so $\dfrac{\partial \psi}{\partial x} = \dfrac{\partial \phi}{\partial y}$ $\quad \dfrac{\partial \psi}{\partial y} = -\dfrac{\partial \phi}{\partial x}$, i.e. satisfies CR equations.

$f(z) = \psi + i\phi$ is analytic in G

$$f'(z) = \frac{\partial \psi}{\partial x} + i\frac{\partial \phi}{\partial x} = -E_y - iE_x = -i(E_x - iE_y)$$

equivalently: $E_x + iE_y = -i\,\overline{f'(z)}$

ψ stream function $\quad \psi(x,y) = \text{const}$ lines of force

ϕ (electrostatic) potential $\quad \phi(x,y) = \text{const}$ equipotentials

Γ conducting surface $\Leftrightarrow$ part of an equipotential

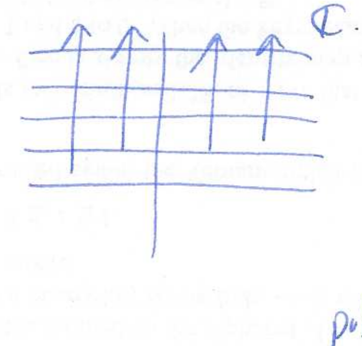
i.e. electric field has no component tangential to Γ (would induce motion of charge # stationarity)

Examples $\quad f(z) = az \quad (a > 0,\ \in \mathbb{R})$

complex potential, electric field $\quad E_x + iE_y = -i\,\overline{f'(z)} = -ia$

$E_x = 0 \quad E_y = -a$

$f(z) = a(x+iy) = ax + iay$

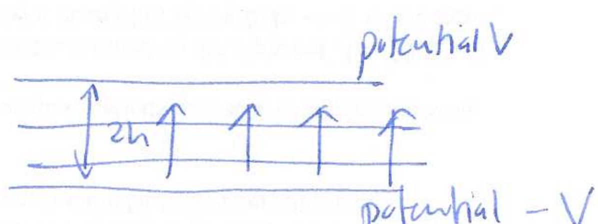


$\psi = ax$ $\quad \phi = ay$

lines of force $\quad$ equipotentials

$ax = \text{const}$ $\quad y = \text{const}$.

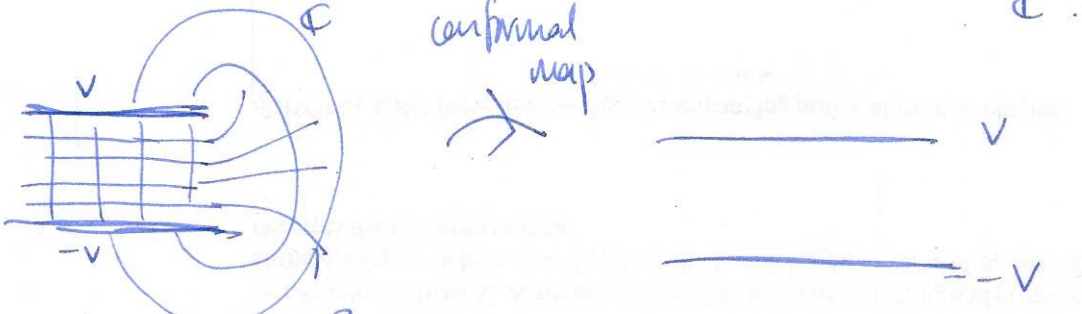
field between two parallel conductors (condenser / capacitor)



choose $a = \dfrac{V}{h}$ $\quad \phi = \dfrac{V}{h}y$

$E = -i\dfrac{V}{h}$

Q: spose your parallel conductors are not infinite?



$z \qquad\qquad w$

$$z = \frac{h}{\pi}\left(e^{\pi w/V} + \frac{\pi w}{V}\right)$$

$$x = \frac{h}{\pi}\left(e^{\pi u/V}\cos\frac{\pi v}{V} + \frac{\pi u}{V}\right) \qquad E = -i\,\overline{\frac{dw}{dz}} = -i\frac{1}{\overline{\frac{dz}{dw}}}$$

$$y = \frac{h}{\pi}\left(e^{\pi u/V}\sin\frac{\pi v}{V} + \frac{\pi v}{V}\right) \qquad = -i\frac{V}{h}\frac{1}{1+e^{\pi u/V}}$$

§13.1 Harmonic functions

<u>Thm</u> $K = |z-z_0| < R$ $u(z)$ harmonic in K w/ harmonic conjugate $v(z)$. Then

$$u(z) = u(z_0 + re^{i\phi}) = a_0 + \sum_{n=1}^{\infty}(a_n\cos n\phi - b_n\sin n\phi)\,r^n$$

$$v(z) = v(z_0 + re^{i\phi}) = b_0 + \sum_{n=1}^{\infty}(b_n\cos n\phi + a_n\sin n\phi)\,r^n$$

for $0 \le r < R$ $\quad 0 \le \phi \le 2\pi$

convergence is uniform for $0 \le r \le R' < R$, $\quad 0 \le \phi \le 2\pi$.

<u>Proof</u> $f(z) = u + iv$ analytic in K.

$$f(z) = c_0 + c_1(z-z_0) + c_2(z-z_0)^2 + \cdots$$

(uniform convergence in $0 < r \le R' < R$).