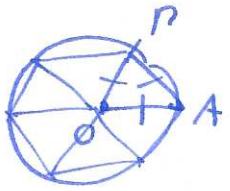
§4.3 Regular polygons

Def: A polygon is regular if all of its sides and angles are equal.

3 sides: equilateral triangle BIP1
4 sides: square BIP46 (3.3.1)

Q: given a circle, can you construct its centre?

6 sides Hexagon. (4.2.1) B4P15 Construct a regular hexagon in a given circle.

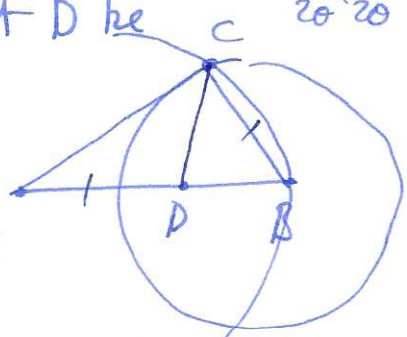


Proof (construction) Given O, and A on circle, draw circle of same radius at A, let B be an intersection! $\triangle OAB$ is equilateral, continue around circle ∴ □.

5 sides Pentagon (4.3.2) B4P10 Construct an isosceles triangle s.t. angles at base are each double remaining angle.

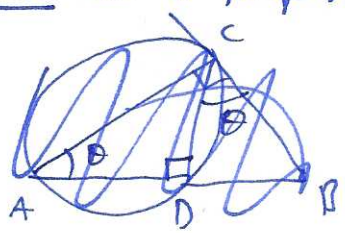
Proof (construction) Draw line segment AB, and let D be a point on AB s.t. $|AB||BD| = |AD|^2$ (Prop 3.4.1)

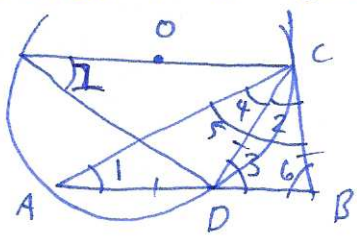
now construct C s.t. $|AC| = |AB|$ and $|BC| = |AD|$.



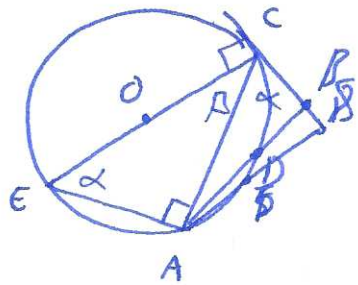
claim $\triangle ABC$ is desired triangle.

note BC is tangent to circle that circumscribes $\triangle ACD$ (~~4.2.4~~; 4.2.5)





$\angle 1 = \angle 2$ (P4.2.4) → angles subtended on circumference are the same.



$\angle 3 = \angle 1 + \angle 4 = \angle 2 + \angle 4 = \angle 5 = \angle 6$ (isosceles)

$|DC| = |BC| = |AD|$

so ADC isosceles ⇒ $\angle 1 = \angle 4$, so $\angle 3 = 2\angle 1$

$\angle 6 = \angle 5$. as required for $\triangle ABC$

so $\angle 1 + \angle 5 + \angle 6 = \pi \Rightarrow \angle 1 = \frac{\pi}{5} = 72°$. □