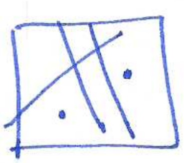
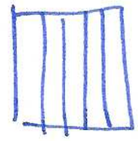
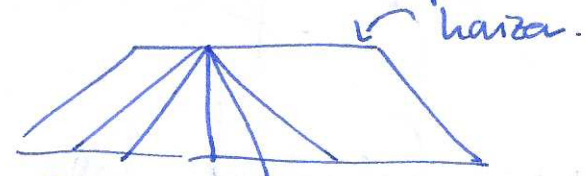
§5.3 Towards projective geometry

$\mathbb{R}^2$: contains points and lines. Look at $\mathbb{R}^2$ as horizontal plane in $\mathbb{R}^3$.



parallel lines don't intersect.





"parallel lines" intersect at infinity"

Q: how do we make this precise? [Desargues' C17].

<u>Def^n</u> An <u>ordinary point</u> is a point of the Euclidean plane.
An <u>ordinary line</u> is a straight line in the Euclidean plane.
Given an ordinary line m, let I_m be the set of all ordinary lines parallel to m. We will call this the <u>ideal point</u> of m or the <u>point at infinity</u> of m.

If m is any ordinary line, let m^* ~~be the exte~~ $= m \cup \{I_m\}$, we call this the <u>extended line</u>. The <u>ideal line</u> or <u>line at infinity</u> consists of the set of all ideal points. A <u>projective point</u> is either an ordinary point, or an ideal point. The <u>projective plane</u> consists of all ordinary and ideal points, i.e. all projective points. A <u>projective line</u> is either an ~~ordinary~~ extended line, or the ideal line.

<u>Prop (5.3.1)</u> Every two distinct projective points are contained in exactly one projective line.

<u>Proof</u>
- P, Q both ordinary points, choose extended line through P, Q.
- P ordinary, Q ideal, choose extended line through P containing Q.
- P, Q both ideal, choose ideal line. $\square$.

<u>Prop (5.3.2)</u> Every two distinct projective lines intersect in exactly one projective point.