Math 431 Complex Analysis Spring 2020 HW 5

(1) Chapter 5 Q 20, 21, 26, 27

(2) Chapter 6 Q 3, 9, 10

(3) A Möbius map is a map of the form $f(z) = \frac{az+b}{cz+d}$, where $ad - bc \neq 0$.

(a) Show that any Möbius map is the composition of maps of the form $z \mapsto 1/z$, $z \mapsto z + a$ and $z \mapsto az$, $(a \in \mathbb{C})$.

(b) Show that Möbius maps take circles and straight lines to circles and straight lines (not necessarily respectively).

(c) Let (a, b, c) be an ordered triple of distinct points in the extended complex plane $\mathbb{C} \cup \{\infty\}$. Show that there is a unique Möbius map f with $f(a) = \infty$, $f(b) = 0$ and $f(c) = 1$. Deduce that for any two ordered triples of distinct points there is a unique Möbius map taking the first triple to the second.

(d) Let γ_1 and γ_2 be two circles or straight lines defined by the equations

$$A_i |z|^2 + B_i z + \overline{B}_i \overline{z} + C_i = 0,$$

for $i = 1, 2$, where $A_i, C_i \in \mathbb{R}$, $B_i \in \mathbb{C}$ and $|B_i|^2 > A_i C_i$. Show that γ_1 and γ_2 cross orthogonally if and only if

$$B_1 \overline{B}_2 + \overline{B}_1 B_2 = A_1 C_2 + A_2 C_1.$$

(e) Use 1(a) and 1(b) to show that Möbius maps take orthogonal circles and straight lines to orthogonal circles and straight lines.