

Math 431 Complex Analysis Spring 2020 HW 6

- (1) Chapter 6 Q 13, 16, 17
- (2) Chapter 7 Q 2, 3, 4, 5, 7
- (3)
 - (a) Suppose a Möbius map takes the circle C to the circle C' . Does it take the center of C to the center of C' ? Are there any restrictions on where the center can go?
 - (b) Let λ be a straight line and γ a circle which does not cross or touch λ . Let μ be the straight line perpendicular to λ which passes through the center of γ . Show that there is a circle δ which crosses γ, λ and μ orthogonally.
 - (c) Let w be one of the points of intersection of δ and μ and let f be a Möbius map taking w to ∞ . Show that f maps γ and λ to two concentric circles.
 - (d) Show that if α and β are any two non-intersecting circles (or straight lines) there is a Möbius map taking α and β to concentric circles.
 - (e) **Steiner's Porism:** If we have two (nonconcentric) circles, one inside the other, and circles are drawn successively touching them and one another, it may happen that the ring of touching circles closes. Steiner's Porism states that if this happens once, it will always happen, regardless of the position of the first circle of the ring. Prove this.