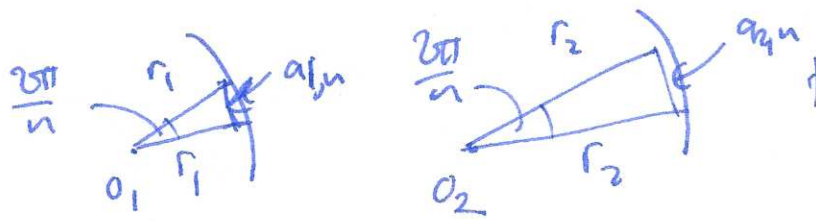
sketch inscribe regular hexagon in each circle.
can bisect angles, can construct regular $n = 3.2^m$-gon.



triangles are similar with ratio $\frac{r_1}{r_2}$.

reasonable assumption, for n large, $c_i \approx n a_{i,n}$.
i.e. circumference is close to perimeter of prescribed polygon

so $\quad \frac{c_1}{c_2} = \frac{n a_{1,n}}{n a_{2,n}} = \frac{r_1}{r_2} \quad \square$. Alternate proof: use calculus.

Euclid proved the following using "method of exhaustion" - Greek version of integration developed by Eudoxus.

(Prop 4.4.3) (B12 P2) The areas of circles are proportional to the squares of their radii.

Proof

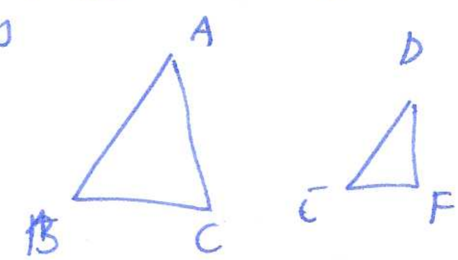
want to show $\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$.

sketch:

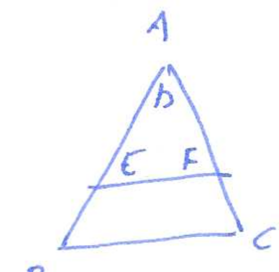
Ex 3.5C #10 (B6 P19) Areas of similar polygons are proportional to the squares of the corresponding sides

Note: enough to show for triangles.

Assume $\triangle ABC \sim \triangle DEF$ similar



want to show $\frac{|\triangle ABC|}{|\triangle DEF|} = \frac{|BC|^2}{|EF|^2}$



Proof: construct DEF inside ABC with D at A

$\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|}$ (3.5.4)