Math 431 Complex Analysis Spring 2020 HW 7

(1) Chapter 8 Q 1, 2, 3, 9, 10, 13, 15, 17, 22, 26

(2) (a) Let γ be the circle $|z| = 1$ so that inversion in γ is the map $g(z) = 1/\overline{z}$. Find a Möbius map $f(z)$ such that $f \circ g \circ f^{-1}(z) = \overline{z}$. What is the image of γ under f?

(b) Let γ and δ be two circles in $\mathbb{C} \cup \{\infty\}$ (including the possibility that one is a straight line). Show that there is a Möbius map f taking γ and δ to exactly one of: (i) a pair of parallel straight lines; (ii) a pair of intersecting straight lines; (iii) a pair of non-intersecting circles.

(c) For each circle C in the plane, let ρ_C denote inversion in C. Show that, if C and C' are two circles which are tangent, then the point of tangency is the unique fixed point of the map $\rho_C \rho_{C'}$ obtained by composing the inversions in C and C'.