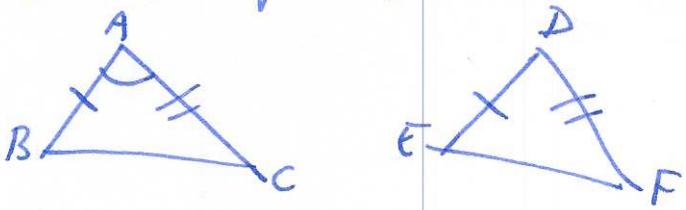
construct circle β from E of radius EF, let $C = \beta \cap EA$. Then $|AC| = |BC|$ ∎.

Propⁿ 4 (SAS for congruent triangles)

If two triangles have two ~~equ~~ sides equal to two sides, respectively, and have the angles contained by the equal straight lines equal, then the bases will be equal, and the remaining angles will be equal to the remaining angles respectively.



Proof

Apply (move) ΔABC to ΔDEF so that A lies on D, AB ~~point~~ lies on DE and ~~AC lies~~ C lies on same side of DE as F.

Then as $|AB| = |DE| \Rightarrow E = B$.

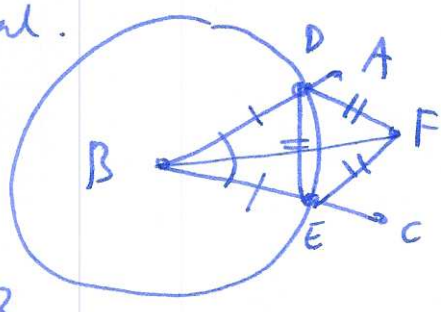
as $\angle BAC = \angle EDF \Rightarrow AC$ lies on ~~AC~~ DF

as $|AC| = |DF| \Rightarrow C = F$.

$\Rightarrow BC$ lies on EF (unique line between two points).

so $|BC| = |EF| \Rightarrow$ two triangles have all sides equal $\Rightarrow$ all angles equal.

Propⁿ 9 To bisect a given rectilineal angle.



Proof Given $\angle ABC$ construct a line BF

s.t. $\angle ABF = \angle CBF$.

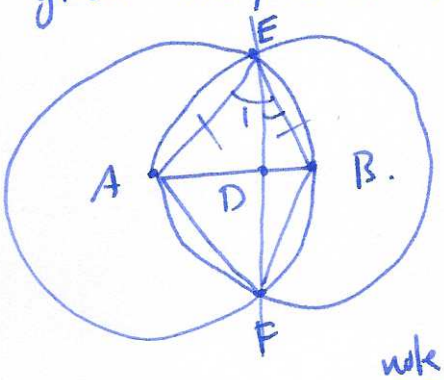
- draw an circle α w/ center B. Let $D = \alpha \cap AB$, $E = \alpha \cap BC$
- construct equilateral triangle on DE let F be third vertex.

claim ~~BF~~ works. proof (of claim) ΔBDF and ΔBEF congruent $\Rightarrow$ $\angle ABF = \angle FBE$ ∎.

Prop^n 10 Line bisector Bisect a given finite line segment

so given AB, construct D on AB s.t. $|AD| = |DB|$.

Pf



construct circle of radius $|AB|$ about A
$|AB|$ B.

Let intersection be E, F.

Let D be $EF \cap AB$.

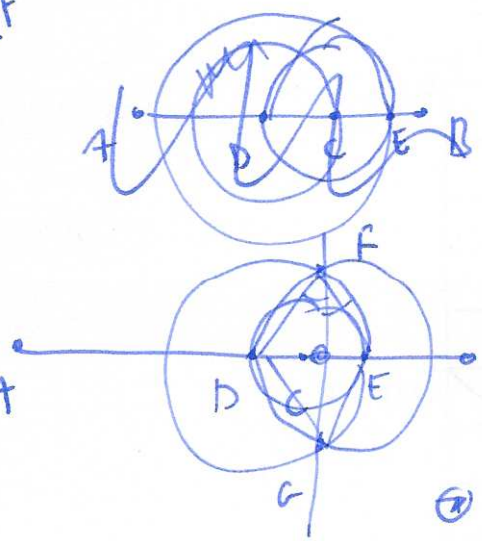
note by Prop^n 9 $\angle AEF = \angle BEF$

so $\triangle AED$ is ~~similar~~ congruent to $\triangle EBD$ $\Rightarrow |AD| = |DB|$ □.

Prop^n 11 (perpendicular bisector) Draw a straight line at right angles to a given straight line.

so given AB and C on AB, construct ⊥ line through C

Proof



draw a circle through C, let intersections be D, E

now draw circles through D, E of radius $= |DE|$ ~~|DC|~~ $= |CE|$.

Let intersections be F, G.

claim FG is ⊥ to AB at C.

by Prop^n 9 $\angle EFD = \angle EFC$

④ [as sum of angles $= \pi$, $\angle FDE = \frac{\pi}{3}$

↗ not neutral! so: $\angle DFC = \pi/6$.

$\angle FCD = \pi - \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{2}$]

⑦ or note by congruent (neutral) triangles $\angle DCF = \angle ECF \Rightarrow$ each equal $\frac{\pi}{2}$. □.

Prop^n 12 To a given inf straight line, and a point not on it, draw ⊥ line through that point.