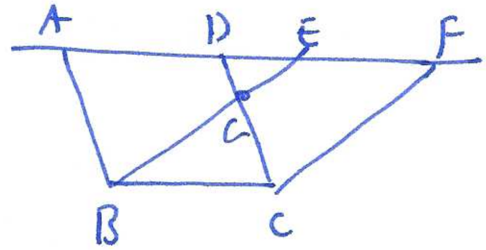
Prop 35 Parallelograms w/ same base and whose top lies in the same line are of equal area



Proof : $AD = BC = EF$

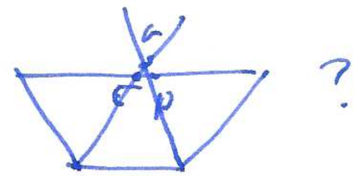
$\Rightarrow AD = EF$

$\triangle BAE = \triangle FDC$ (SAS)

so $\text{area}(ABCD) + \triangle FDC = \text{area}(BCFE) + \triangle ABE$

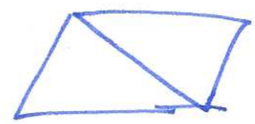
$\Rightarrow \text{area}(ABCD) = \text{area}(BCFE)$. $\square$.

Note : proof incomplete, suppose G not in ABCF?

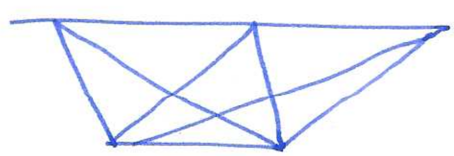


(modern) **Prop 3.2.5** Area of triangle is $\frac{1}{2}$ base $\times$ height

Proof complete to parallelogram and use 3.1.8



Prop 37 Triangles with same base and on the same parallels are equal to each other.

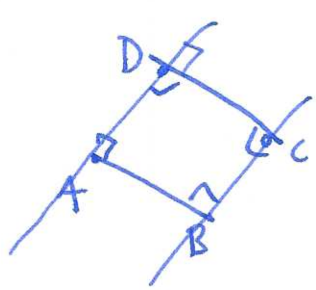


Pf : construct parallelograms $\square$.

§3.3 Pythagoras thm

Prop 46 On a given straight line construct a square.

Proof

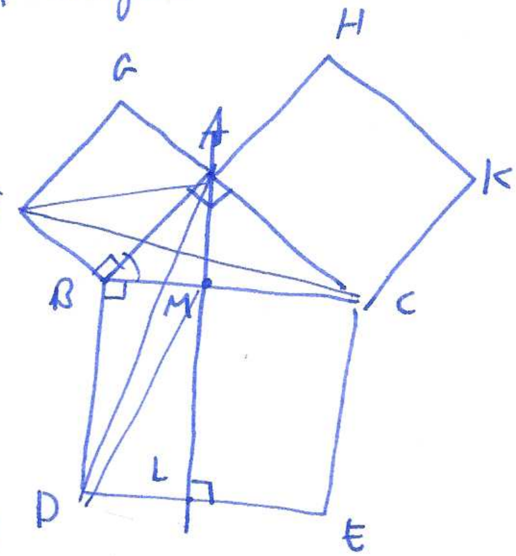


- construct perpendiculars at A, B
- draw circle at A, B to construct C, D s.t. $|AB| = |BC| = |AD|$
- connect DC

claim : ABCD is a square : $|DC| = |AB|$ by parallel lines.

corresponding angles equal by parallel lines
$\Rightarrow$ all angles $= \frac{\pi}{2}$ $\square$.

Prop 47 (Pythagoras' Thm) In a right angled triangle, the square of the side opposite the right angle is equal to the sum of the squares on the sides containing the right angle.



Pf construct $\perp$ line to BC through A.

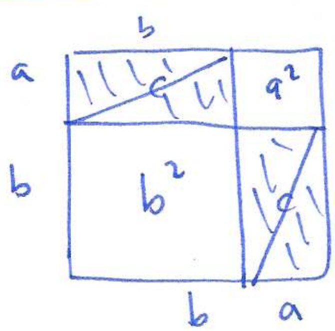
$\triangle ABD \cong \triangle FBC$ by SAS $|FB| = |AB|$
$|BD| = |BC|$

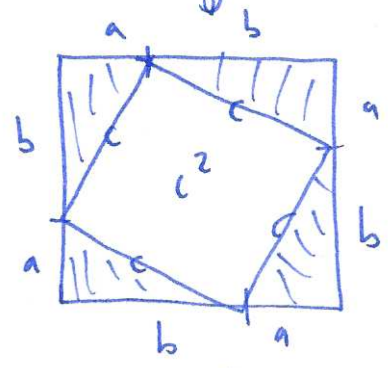
$\therefore$ $\square$ BDLM $=$ $\square$ ABFG (area of parallelogram twice area of triangle)

similarly $\square$ CEML $=$ $\square$ ACKH

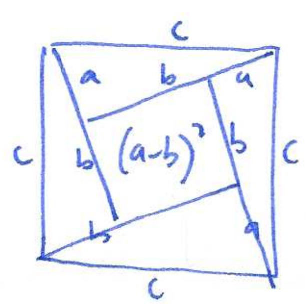
So $\square$ ABFG $+$ $\square$ ACKH $=$ $\square$ BDEC. $\square$.

Proof ② (Chou-Pei Suan-ching ~ 250 BC)





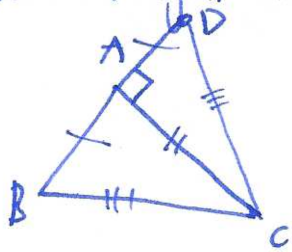
Proof ③ (Bhaskara 1114 – 1185)



$$c^2 = 4\frac{ab}{2} + (a-b)^2 = a^2 + b^2 \quad \square.$$

Prop 48 (converse) If a triangle has the square of one side equal to the sum of the squares on the other two, then the other two sides form a right angle.

Proof



assume $|BC|^2 = |AB|^2 + |AC|^2$

construct $AD \perp$ to AC with $|AD| = |AB|$.

as ADC right triangle, $|DC|^2 = |AD|^2 + |AC|^2$.

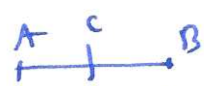
$= |AB|^2 + |AC|^2$

$= |BC|^2$.

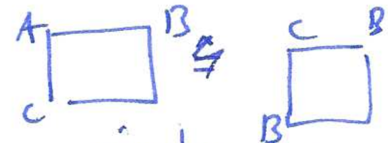
so $ADC \cong ABC$ by SSS $\Rightarrow \angle BAC = \angle CAD = \frac{\pi}{2}$ $\square$.

End of book 1 !

§3.4 Consequences of Pythagoras' Thm.

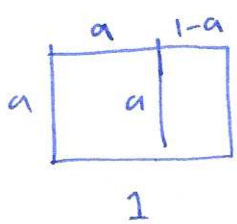


Book 2 **Prop 6** cut a given line segment s.t.



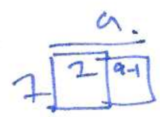
similar.

i.e.



want $\frac{a}{1} = \frac{1-a}{a}$

or



$\frac{a}{1} = \frac{1}{a-1} \Rightarrow a = \tau$.

$a^2 = 1 - a$

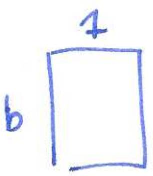
$a^2 + a - 1 = 0$

$a = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1+\sqrt{5}}{2}$

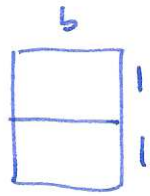
golden ratio $\tau = \frac{1}{a} = \cancel{\frac{+1+\sqrt{5}}{2}} \cancel{\frac{-1-\sqrt{5}}{-1 \mp \sqrt{5}}}$.

$= \frac{2}{-1+\sqrt{5}} = \frac{2(-1-\sqrt{5})}{(-1+\sqrt{5})(-1-\sqrt{5})} = \frac{-2-2\sqrt{5}}{-4} = \frac{1+\sqrt{5}}{2}$.

A4 paper ratio:



s.t. similar to



i.e. $\frac{b}{1} = \frac{2}{b}$ $\quad b^2 = 2$

$b = \sqrt{2}$.

Prop 14 construct a square equal to a given rectangle.

($\Rightarrow$ i.e. can construct a line of length $\sqrt{a}$!).