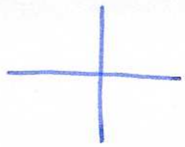
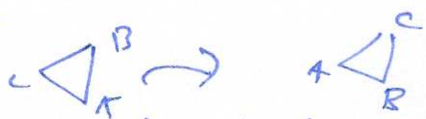
Isometries



eg. • translations
• rotations
• reflections
• anything else ?

observation: form a group
— can we explicitly describe it ? subgps? etc.

Q: • classify
• describe ? claim: image of 3 distinct non-colinear points is enough.



describes isom

coordinate systems on $\mathbb{R}^2$: cartesian (x,y)
$\mathbb{C}$ complex $z = x+iy$.
vector $\begin{bmatrix} x \\ y \end{bmatrix}$.

• isometries are real functions $f: \mathbb{R}^2 \to \mathbb{R}^2$

$$(x,y) \longmapsto (f_1(x,y), f_2(x,y)).$$

• can be written as complex functions (conjugate) $f: \mathbb{C} \to \mathbb{C}$

$$z \longmapsto f(z)$$
$$z \longmapsto f(\bar{z}).$$

• can be written as matrix + translation vector

$$f: \mathbb{R}^2 \to \mathbb{R}^2$$
$$\underline{v} \longmapsto A\underline{v} + \underline{b}$$

geometric setup group $G \curvearrowright X$ space.

properties: (1-) transitive : can take any pt to any other pt.
(2-) transitive : pairs of points.
(3-)-transitive etc.

natural subgroups: stabilizers:

$$\mathrm{stab}(p) = \{g \in G \mid gp = p\}.$$
$$\mathrm{stab}(\{p,q\}) = \{g \in G \mid g\{p,q\} = \{p,q\}\}.$$
$$\mathrm{stab}(\text{line } L)$$
$$\mathrm{stab}(\text{3 distinct non-colinear pts}).$$

§2 Neutral geometry of triangles

neutral: don't use parallel postulate (5), so hold in Euclidean, spherical, hyperbolic geometry.

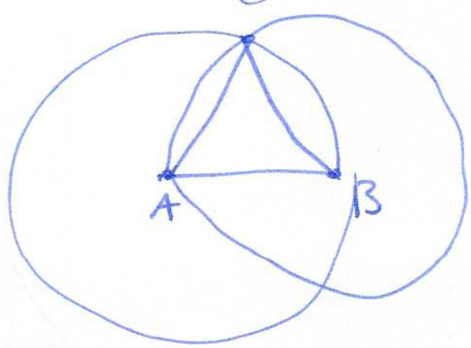
Example.

Prop 1 (2.3.1 p48) On a given finite straight line to construct an equilateral triangle ⟺ Given a finite line segment, one may construct an equilateral triangle whose base is the given line segment.

Proof (will consist of an explicit construction) : . . . QED □.

construction " straight edge and compass"
↗
can draw straight lines, but not nec. construct lines of arbitrary length.

compass : rigid — remembers distances. } equivalent! (**)
collapsible — does not remember distance



Proof (of Prop 1) given AB

construct circle at A of radius AB.
B AB

let C be one of the intersection points (*).

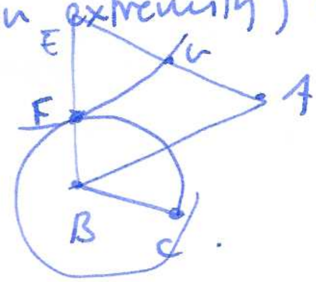
draw lines AC, AB

claim: ABC is an equilateral triangle

circle at A, so |AB| = |AC| both radii
circle at B so |BC| = |AB| both radii □.

(*) how do you know circles intersect? ⇐ uses extra axiom S (separation)

(**) Prop 2 To place at a given point (as an extremity) a straight line equal to a given straight line)



Proof find D s.t. |AD| = |BC|.

construct ABE equilateral

construct circle α through B of radius BC, let $F = \alpha \cap EB$