

Math 431 Complex Analysis Spring 2021 HW 10

- (1) Chapter 11 Q 11, 15, 16 23, 24, 25, 26, 30, 32
- (2) If G is a group and $g, h \in G$ then the *conjugate* of g by h is the element hgh^{-1} and the *conjugacy class* of g is the set $c_G(h) = \{hgh^{-1} \mid h \in G\}$ (in other words it is the set of conjugates of g). Recall that if G acts on a set X and $k = hgh^{-1}$, then whenever g takes x to y , k takes $h(x)$ to $h(y)$. Thus g has the same effect on X as k has on X adjusted by h , so k and g are really the same sort of element.

What are the conjugacy classes in the following groups.

- (a) $O(2)$, which is the group of transformations of the form $z \mapsto e^{i\theta}z$ or $z \mapsto e^{i\theta}\bar{z}$ where $\theta \in [0, 2\pi)$.
- (b) The group of transformations of the form $z \mapsto e^{i\theta}z + a$ or $z \mapsto e^{i\theta}\bar{z} + a$, where $\theta \in [0, 2\pi)$ and $a \in \mathbb{C}$ (this is the full isometry group of the plane, the group of all distance preserving transformations).
- (c) The group of Möbius transformations acting on $\mathbb{C} \cup \{\infty\}$.