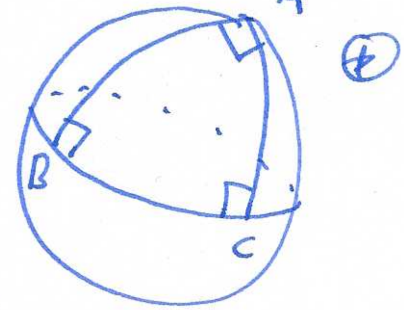
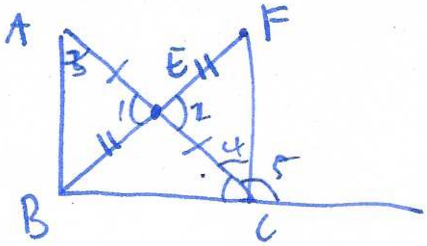
Prop 16 (exterior angle) Exterior angle of triangle greater than either of the opposite angles.

False in spherical geometry!





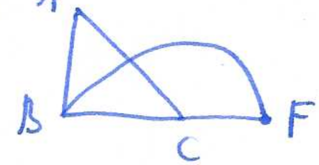
$\angle 4$ bigger than $\angle 5$.

proof uses F does not lie on extension of BC.

E midpoint of AC, extend BE to EF with $|BE| = |EF|$ … .

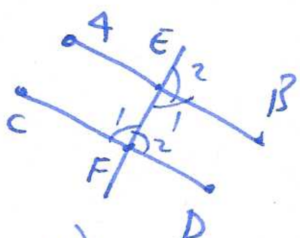
In example (*) F lies on extension of BC!

so $\angle 4 = \angle 5$!

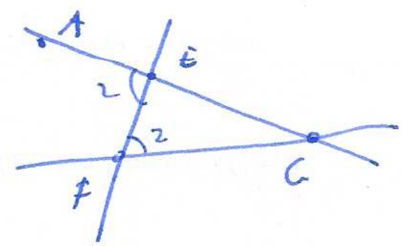


(2.5.35)

Prop 27 If a straight line falling on two straight lines makes the alternate angles equal, the straight lines are parallel to each other. (parallel means don't intersect)



Proof suppose not, wlog intersect on right:



$\angle AEF$ exterior to $\triangle EFC$

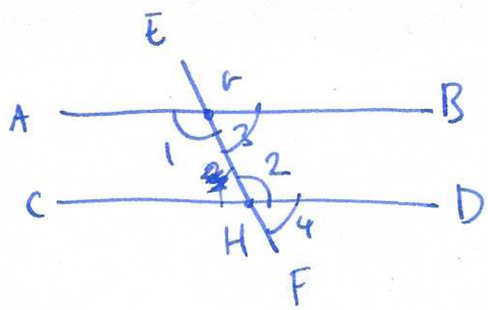
but equal to $\angle EFC$ # Prop 16 $\square$.

Chap 3 Non-neutral (euclidean) geometry

↑ Prop 28 if 1) + 2) $= \pi$ then parallel.

§3.1 Parallelism

Prop 29 (3.1.1) (converse of P27/28) A straight line falling on parallel straight lines makes the alternate angles equal, the corresponding angles equal, and the interior angles on the same side sum to π.



Proof suppose $\angle 1 \neq \angle 2$ wlog $\angle 1 > \angle 2$

$\cancel{\angle 1 + \angle 3 = \angle 2 + \angle 3}$.

so $\underbrace{\angle 1 + \angle 3}_{= \pi} > \angle 2 + \angle 3$.

so $\pi > \angle 2 + \angle 3$.

(P5) parallel postulate $\Rightarrow$ lines $\substack{AB \\ CD}$ intersect $\#$.

$\therefore$ $\angle 1 = \angle 2$. ~~done~~ $\cancel{\angle 2 + \angle 3 = \pi}$.

$\cancel{\Rightarrow \angle 3 = \angle 4}$.

Furthermore: $\angle 1 + \angle 3 = \angle 2 + \angle 4 = \pi$

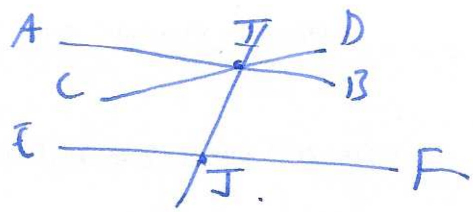
$\Rightarrow$ $\angle 3 = \angle 4$ and $\angle 2 + \angle 3 = \pi$. $\square$.

Prop 3.1.2 Parallel lines stay same distance apart.

Prop 30 straight lines parallel to same the same straight line are parallel to each other.

Proof (Euclid): $AB \parallel EF$, $CD \parallel EF$ prove $AB \parallel CD$

spose not, then AB intersects CD.

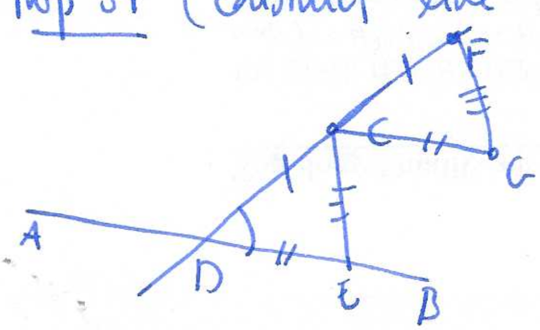


construct IJ....

can't do this in $\mathbb{H}^2$!

counterexample to P5O $\therefore$ $\square$.

Prop 31 (construct line parallel to AB through C).



construct line w/ same angle through C.

just draw some extra line construct cong triangle w/ side $|CF| = |DC|$. $\square$.