Math 431 Complex Analysis Spring 2021 HW 8

(1) Chapter 9 Q 3, 4, 5, 6, 10, 11

(2) Chapter 10 Q 1, 2, 3, 4, 5, 6, 8

(3) If G is a group *acting on* a set A (in other words the elements of G permute the elements of A) then for any $a \in A$, the orbit of G containing a is $\{g(a) \mid g \in G\}$.

For example, the orthogonal group $O(2)$ is the set of all transformations of the form $z \mapsto e^{i\theta}z$ and $z \mapsto e^{i\theta}\overline{z}$ for $\theta \in [0, 2\pi)$. It permutes the set of points, the set of pairs of distinct points, the set of straight lines and the set of circles in $\mathbb{C}$. Two points $z, w \in \mathbb{C}$ are in the same orbit if and only if $|w| = |z|$. All of these orbits are infinite except for the orbit of $\{0\}$ which just has one point. Two (unordered) pairs $\{u, v\}$ and $\{w, z\}$ are in the same orbit if and only if the (possibly degenerate) triangle $0uv$ is congruent to $0wz$ with 0 representing the same vertex. Two straight lines are in the same orbit if and only if their (perpendicular) distance to the origin is the same. Two circles are in the same orbit if and only if they have the same radius and their centers are the same distance from 0.

(a) Let G be the group of transformations of the form $z \mapsto e^{i\theta}z + a$ or $z \mapsto e^{i\theta}\overline{z} + a$, where $a \in \mathbb{C}$ and $\theta \in [0, 2\pi)$ (this is the full isometry group of the plane: the set of distance preserving transformations).
(b) Let G be the group of transformations of the form $z \mapsto az$ or $z \mapsto a\overline{z}$, where $a \in \mathbb{C} \setminus \{0\}$ (this is the group of all similarities of the plane which fix the origin: it preserves angles everywhere).
(c) Let G be the group of transformations of the form $z \mapsto az + b$ or $z \mapsto a\overline{z} + b$, where $a \in \mathbb{C} \setminus \{0\}$ and $b \in \mathbb{C}$ (this is the full group of similarities of the plane).

For each of the above groups of transformations of $\mathbb{C}$ give similar descriptions of the orbits on: (i) points; (ii) unordered pairs of distinct points; (iii) straight lines; (iv) circles; and for (a) and (c) only: (v) unordered triples of distinct points; unordered pairs of distinct straight lines.