

# MTH 623 Geometry for high school teachers

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-students w/ disabilities

Text: Geometry from Euclid to Kup, Saul Stahl

HW: hand in on paper. → read some of these from text.

Euclid's elements: 23 axioms, 5 postulates, 5 common notions (p34)

Postulates: 1) can draw a straight line between any two points (unique!)

2) can extend lines indefinitely

3) can draw a circle with any center and radius

4) All right angles are equal

5) Parallel postulate:  ← lines meet.

There are geometries where 5) does not hold: hyperbolic and spherical geometry

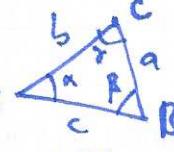
## Spherical geometry



- if A, B are not opposite points, there is a unique line between them.
- parallel postulate as stated true, but all lines intersect so no parallel lines!
- angles ↔ angle between tangent lines.

There are analogies of the Euclidean triangle relations

in spherical geometry.



sine rule:

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

## spherical

$$\frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c}$$



→ triangle can have 3 right angles!

cosine rule:

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$\cos \alpha = \cos b \cos c + \cos a \sin b \sin c \cos \gamma$$

$$\cos \alpha = \cos a \sin b \sin \gamma - \cos b \cos c \cos \gamma$$