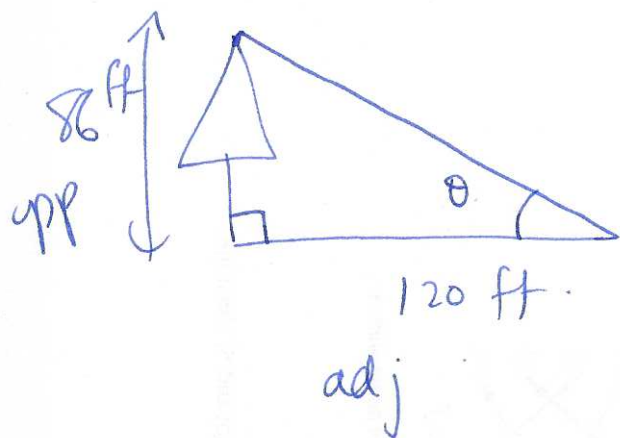


①



$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

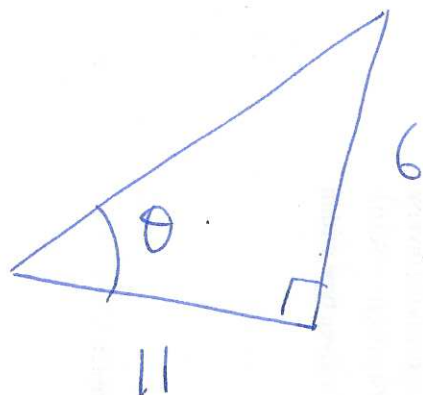
$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{86}{120}$$

$$\theta = \tan^{-1}\left(\frac{86}{120}\right)$$

$$\theta \approx 0.622 \times \frac{180}{\pi} \text{ radians} \xrightarrow{\text{atan, arc tan}} \text{degrees} \rightarrow 35.6^\circ$$

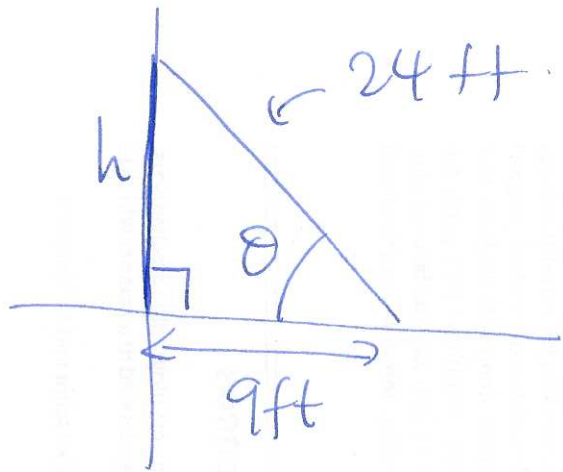


find θ (in degrees) 2

$$\tan \theta = \frac{6}{11}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{6}{11}\right) \left(\neq \frac{180}{\pi} \right) \\ &= 28.6^\circ\end{aligned}$$

③



$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{9}{24}$$

$$\theta = \cos^{-1}\left(\frac{9}{24}\right) \times \frac{180}{\pi}$$

acos
arccos.

$$\theta \approx 68.0^\circ \quad (1)$$

$$h^2 + 9^2 = 24^2$$

$$h^2 = 24^2 - 9^2$$

$$(2) \quad \frac{h}{24} = \sin(68.0)$$

$$h = 24 \times \sin\left(68.0 \times \frac{\pi}{180}\right) \quad h = \sqrt{24^2 - 9^2} = 22.2 \text{ ft}$$

22.2...

$$\theta = \frac{\pi}{3}$$

radians.

$$\sin(2\theta)$$

$$2\sin(\theta)$$

(4)

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$2\sin(\theta) = 2\sin\left(\frac{\pi}{3}\right)$$

$$\sin\left(\frac{2\pi}{3}\right)$$

$$= \frac{2\sqrt{3}}{2} = \sqrt{3}$$

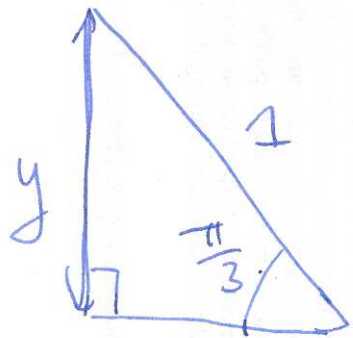
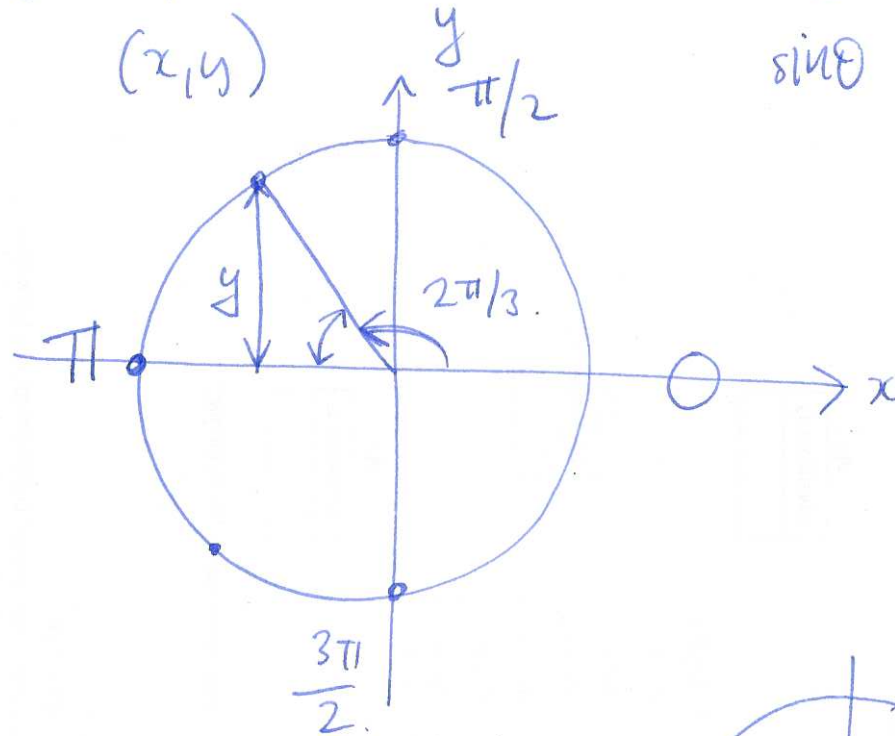
$$\sin\left(\frac{2\pi}{3}\right)$$

$$(\cos\theta, \sin\theta)$$

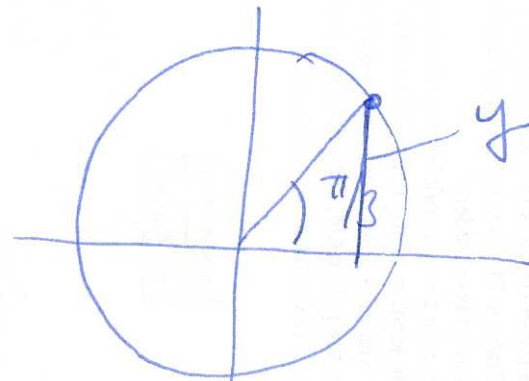
$$(x, y)$$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$

5



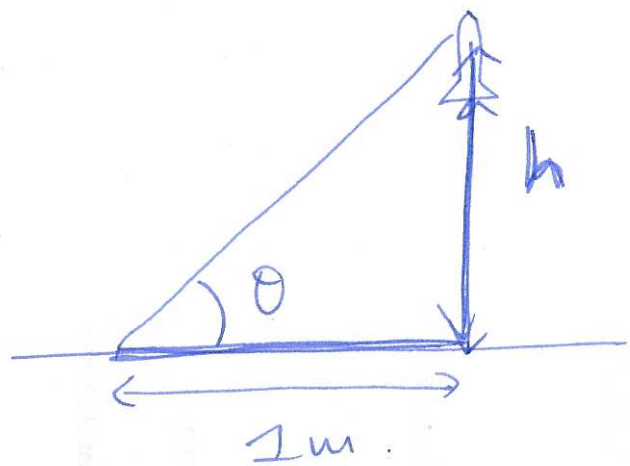
$$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$$



$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

⑥

$$1 \text{ mile} = 5280 \text{ ft}$$



$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

40°
60°
80°
85°

$$= \frac{\cancel{\text{opp}}/\cancel{\text{hyp}}}{\cancel{\text{adj}}/\cancel{\text{hyp}}} = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{h}{5280}$$

$$h = 5280 \tan \theta$$

θ	40°	60°	80°	85°	degrees
---	-----	-----	-----	-----	---------

h	4430 ft	9145	29,944	60,351	ft.
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$\operatorname{cosec}(\theta)$ $\cot(\theta)$ in quadrant 2.

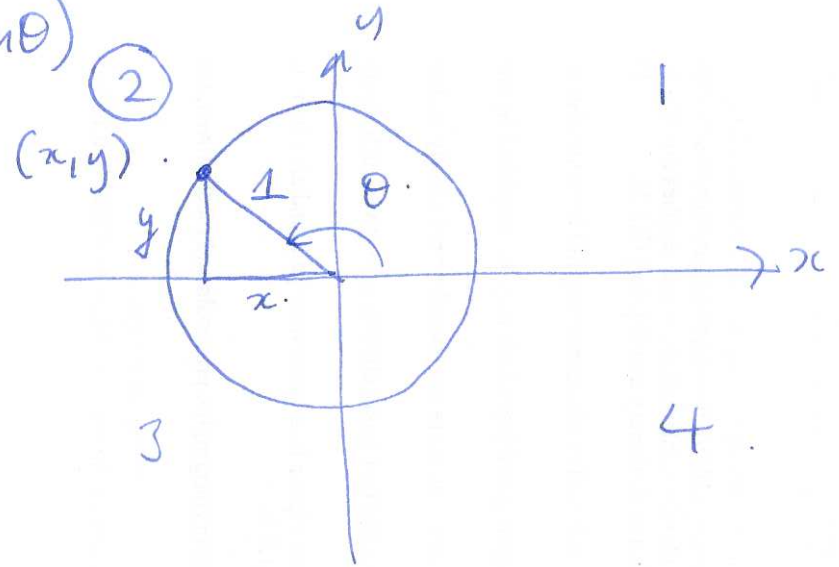
$$\frac{1}{\sin \theta} = \operatorname{cosec}(\theta)$$

$$(\cos \theta, \sin \theta)$$

$$\frac{1}{\cos \theta} = \sec(\theta)$$

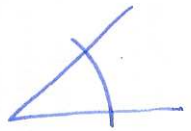
$$\frac{1}{\tan \theta} = \cot(\theta)$$

$$x^2 + y^2 = 1$$



$$\operatorname{cosec}(\theta) = \frac{1}{\sin \theta} = \frac{1}{y}$$

$$\cot(\theta) = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta} = \frac{x}{y}$$


eigen value.

$$\operatorname{cosec}(\theta) = \frac{1}{\sin \theta} = \frac{1}{y}$$

$$\cot(\theta) = \frac{\cos \theta}{\sin \theta} = \frac{x}{y}$$

$$\operatorname{cosec}^2(\theta) = \cot^2(\theta) + 1$$

$$\operatorname{cosec}(\theta) = (\pm) \sqrt{\cot^2(\theta) + 1}$$

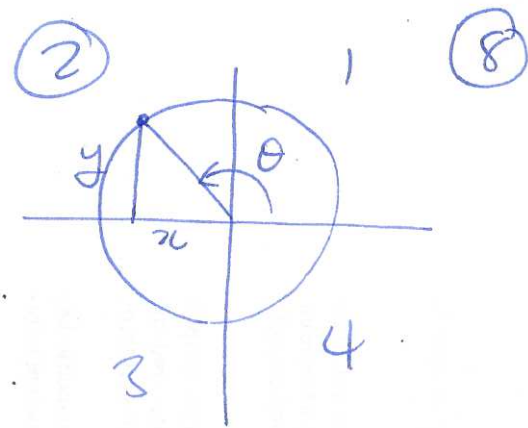
which one? depends on quadrant.

Q(2)

$$\operatorname{cosec}(\theta) = \sqrt{\cot^2 \theta + 1}$$

$$\frac{x^2 + y^2}{y^2} = \frac{1}{y^2}$$

$$\frac{x^2}{y^2} + 1 = \frac{1}{y^2}$$



$\cot(\theta)$ $\sin(\theta)$ Q2

$$\frac{y}{x}$$

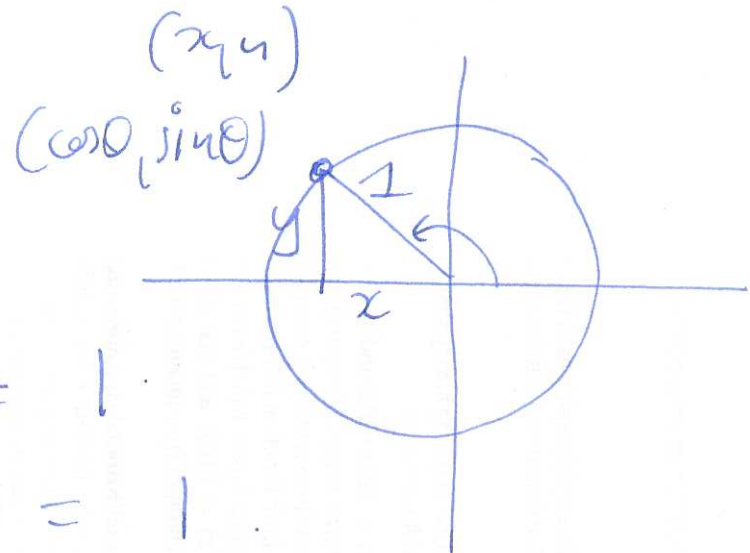
$$y$$

-ve
+ve

$$x^2 + y^2 = 1$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$\frac{x^2 + y^2}{y^2} = \frac{1}{y^2}$$



Q: which one?

$$\cot^2(\theta) + 1 = \frac{1}{\sin^2 \theta}$$

$$\cot^2(\theta) = \frac{1}{\sin^2 \theta} - 1$$

$$\cot(\theta) = \pm \sqrt{\frac{1}{\sin^2 \theta} - 1} = -\sqrt{\frac{1}{\sin^2 \theta} - 1}$$

$$\theta = \frac{\pi}{3}$$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$

$$\sin(2\theta)$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

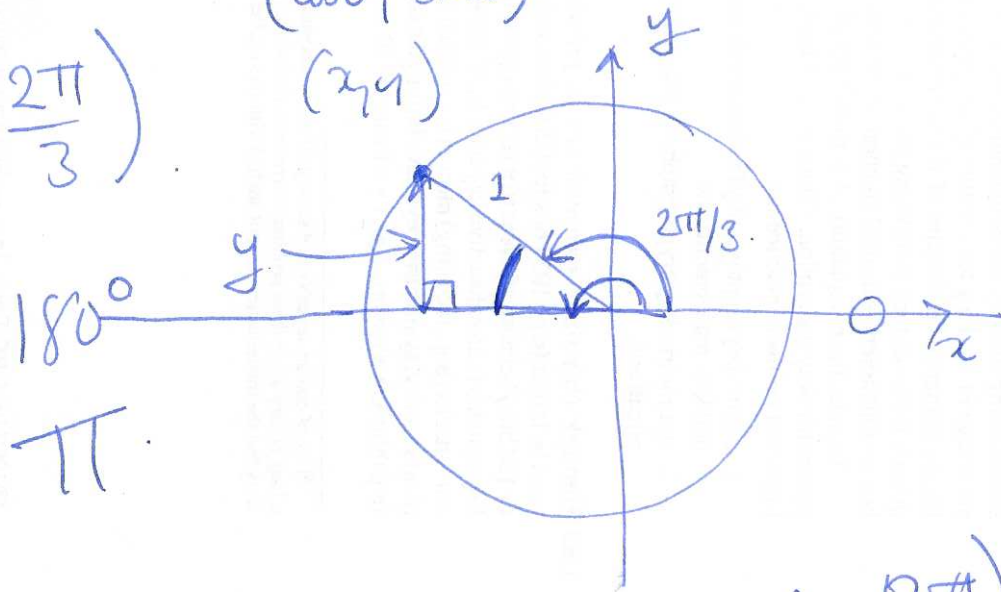
$$6 \sin(\theta)$$

$$6 \times \sin\left(\frac{\pi}{3}\right) = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$$

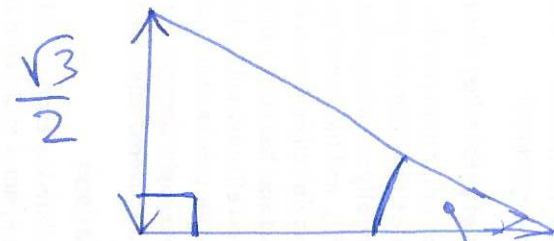
$(\cos \theta, \sin \theta)$

(x, y)

$$\sin\left(\frac{2\pi}{3}\right)$$

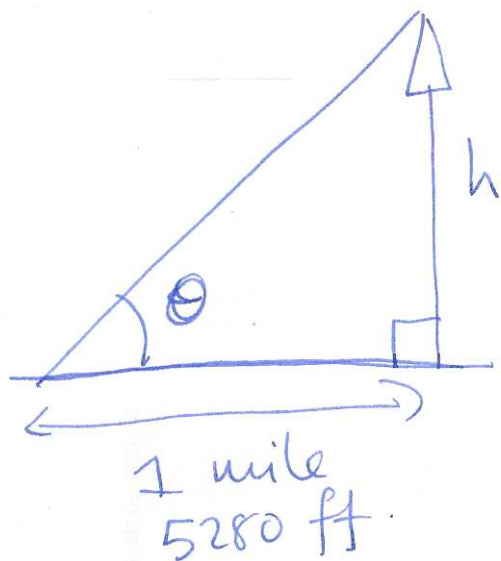


$$360^\circ = 2\pi$$



$$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

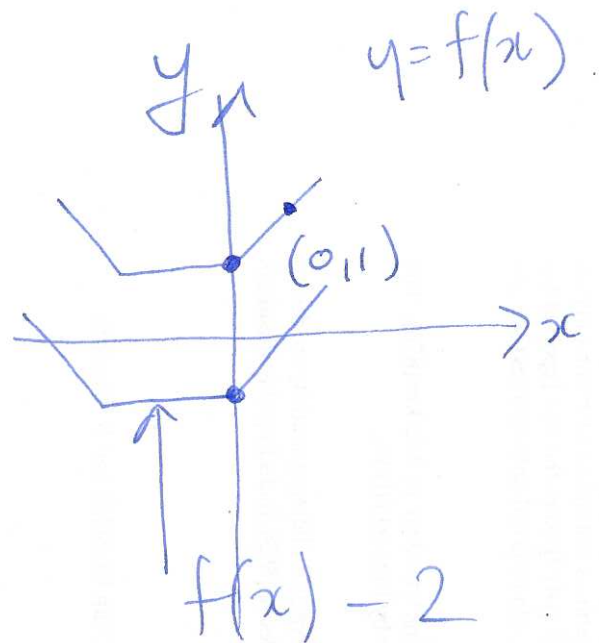


θ	35°	60°	75°	85°
	3697	9145	19,705	60,351

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{h}{5280}$$

$$h = 5280 \tan \theta$$

$$5280 \times \tan\left(\theta \times \frac{\pi}{180}\right)$$



① draw $f(x) - 2$

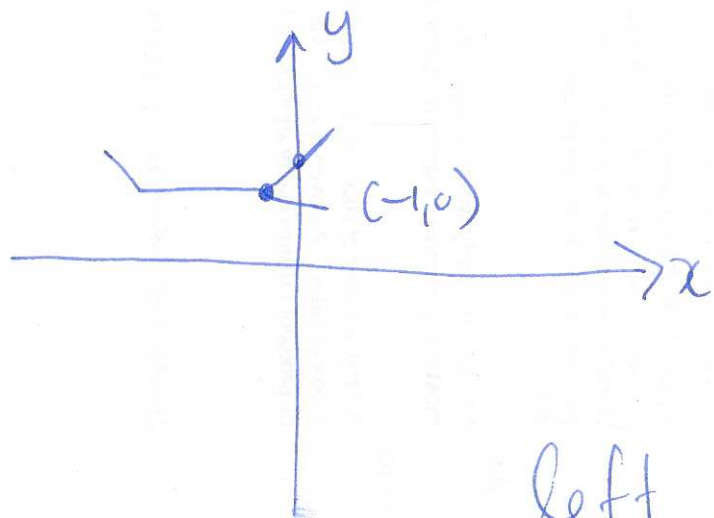
$(0, 1) \quad f(0) = 1$

$$f(0) - 2 = 1 - 2 = -1$$

② draw $f(x+1)$

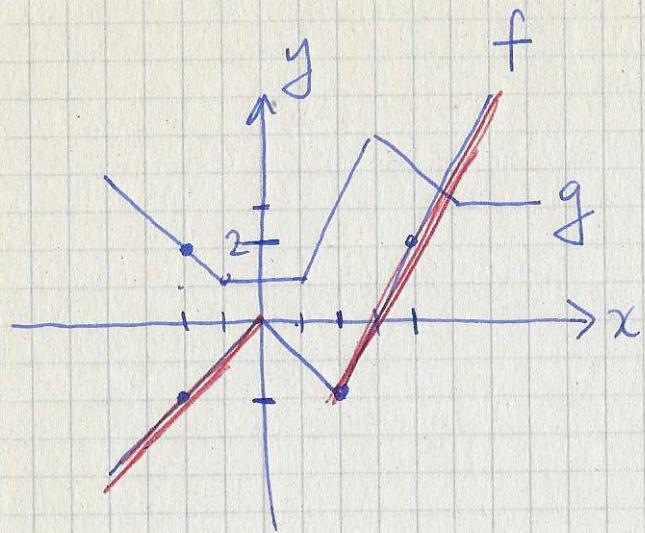
$(0, 1) \quad f(0) = 1$

$$f(0+1) = f(1) = 2$$



left shift by 1

$$f(-1+1) = f(0) = 1$$



- i) $(f-g)(2)$ ii) $(g \circ f)(4)$ (13)
- iii) $(f \circ g)(-2)$ iv) $f(x)$
increasing

$$\text{i) } (f-g)(2) = f(2) - g(2) \\ = 4 - 9 = -5.$$

$$\text{ii) } (g \circ f)(4) = g(f(4)) = g(2) = 3.$$

$$\text{iii) } (f \circ g)(-2) = f(g(-2)) = \cancel{f(-2)} \\ = f(2) = 4$$

$$\text{iv) } f \text{ increasing } (-4, 0) \cup (2, 4) \\ (-\infty, 0) \cup (2, \infty).$$

$$2x - 3y = 6$$

$$+3y \quad +3y$$

$$2x = 6 - 3y$$

$$-6 \quad -6$$

$$\frac{2x - 6}{-3} = \frac{-3y}{-3}$$

$$-\frac{2}{3}x + 2 = y$$

y-intercept: $x = 0$

$$y = 2 \quad (0, 2)$$

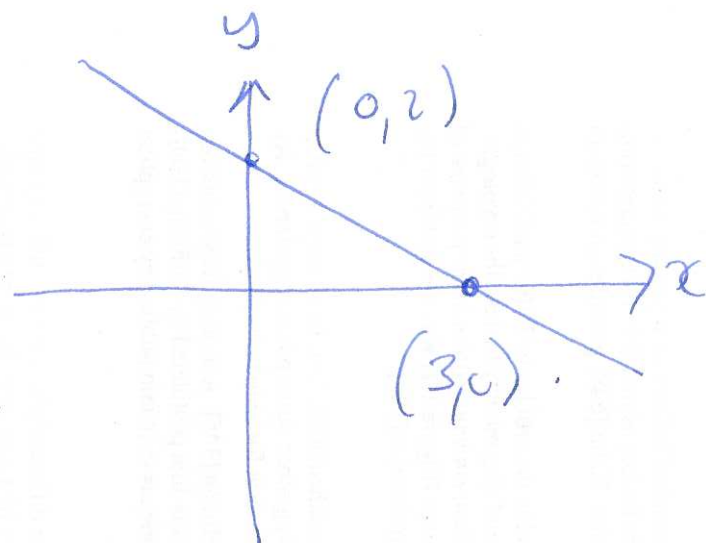
$$0 = -\frac{2}{3}x + 2$$

$$+\frac{2}{3}x \quad +\frac{2}{3}x$$

$$\frac{3}{2} \times \frac{2}{3}x = 2 \times \frac{3}{2}$$

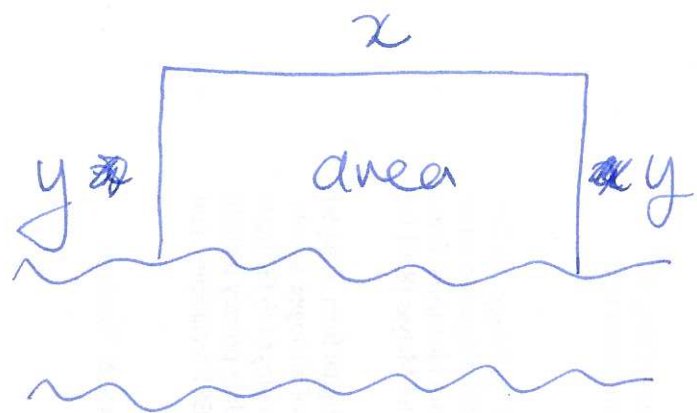
$$x = 3$$

draw, find x, y intercepts



$$y = -\frac{2}{3}x + 2$$

x-intercept (3)
(3, 0)



400 ft of fence.

largest area.

$$x + 2y = 400$$

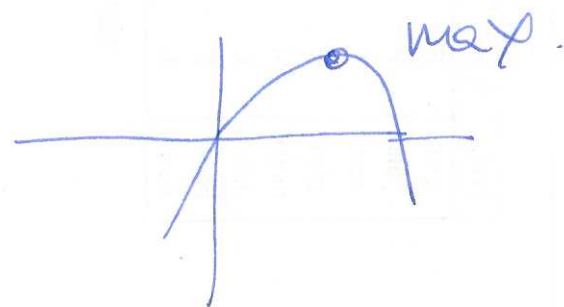
$$x = 400 - 2y$$

area $A = xy =$

max

$$A = (400 - 2y)y = 400y - 2y^2$$

complete the square.



$$400y - 2y^2$$

$$-2y^2 + 400y$$

$$-2 \left[y^2 - 200y \right]$$

$$-2 \left[(y - 100)^2 - 100^2 \right]$$

$$-2 \left[y^2 - 200y + 100^2 - 100^2 \right]$$

$$y = 100$$

$$x + 200 = 400$$

$$x = 200$$

$$A = \pi y = 100 \times 200 = 20,000$$

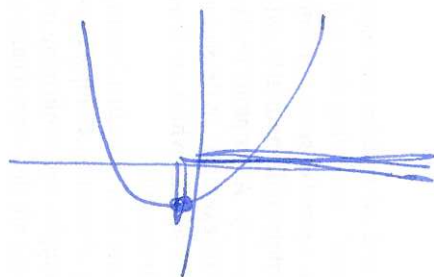
$$x + 2y = 400$$

find inverse functions

$$f(x) = (2x+1)^2$$

$$y = (2x+1)^2$$

$$\sqrt{y-1} = 2x+1$$



$$\frac{\sqrt{y}-1}{2} = \frac{2x}{2}$$

$$x = \frac{\sqrt{y}-1}{2}$$

✓ inverse on $(-\frac{1}{2}, \infty)$

$$f(x) = e^{3x-2} - 6$$

$$y + 6 = e^{3x-2} + 6$$

$$y + 6 = e^{3x-2}$$

$$\ln(y+6) = \ln(e^{3x-2})$$

$$\ln(y+6) + 2 = 3x - 2 + 2$$

$$\ln(e^x) = x$$

$$e^{\ln(x)} = x$$

$$\frac{\ln(y+6) + 2}{3} = \frac{3x}{3}$$

$$x = \frac{\ln(y+6) + 2}{3}$$

$$f(x) = \sin 2x$$

$$g(x) = 3^{x-1}$$

19

$$\text{i) } f \circ g(x) = f(g(x)) = \sin(2 \times 3^{x-1})$$

$$\text{ii) } g \circ f(x) = g(f(x))$$

$$= 3^{(\sin(2x) - 1)}$$