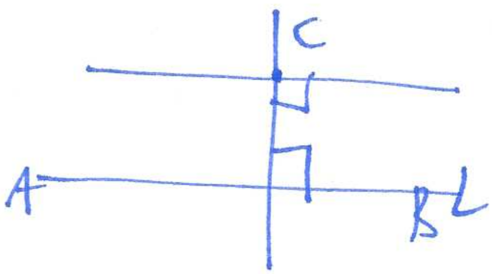
Playfairs postulate (equivalent to Euclids P5)
Given a line L and point C not on L, there is a unique parallel line through C

Postulate H (negation of Playfair) There is a straight line that is parallel to two distinct intersecting lines.

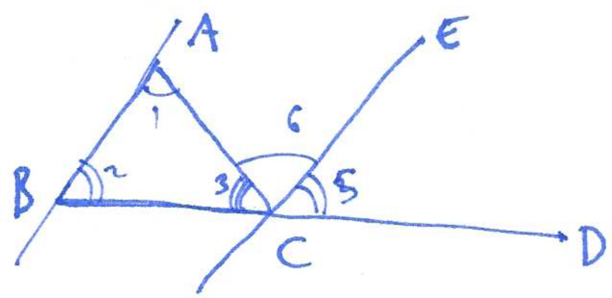
Hyperbolic geometry: Euclid P1–4 (A, J) and H.

Alternate construction for P31:



Prop 32. Sum of angles of triangle = π.

Proof



extend BC to D
construct CE parallel to AB
$\angle 2 = \angle 5$ parallel lines
$\angle 6 = \angle 1$ "

$\Rightarrow \angle 1 + \angle 2 + \angle 3 = \pi$. $\square$.

Prop 33 A quadrilateral in which two opposite sides are both equal and parallel to each other is a parallelogram.

Proof Assume $AB \parallel CD$ $|AB| = |CD|$

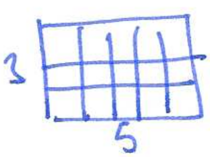
draw BC
parallel lines make equal angles
$\Rightarrow$ triangles congruent $\square$.

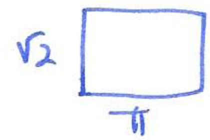
§3.2 Area

Modern axioms: assume chosen a unit length 

- **unit**: the unit square has area 1 unit^2
- **additivity**: if a figure is divided by a line into two subfigures, then the area of the figure is the sum of the area of the subfigures.
- **invariance**: congruent polygons have equal areas.

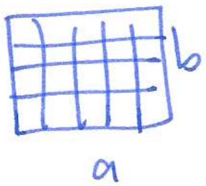
Euclid: avoids defining a unit, so can only compare areas of shapes.



$3 \times 5 = 15$ unit^2. problem: 

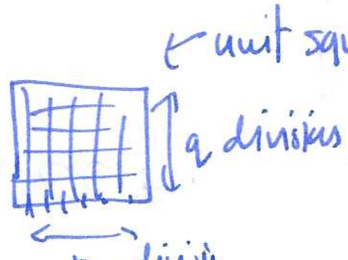
area $= \sqrt{2}\,\pi$.

Proposition 3.2.1. Area of rectangle sides $a, b \in \mathbb{N}$, area ab unit^2.

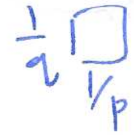


# of squares is $a \times b$.

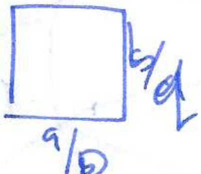
note $\frac{1}{p} \times \frac{1}{q} = \frac{1}{pq}$



so area of pq squares $= 1$ unit^2

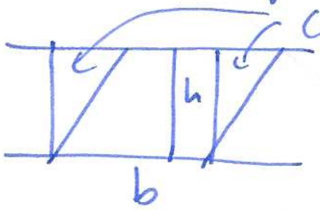
so 

$= \frac{1}{pq}$ unit^2.

so for any fractions 

area is $\frac{ab}{pq}$. □.

Prop 3.2.2 Area of parallelogram of base b and height h is bh unit^2.

Proof 

congruent triangles, so area $= bh$.