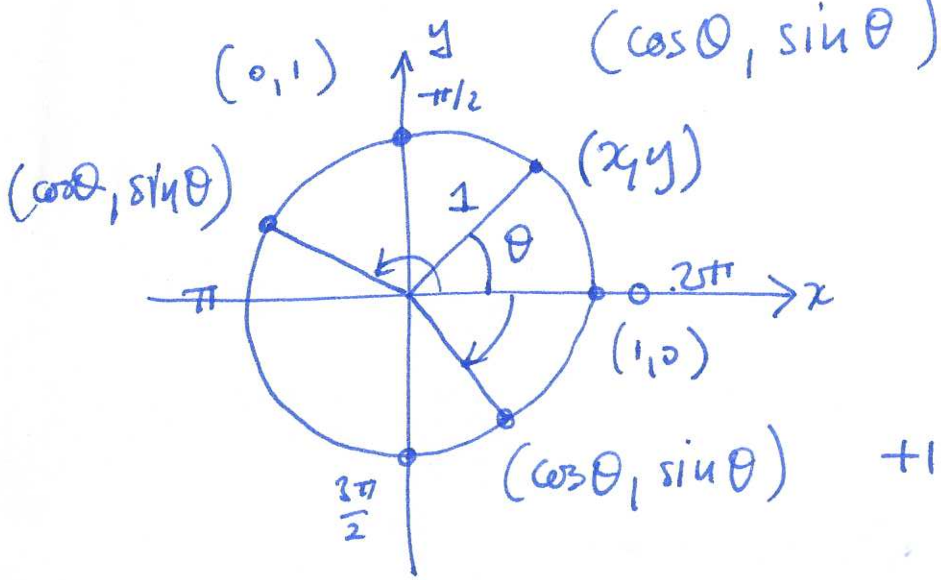
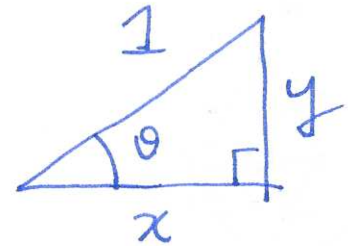
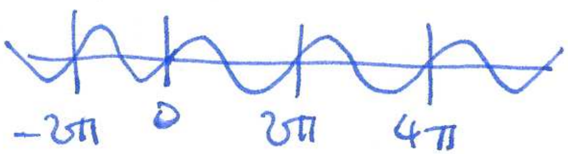


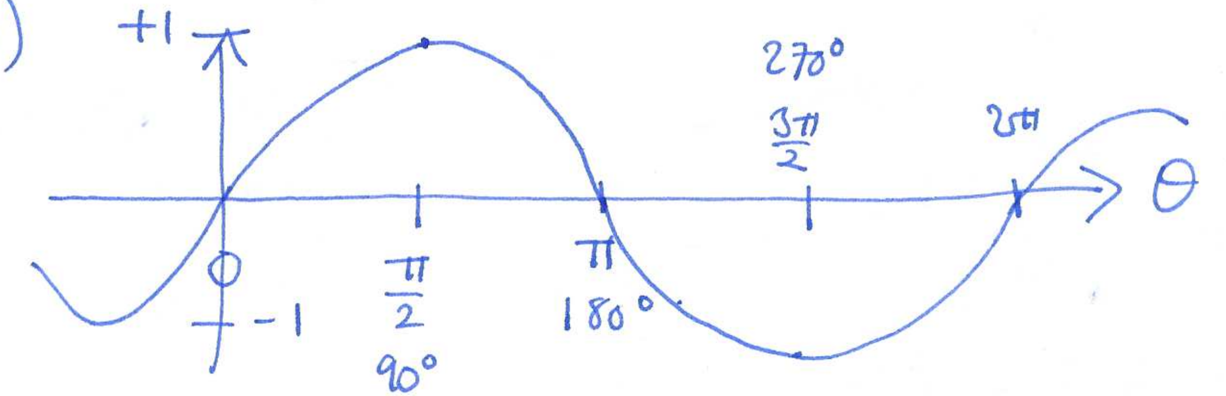


$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$.

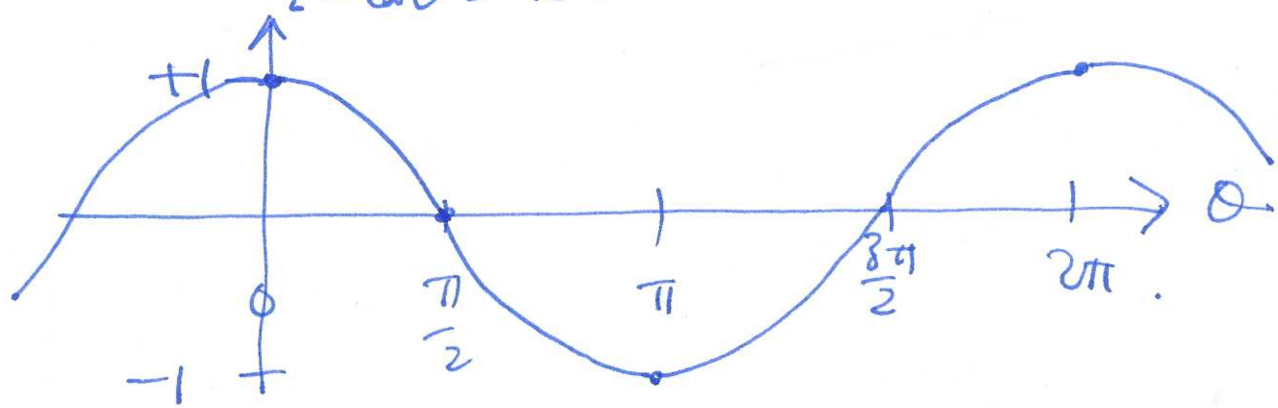
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$.



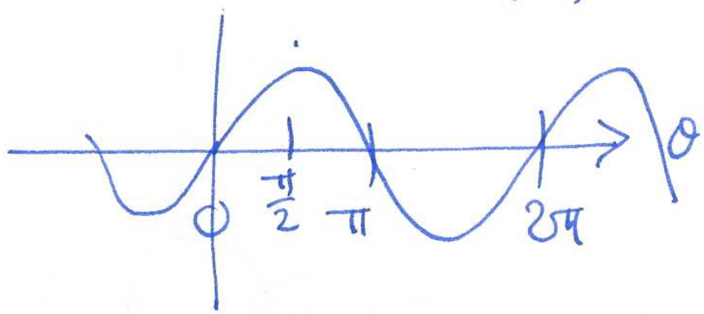
$z = \sin(\theta) = y$.



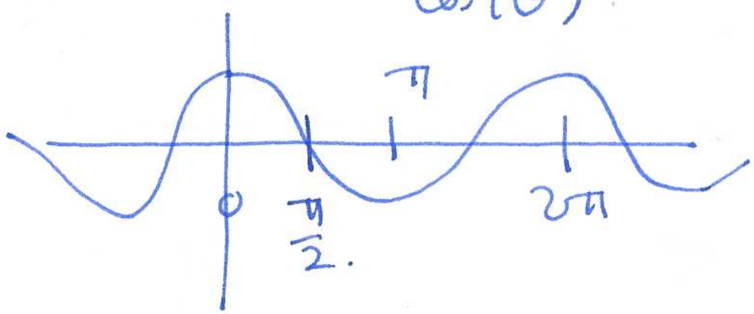
$z = \cos\theta = x$.



$\sin(\theta)$



$\cos(\theta)$



shift $\sin(\theta)$ $\frac{\pi}{2}$ units to the left, looks like you get $\cos\theta$.

<u>recall</u> $\sin(\theta) + \frac{\pi}{2}$.

vertical shift by $\frac{\pi}{2}$.

horizontal shift $\sin(\theta + \frac{\pi}{2})$.

<u>check</u> $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

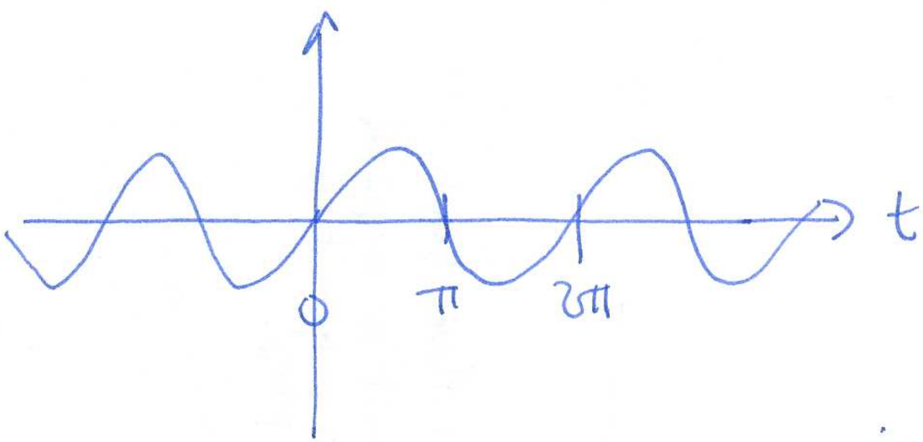
$$\sin(\theta + \tfrac{\pi}{2}) = \sin(\theta) \underbrace{\cos(\tfrac{\pi}{2})}_{0} + \cos\theta \underbrace{\sin(\tfrac{\pi}{2})}_{1}$$

$$= \cos\theta.$$

$y = \cancel{8} \sin(2t).$

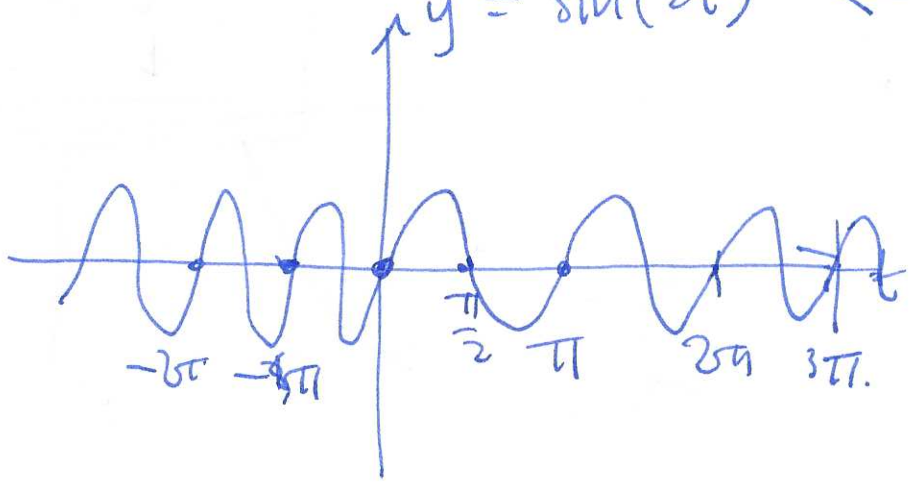
amplitude
period
frequency
phase shift.

$y = \sin(t)$



scaling
horizontal expansion
or shrink

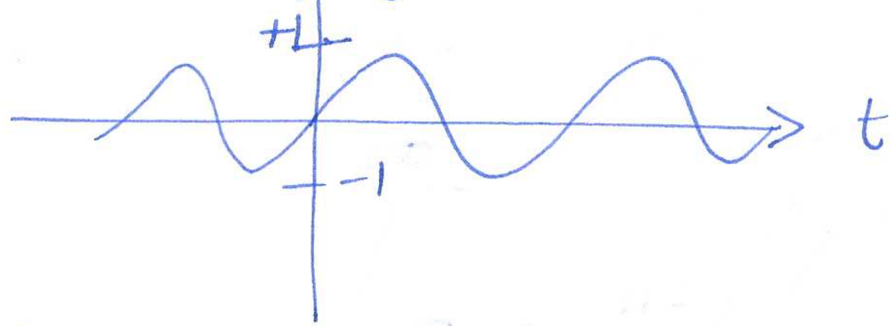
$y = \sin(2t)$ ←



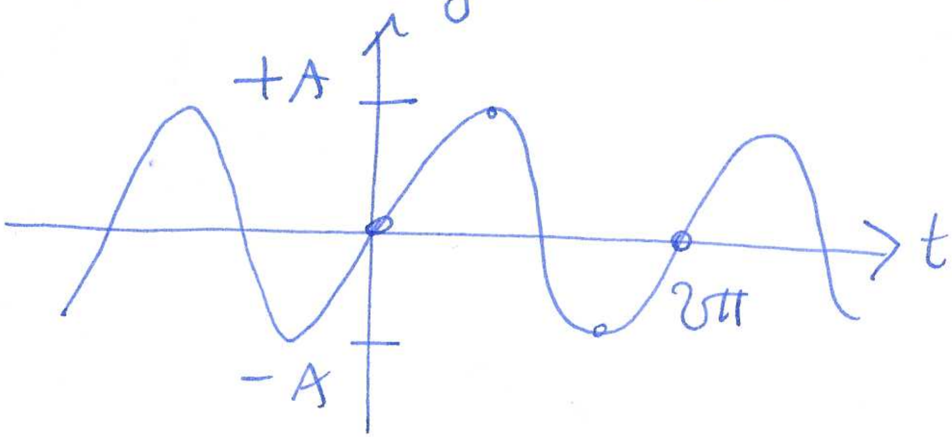
$A \sin(b(t+c))$.

phase shift

$y = \sin(t)$.

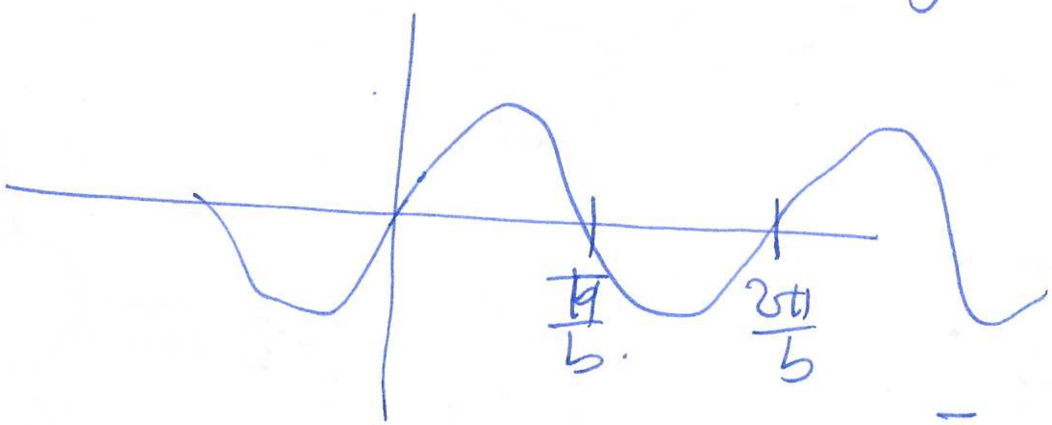


y $A \sin(t)$.



A = amplitude.

$y = A \sin(bt)$



period. $\frac{2\pi}{b}$.

frequency $= \frac{2\pi}{\text{period}} = b$.

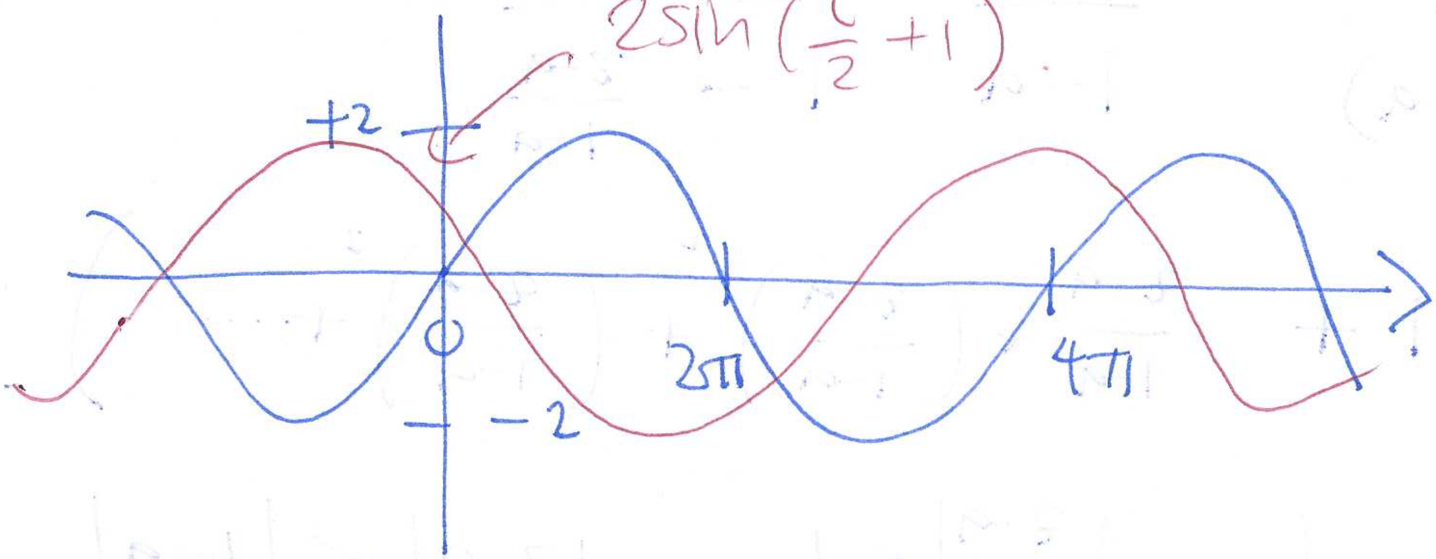
$$y = 2\sin\left(\frac{t}{2}+1\right) = 2\sin\left(\frac{1}{2}(t+2)\right)$$

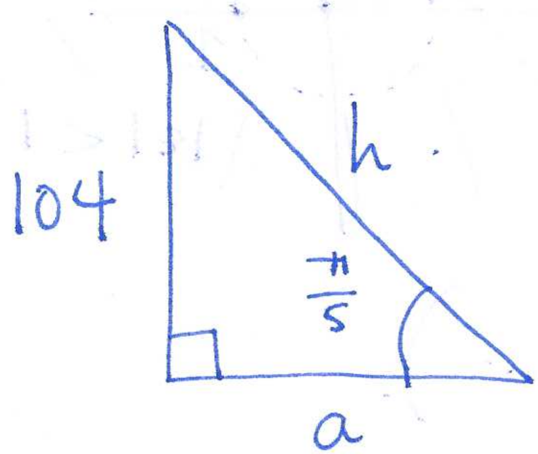
amplitude 2

frequency 1/2

phase shift 2

period is $\frac{2\pi}{1/2} = 4\pi$.





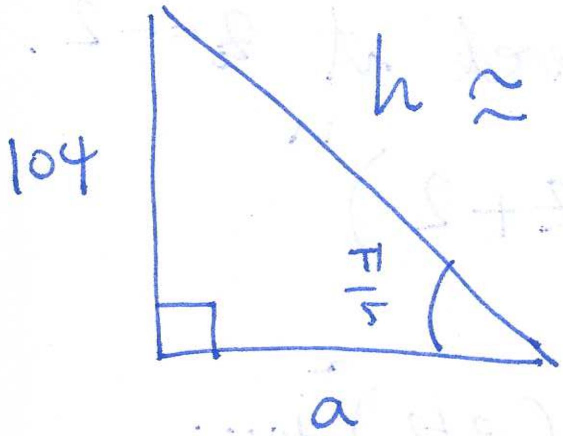
$$\frac{\text{opp}}{\text{adj}} = \frac{104}{a} = \tan\left(\frac{\pi}{5}\right)$$

$$= \frac{\sin\left(\frac{\pi}{5}\right)}{\cos\left(\frac{\pi}{5}\right)}$$

$$\frac{\text{opp}}{h} = \boxed{\frac{104}{h} = \sin\left(\frac{\pi}{5}\right)}$$

$$104 = h \sin\left(\frac{\pi}{5}\right)$$

$$\frac{104}{\sin(\pi/5)} = h \approx 176.94$$



$h \approx 176.94.$

(1)

$$104^2 + a^2 = h^2.$$

$$a^2 = h^2 - 104^2.$$

$$a = \sqrt{h^2 - 104^2}.$$

$$a \approx 143.15.$$

(2)

$$\frac{opp}{hyp} = \sin\theta$$

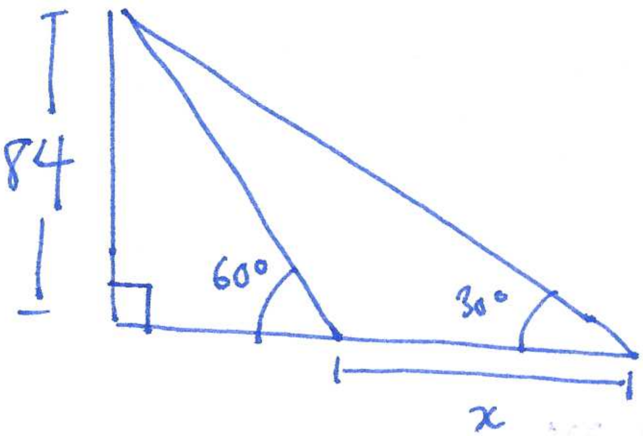
$$\frac{adj}{hyp} = \cos\theta$$

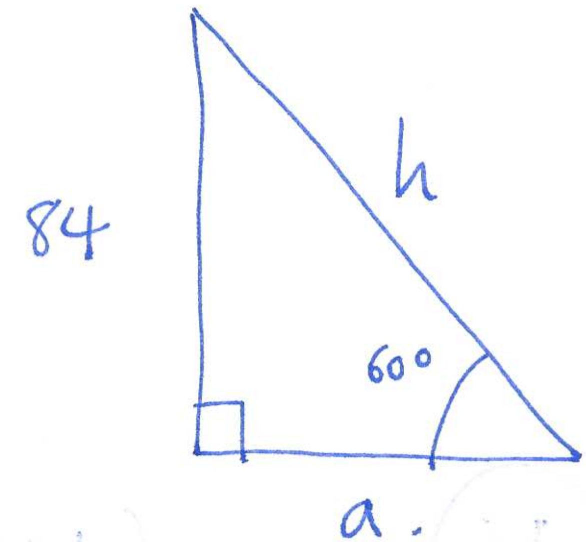
$$\frac{\sin\theta}{\cos\theta} = \frac{opp/hyp}{adj/hyp} = \frac{opp}{adj}.$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{104}{a} = \tan\left(\frac{\pi}{5}\right) \qquad \frac{104}{\tan\left(\frac{\pi}{5}\right)} = a$$

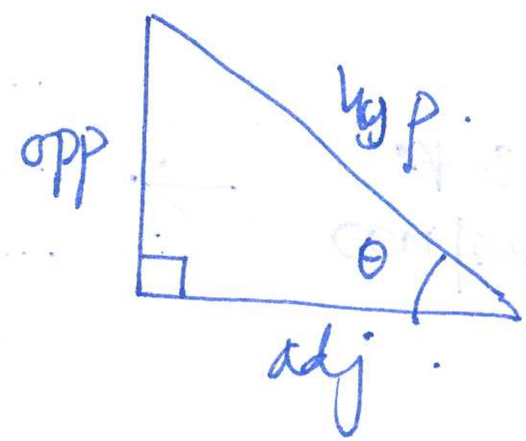
$$a \approx 143.14.$$





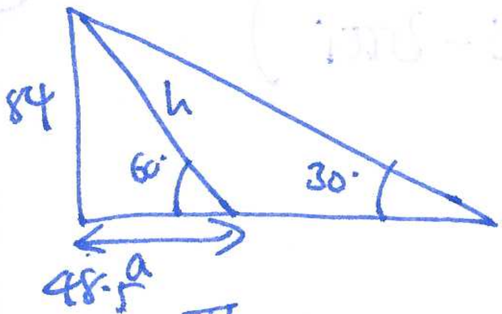
$$h^2 = 84^2 + a^2$$

$$\sin(60°) = \frac{opp}{hyp} = \frac{84}{h}.$$



$$\cos(60°) = \frac{adj}{hyp} = \frac{a}{h}.$$

$$\tan(60°) = \frac{\sin(60°)}{\cos(60°)} = \frac{opp}{adj} = \frac{84}{a}.$$

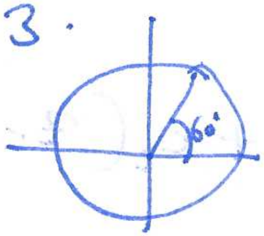


$$\sin(60^\circ) = \frac{84}{h}.$$

$$h\sin(60^\circ) = 84$$

$$h = \frac{84}{\sin(60^\circ)} = \frac{84}{\sqrt{3}/2}.$$

$$60^\circ \leftrightarrow \frac{\pi}{3}.$$



θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$.
$\sin\theta$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$.

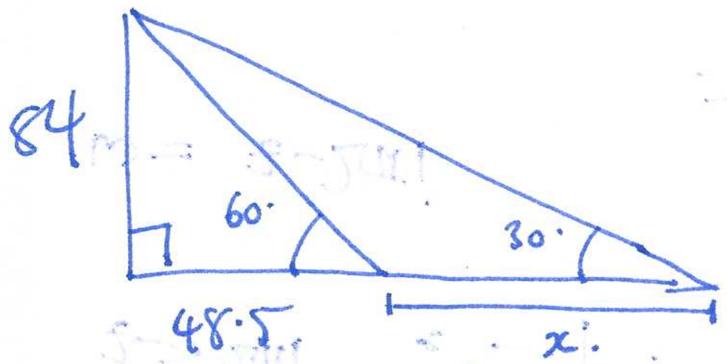
$$\sin(60^\circ) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}.$$

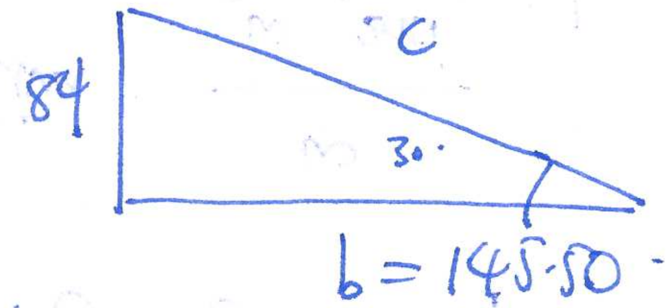
$$h^2 = 84^2 + a^2.$$

$$a^2 = h^2 - 84^2.$$

$$a = \sqrt{h^2 - 84^2} = \sqrt{\frac{84^2}{(\sqrt{3}/2)^2} - 84^2}.$$

$$\approx 48.50.$$





$30° \leftrightarrow \frac{\pi}{6}$.

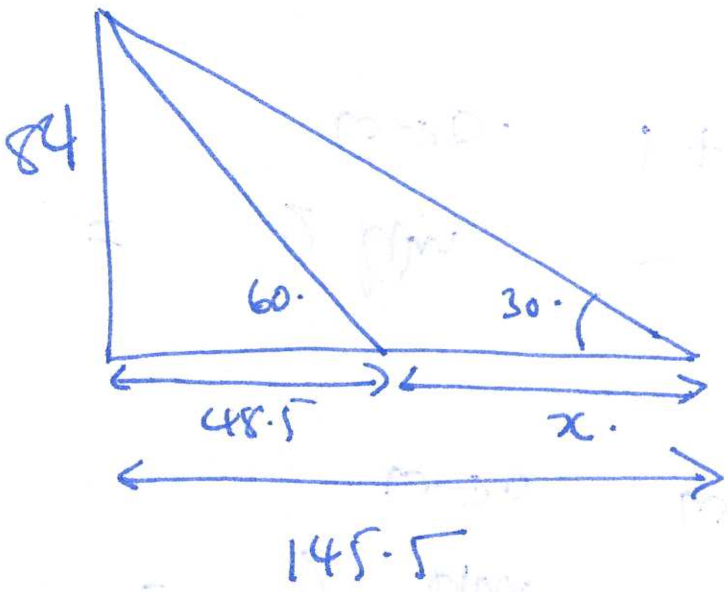
$$\frac{\sin(30°)}{\cos(30°)} = \tan(30°) = \frac{84}{b}.$$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos\theta$	1	$\sqrt{3}/2$			

$$\tan\left(\frac{\pi}{6}\right) = \frac{\sin(\pi/6)}{\cos(\pi/6)} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}.$$

$$\frac{1}{\sqrt{3}} = \frac{84}{b}.$$

$$\frac{b}{\sqrt{3}} = 84 \qquad \boxed{b = \sqrt{3} \times 84}$$



$$x = 145.5 - 48.5$$

$$x = 97.$$

Q1 a) $f(x) = \sin(x^2+1)$.

Q: is this even, odd, neither?

a function $f(x)$ is <u>even</u> if $f(x) = f(-x)$

<u>odd</u> if $f(-x) = -f(x)$.

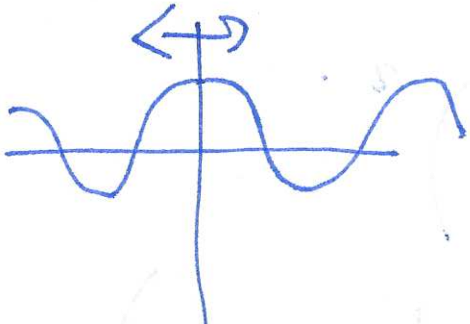
<u>otherwise</u> its <u>neither</u>.

even	odd	neither
$f(-x) = f(x)$	$f(-x) = -f(x)$	

$\boxed{f(x) = x^2}$ even

$f(-x) = (-x)^2 = x^2$

$= f(x)$



symmetric in reflection in y-axis

even $\boxed{\cos(x)}$

$\cos(-x) = \cos(x)$.

$\boxed{f(x) = x}$ odd.

$f(-x) = -x = -f(x)$

$\boxed{f(x) = x^3}$.

$f(-x) = (-x)^3$.

$= (-1)^3 \times x^3$.

$= -x^3$.

$= -f(x)$.

$\boxed{\sin(x)}$ odd.

half-turn symmetry.

$\boxed{f(x) = x+1}$.

$f(-x) = -x+1$.

$\neq f(x)$.

$\neq -f(x)$

$-x-1$.

neither.

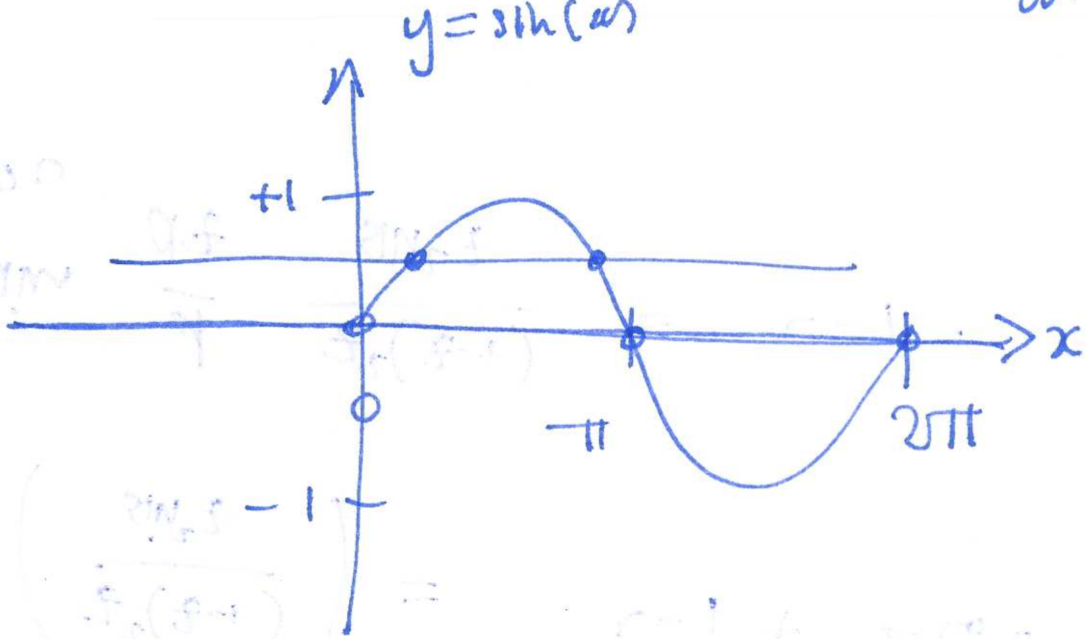
Q $\sin(x^2+1) = f(x)$ even? odd? neither?

$$f(-x) = \sin((-x)^2+1)$$

$$= \sin(x^2+1) = f(x).$$

so $\sin(x^2+1)$ is <u>even</u>.

b) $f(x) = \sin(x)$ Q: is this one-to-one on the interval $[0, 2\pi]$.



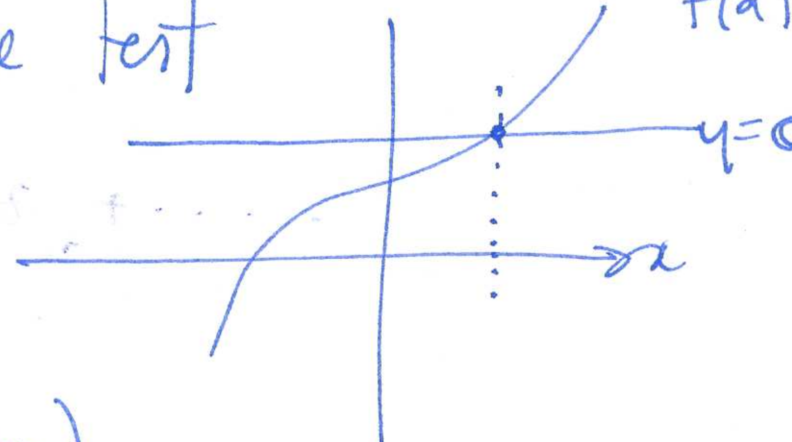
one-to-one

injective

$f(a) = f(b)$

$\Rightarrow a = b$.

horizontal line test



not one-to-one.

$\sin(0) = 0 = \sin(\pi) = \sin(2\pi)$.

c) for any functions $f(x)$ and $g(x)$

$$f \circ g(x) = g \circ f(x)$$

$$f(g(x)) \stackrel{?}{=} g(f(x)) \qquad \underline{\text{no}}.$$

example. $f(x) = x+1$

$g(x) = x^2$.

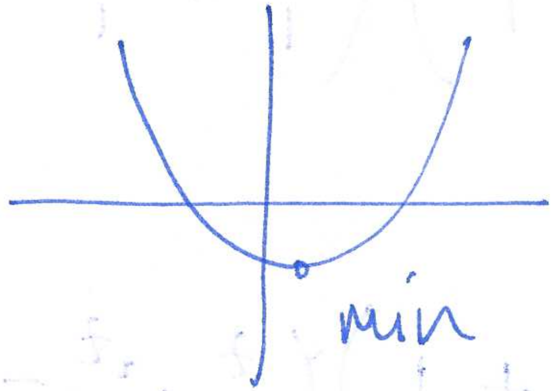
$f(g(x)) = x^2+1$ $\qquad$ $g(f(x)) = (x+1)^2$

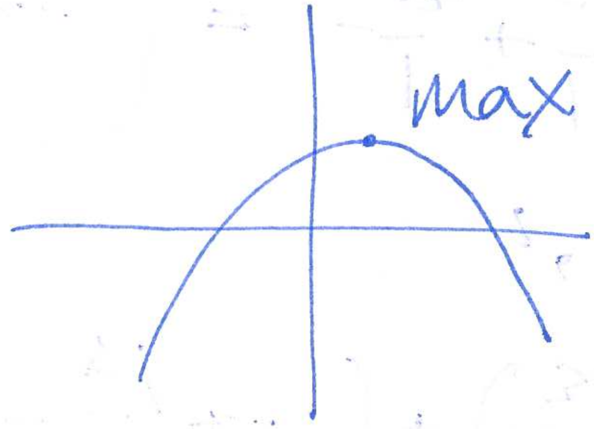
$= x^2+2x+1$

$\neq$

d) A quadratic function always has an absolute max or min on the interval $(-\infty, \infty)$.

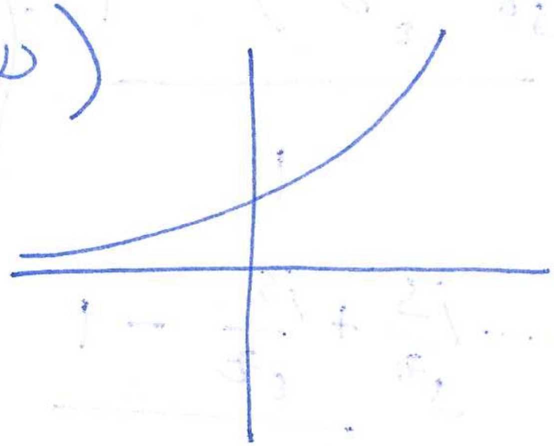
$\boxed{\text{yes}}$

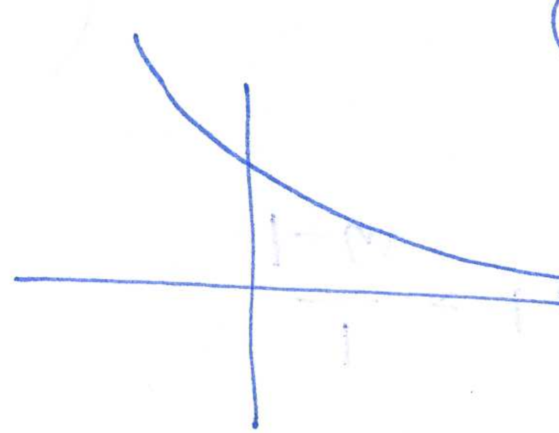




e) exponential functions have abs max/min on $(-\infty, \infty)$

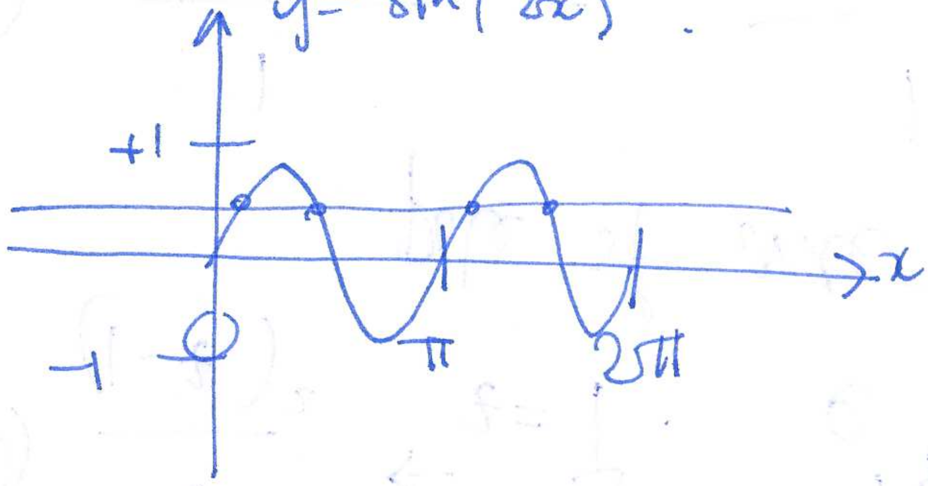
no





$f(x) = \sin(2x)$ Q: does this have an inverse on $[0, \cancel{2\pi}]$
2π.

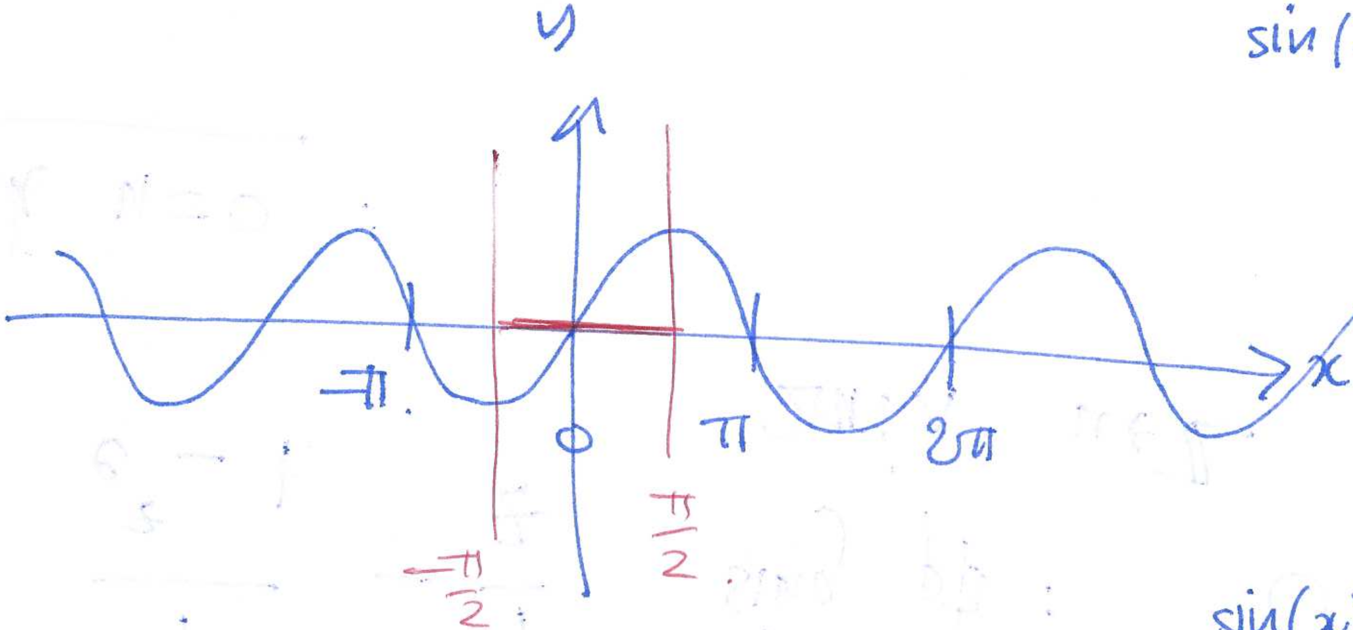
$y = \sin(2x)$.



$f(x)$ has an inverse if it is one-to-one.

horizontal line test

<u>not</u> one-to-one $\Rightarrow$ no inverse on $[0, 2\pi]$.



sin(x)

not one-to-one

no inverse.

sin(x) not one-to-one on $(-\infty, \infty)$.

$$\sin^{-1}(x) = \arcsin(x)$$

restrict $\sin(x)$ to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

then it is one-to-one, and has an inverse.

$$\sin(x) : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1].$$

$$\sin^{-1}(x) : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

20

9) simplify $\sqrt[3]{(x^3y)^2 y^4}$

$\sqrt[4]{x} = x^{1/4}$.

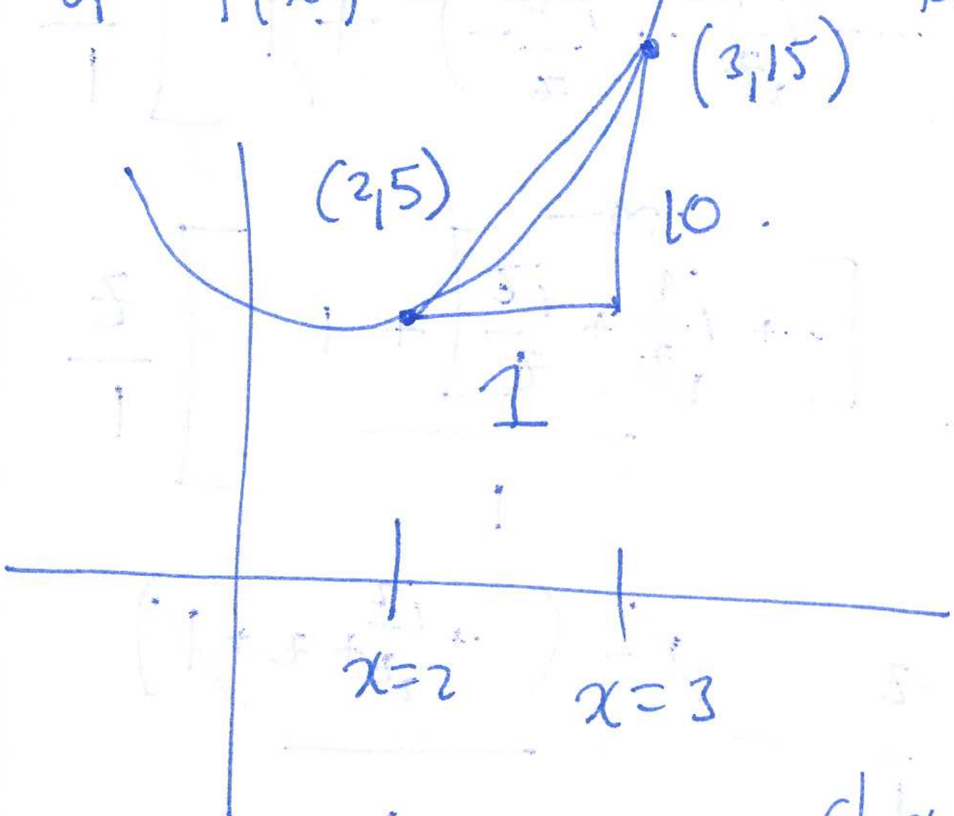
$= \left(x^6 y^2 y^4\right)^{1/3}$.

$(x^a)^b = x^{ab}$.

$= \left(x^6 y^6\right)^{1/3} =$

$= x^{6/3}\, y^{6/3} = x^2 y^2$

h) what is the average rate of change of $f(x) = 2x^2 - 3$ between $x = 2$ and $x = 3$.

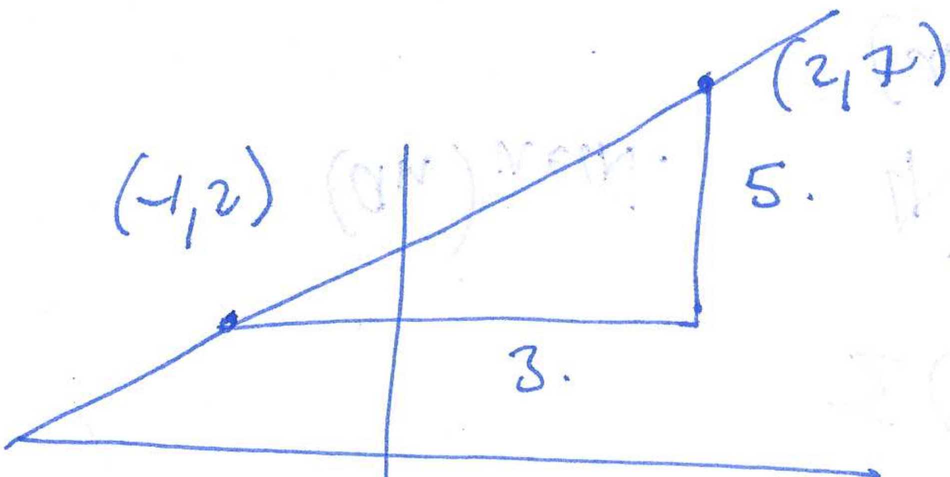


$$f(2) = 2(2)^2 - 3$$
$$= 2 \times 4 - 3 = 5$$

$$f(3) = 2(3)^2 - 3$$
$$2 \times 9 - 3$$
$$= 15$$

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{10}{1} = 10.$$

Q2 a) find the slope and equation of the line through $(-1,2)$ and $(2,7)$



$y = mx+b.$

$y - y_0 = m(x - x_0).$

find slope

$$m = \frac{\text{diff } y}{\text{diff } x} = \frac{7-2}{2-(-1)}.$$

$$\boxed{y = \frac{5x}{3} + \frac{11}{3}}$$

$$m = \frac{5}{3}.$$

$$y - 7 = \frac{5}{3}(x-2) \rightsquigarrow$$

$$y - 7 = \frac{5x}{3} - \frac{10}{3}$$

$$y = \frac{5x}{3} + 7 - \frac{10}{3} \quad \frac{11}{3}.$$

$$f(x) = 7 - 8x - 2x^2$$

complete the square
/ put in standard form
$a(x-h)^2 + k$.

$$-2x^2 - 8x + 7$$

$$-2\left[x^2 + 4x - \frac{7}{2}\right] \qquad \boxed{-2(x+2)^2 + 1}$$

$$-2\left[(x+2)^2 - \frac{1}{2}\right]$$

$$(x+2)^2 \qquad -4 + \frac{7}{2}$$

$$x^2 + 4x + 4 - 4 + \frac{7}{2}$$

$$\frac{-8+7}{2} = -\frac{1}{2}$$

max

$(-2, 1)$