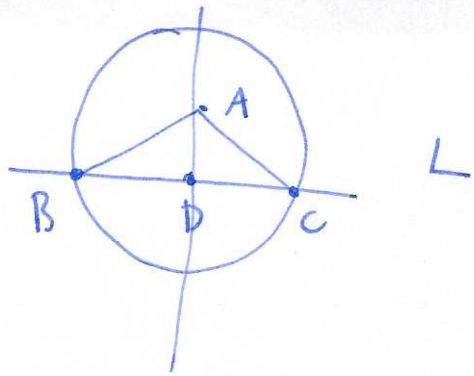
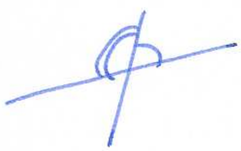
Proof



draw a circle through A which hits L twice at B, C say
now find midpoint D of BC.
claim $AD \perp BC$:
$\triangle ABD$ congruent to $\triangle ACD$
so $\angle BDA = \angle ADC = \frac{\pi}{2}$. □.

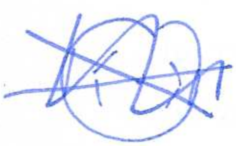
Prop^n 13 The sum of two supplementary angles is equal to two right angle



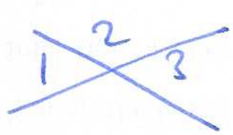
□.

Prop^n 15 If two straight lines cut one another, opposite angles are equal.

Pf

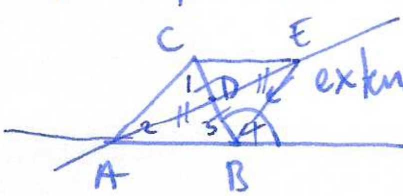


~~SAS~~



$1+2 = \pi$ ①
$2+3 = \pi$ ②
①−②: $1-3=0$
$1=3$. □.

Prop^n 16 (exterior angle) an exterior angle is greater than either of the interior ~~or~~ and opposite angles.

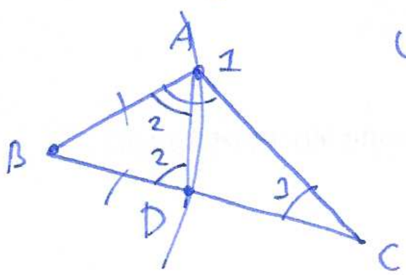


exterior angle

show $\angle 1 > \angle 4$.
construct midpoint D of CB
construct E on line through AD s.t $|AD| = |DE|$
by congruent triangles, $\angle 1 = \angle DBE$
(this uses separation). □.

Prop^n 18 In any triangle the greater side subtends the greater angle.

Pf

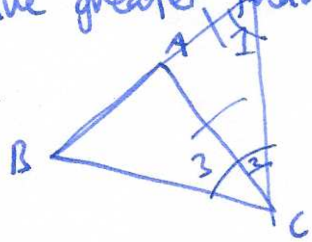


cut $|AB| = |BD|$.
now $\angle 1 > \angle 2$
$\angle 2 = \angle DAD = \angle BDA$ by isosceles triangle.
$\angle 2 > \angle 3$ as angle 2 ext angle for $\triangle ADC$
so $\angle 1 > \angle 3$. □.

Prop$^{\underline{n}}$ 19 In any triangle the greater angle is subtended by the greater side.

Prop$^{\underline{n}}$ 20 (triangle inequality) In any triangle, two sides taken together are greater than the remaining one.

Pf



want: $|AB| + |AC| > |BC|$.

extend ~~AB~~ BA to D s.t. $|AD| = |AC|$

draw CD

$\angle 3 > \angle 2$

$\angle 2 = \angle 1$ isoceles.

so $\angle 3 > \angle 1$.

so $|AB| + |AD| > |BC| \xrightarrow{\text{Prop 19}}$. $\square$.

Prop$^{\underline{n}}$ 26 (ASA cong. test) (or AAS).

① If two triangles have two angles equal to two angles resp. and the sides joining the angles are also equal, then they are congruent

② If two triangles have two angles equal to two angles resp and the side opposite to one of the angles is equal resp, then the triangles are congruent

Proof ①

suppose (wlog) $|AB| < |DE|$, extend BA to G s.t. $|BG| = |ED|$.

then $\triangle GBC$ cong to $\triangle DEF$ by SAS.

So $\angle GCB = \angle 2$ # $\Rightarrow G = A$. $\Rightarrow \triangle ABC$ cong $\triangle DEF$ $\square$①