 $\square$.

Remark set of ideal points is a circle!
set of directed lines is S^1, but set of lines is $S^1/\sim = S^1$.
Fact Can also construct projective geometry as $S^2/\sim$!
Fact : projective plane is $\mathbb{R}P^2$ ← non-orientable surface.
Exercise : make Möbius strips, cut them up.

§6 Planar symmetries

Motion / isometry of the plane : map which preserves distances.

Def^{n} A motion / isometry of the plane is a ~~map~~ function $f: \mathbb{R}^2 \to \mathbb{R}^2$
s.t for any points $P, Q \in \mathbb{R}^2$, $|PQ| = |f(P) f(Q)|$.

Remarks / Prop^{n}.

① composition of two isometries is an isometry
② identity map is an isometry
③ isometries have inverses (check!) which are also isometries.
↳ bijection ↗ one-to-one ↘ onto.
④ associativity

Corollary The collection of all isometries forms a group.

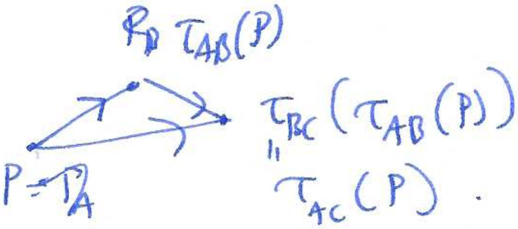
Examples of isometries

Translations geometric : choose two points $A \to B$. T_{AB} for any point P
construct line of length $|AB|$ parallel to AB in direction $\overrightarrow{AB}$ at P. Endpoint
is $f(P)$ $\quad P \to f(P)$. coordinates : $(x,y) \mapsto (x,y) + (a,b)$ $\quad (\mathbb{R}^2)$

$z \mapsto z + a \quad (\mathbb{C})$.

Prop^{n} If A, B, C are three points in the plane, then $\tau_{BC} \circ \tau_{AB} = \tau_{AC}$

Proof (geometric)

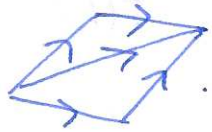


coordinates: $(x,y) \mapsto (x,y) + (x_1,y_1) \mapsto (x,y) + (x_1,y_1) + (x_2,y_2)$

$= (x,y) + (x_1+x_2, y_1+y_2)$. □

$\left.\begin{array}{l} z \mapsto z+a \\ z \mapsto z+b \end{array}\right\}$ $z \mapsto z + (a+b)$.

Prop^{n} $\tau_{AB} \circ \tau_{BC} = \tau_{BC} \circ \tau_{AB}$.



□.

Rotations **Def^{n}** (geometric) $R_{C,\alpha}$: C point in plane, α angle.

$R_{C,\alpha}(P)$: construct angle α at C, then construct $|C R_{C,\alpha}(P)| = |CP|$ □.

Coordinates! Exercise.

Prop^{n} • if $\alpha = 2\pi / 360°$ $R_{C,\alpha} = R_{C,0}$.

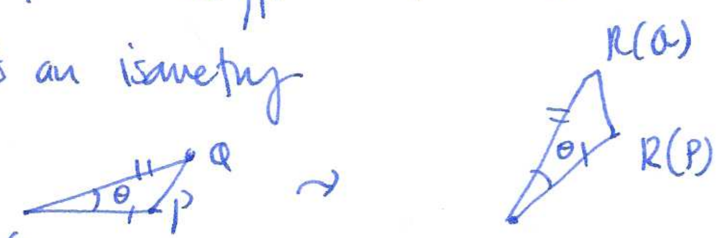
• $(R_{C,\alpha})^{-1} = R_{C,-\alpha}$

• $R_{C,\alpha} \cdot R_{C,\beta} = R_{C,\alpha+\beta}$.

Q: what is $R_{C,\alpha} \cdot R_{C',\beta}$ if $C \neq C'$?

Prop^{n} Every rotation is an isometry

Proof (geometric)



$|PQ| = |R(P)R(Q)|$

by SAS. □.