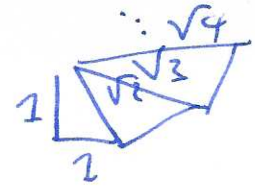
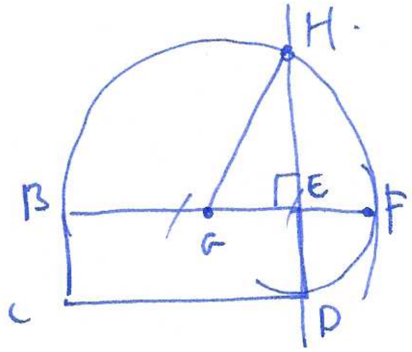
Remark iterative construction of $\sqrt{n}$:



Construction



special case $|BE| = |BC| \Rightarrow$
$\square$ BCDE is a square, just choose $|BC|$.

wlog $|BE| > |BC|$.

extend BE to F s.t. $|ED| = |EF|$, construct G midpoint of BF
now extend ED vertically and intersect with circle about G of radius BC.

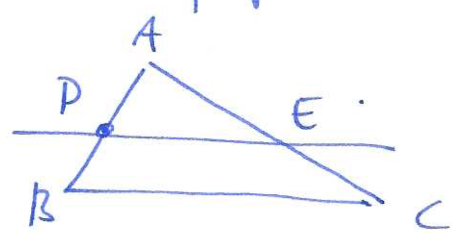
claim: $|EH|^2 = |BC||BE|$.

proof $|GH|^2 = |EH|^2 + |EG|^2$, so
$$\begin{aligned}|EH|^2 &= |GH|^2 - |EG|^2\\ &= (|GH| + |EG|)(|GH| - |EG|)\\ &= (|BG| + |EG|)(|GF| - |EG|)\\ &= |BE|\,|EF| = |BE||BC| \quad \square.\end{aligned}$$

§3.5 Similarity

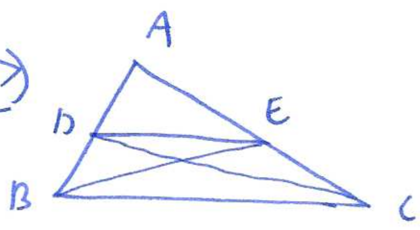
Book 6 Prop 2 (attributed to Thales)

If a straight line meets two sides of a triangle, then it is parallel to the third side iff it cuts them into proportional segments.



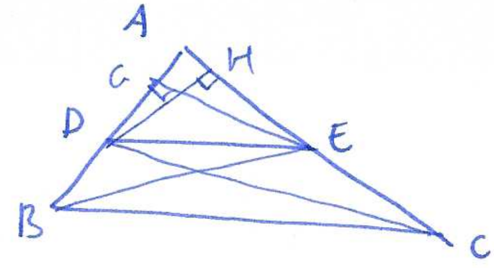
i.e. $DE \parallel BC \Leftrightarrow \dfrac{AD}{BD} = \dfrac{AE}{CE} \Leftrightarrow \dfrac{AD}{AB} = \dfrac{AE}{AC}$

Proof $(\Rightarrow)$



Join CD and BE, consider areas:
$|\triangle BDE| = |\triangle CDE|$
$|\triangle ABE| = |\triangle ADC|$ so $\dfrac{|\triangle ADE|}{|\triangle ADE|} = \dfrac{|\triangle ACD|}{|\triangle ADE|}$

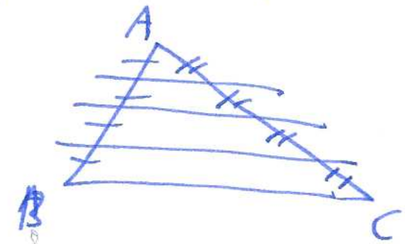
construct heights of ΔADE



$$\frac{\Delta ABE}{\Delta ADE} = \frac{|AB||GE|}{|AD||GE|} = \frac{|\Delta ACD|}{|\Delta ADE|} = \frac{|\overset{AC}{\cancel{CD}}||DH|}{|AE||DH|} \Rightarrow \frac{|AB|}{|AD|} = \frac{|\overset{AC}{\cancel{CD}}|}{|AE|}. \square.$$

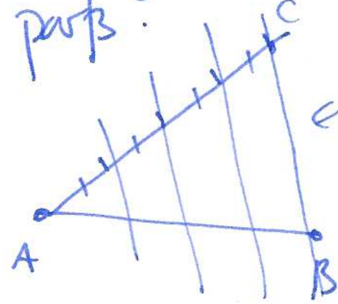
$\Leftarrow$ fact: these steps are reversible, as Euclid Prop 39: if triangles on same base between two lines have equal area, then lines are parallel. $\square$.

(Ex 8 p126) Corollary If straight lines $m_1, m_2, \dots, m_k$ are all parallel to one side of a triangle, and they cut off equal segments on a second side, then they cut off equal segments on the third side:



(Ex 10 p126) Divide a given line segment into n equal parts.

Construction:



$\Leftarrow$ just construct parallel lines to BC.

Defn

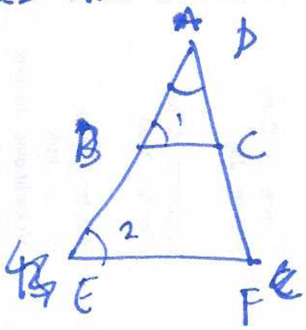
Similar polygons are those whose corresponding angles are equal, and whose corresponding sides are proportional. (e.g. congruence $\Rightarrow$ similar)

Book 6 Prop 4 (AAA test for similarity.) Equiangular triangles are similar.

Book 6 Prop 4 (AAA ⇒ similar)

Thm Equiangular triangles are similar.

Proof wlog $|AB| < |DE|$



$\angle 1 = \angle 2$

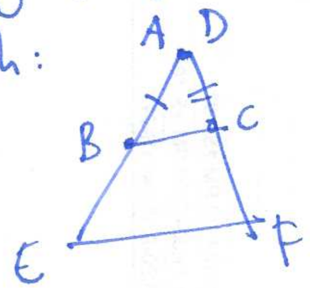
$\Rightarrow BC \parallel EF$

$\Rightarrow$ sides proportional □.

B6 Prop 5 (SSS ⇒ similar)

Thm If two triangles have their sides proportional, then they are similar

Proof wlog $|AB| < |DE|$ construct subedges in DEF of right length:



proportional ⇒ $BC \parallel EF$

⇒ $|BC|$ proportional to EF.

⇒ ABC constructed in DEF is congruent to original ABC

⇒ same angles □.

B6 Prop 6 (SAS ⇒ similar).

If two triangles have one angle equal to one angle, and the two corresponding sides proportional, then the triangles are similar.

Proof □.

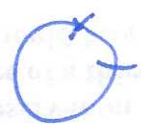
§4 Circles and regular polygons (Books III, IV)

§4.1 Neutral geometry of the circle

Equal circles $\Leftrightarrow$ equal radii.

chord:  line segment joining two points

diameter:  chord containing the center

arc:  portion of circle between two points.

segment:  portion between chord and corresponding arc.

sector:  portion between two radii.

central angle:  angle between two radii

(4.1.1) B3 P26, 27, 28, 29 In equal circles

1) equal angles stand on equal arcs
2) central angles on equal arcs are equal
3) equal chords cut off equal arcs
4) equal arcs subtended by equal chords

Proof Let c_1 c_2 be equal circles (radius same) centred at E, E'.