half life / expaential decay: radon-222

in 3 days decays to 50%. a) half life?

b) 20%.?

t time

R(t) amount of radon 222.

$R(0) = 1$ $\qquad R(t) = \underbrace{A}_{=1} e^{-bt}$

$R(t) = e^{-bt}$.

$$R(t) = e^{-bt}$$

$t = 3 \qquad R(t) = 0.58$

$$0.58 = e^{-3b}.$$

$$\frac{\ln(0.58)}{-3} = \ln\left(e^{-3b}\right). = \frac{-3b}{-3}.$$

$$b = \frac{\ln(0.58)}{-3.} \simeq 0.18\ldots$$

$$R(t) = e^{\frac{\ln(0.58)}{3} t}.$$

a) half ~~life~~ life: $R(t) = \frac{1}{2}$.

$$e^{\frac{\ln(0.58)}{3} t} = \frac{1}{2}.$$

$$\ln\left(e^{\frac{\ln(0.58)}{3} t}\right) = \ln(1/2).$$

$$\frac{\ln(0.58)}{3} t = \ln(1/2).$$

$$t = \frac{3\ln(1/2)}{\ln(0.58)} \approx 3.82\ldots$$

↑ half life in days.

b) How long until 20%. ?

$$R(t) = 20\% = 0.2$$

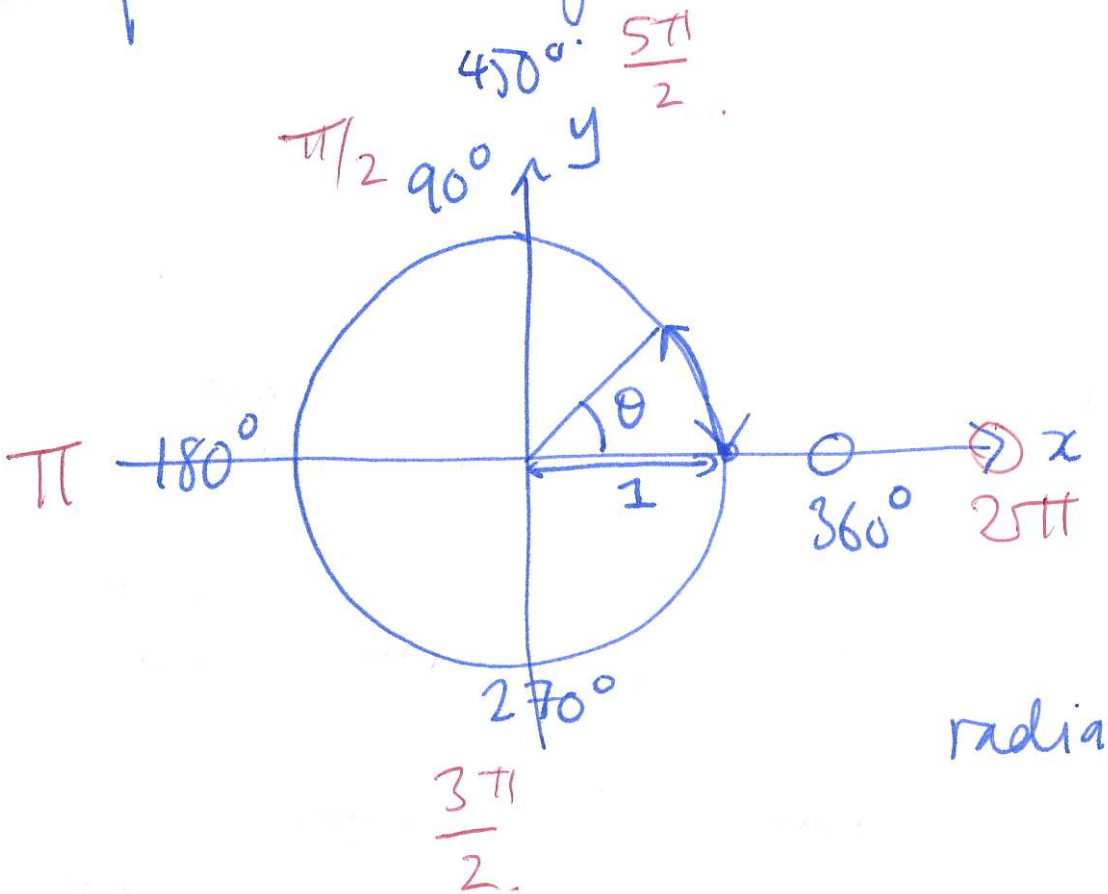
$$e^{\frac{\ln(0.58)}{3}t} = 0.2.$$

$$\ln\left(e^{\frac{\ln(0.58)}{3}t}\right) = \ln(0.2)$$

$$\frac{\ln(0.58)}{3}\, t = \ln(0.2)$$

$$t = \frac{3\ln(0.2)}{\ln(0.58)} \approx 8.86 \text{ days.}$$

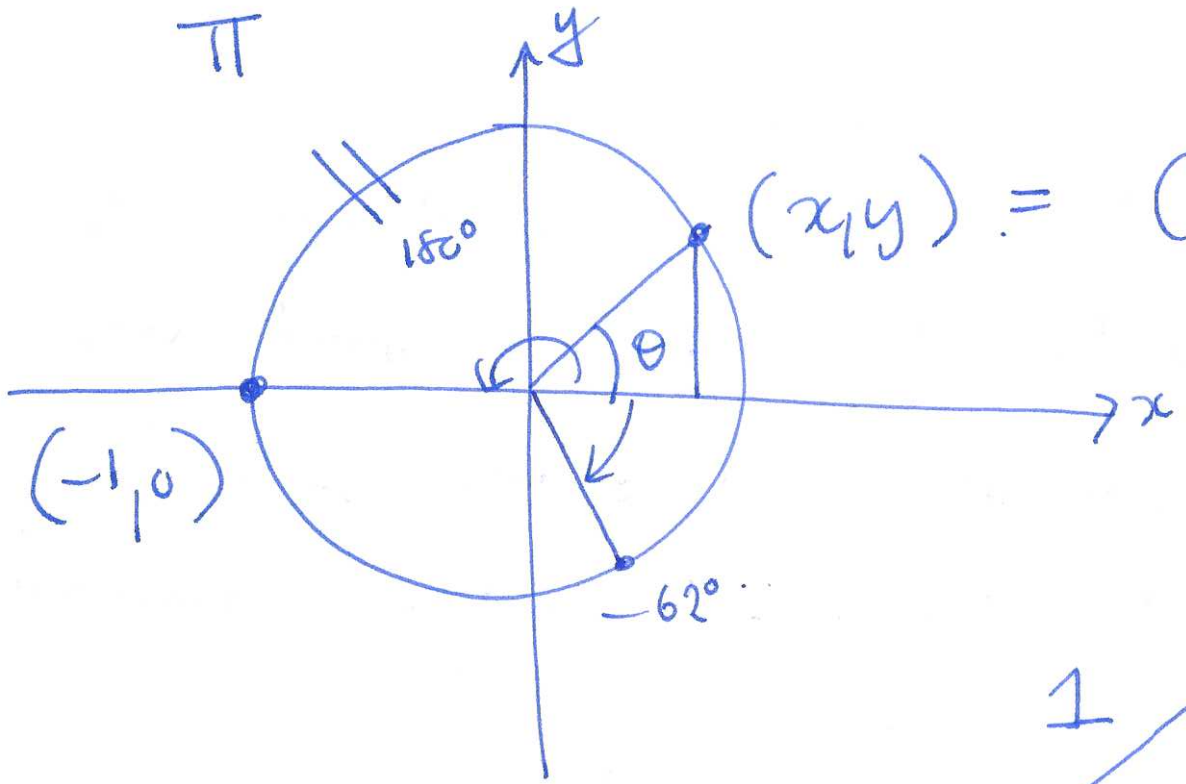
angles : degrees vs radians



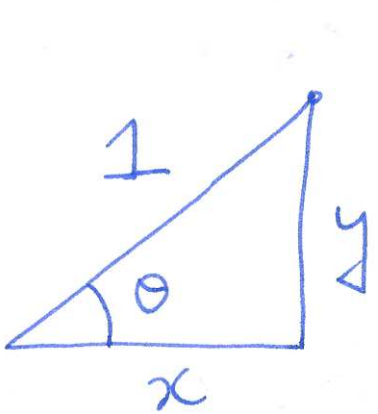
$360^\circ = 2\pi$ radians.

$$\text{radians} = \frac{\text{degrees}}{360} \times 2\pi.$$

$$\text{degrees} = \frac{\text{radians} \times 360}{2\pi}$$



$(x, y) = (\cos\theta, \sin\theta)$

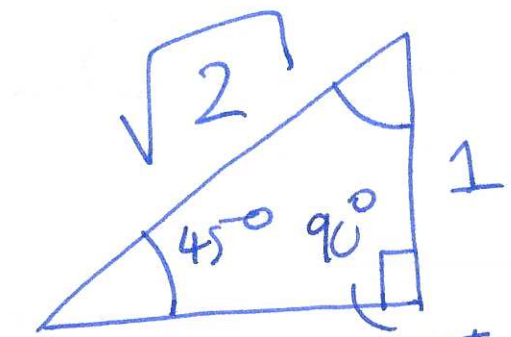


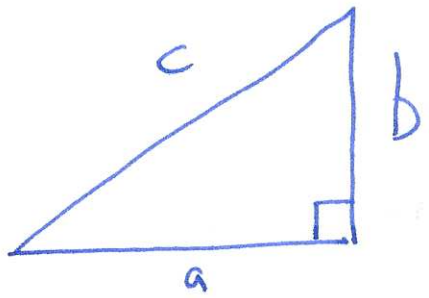
$\sin\theta = y$

$\cos\theta = x$

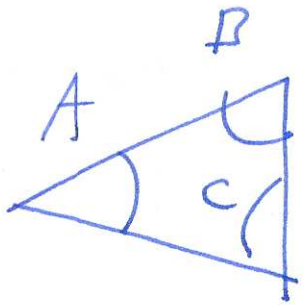
$\cos(180°) = \cos(\pi) = -1.$

$\sin(180°) = \sin(\pi) = 0.$





$$a^2 + b^2 = c^2$$



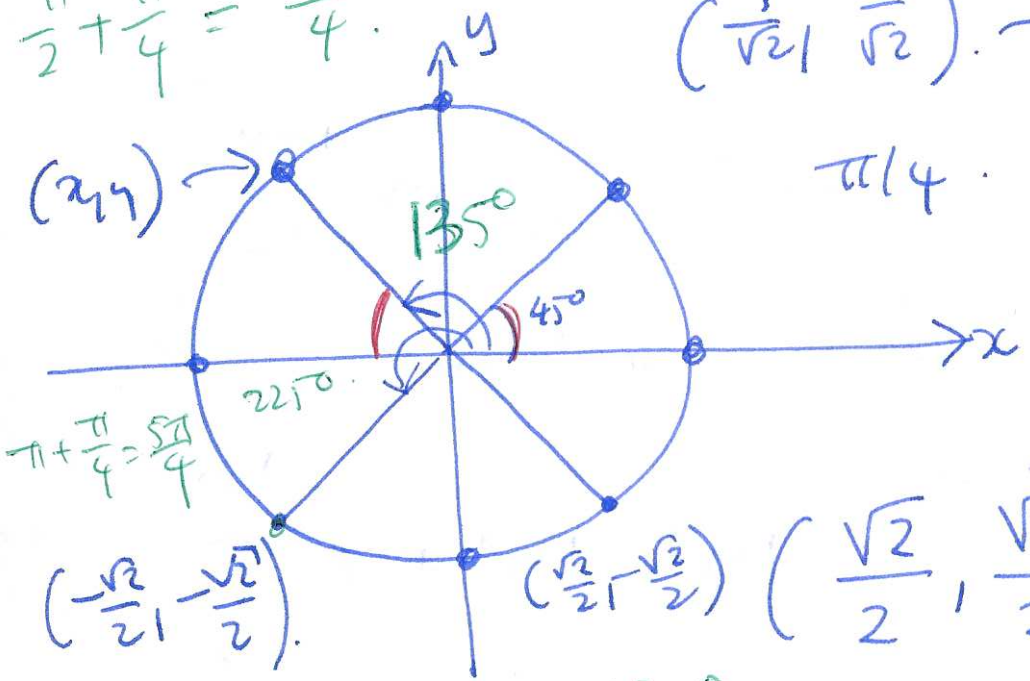
$$A + B + C = 180^\circ$$

$$\pi.$$

$$\sin(45^\circ) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$$

$\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$.



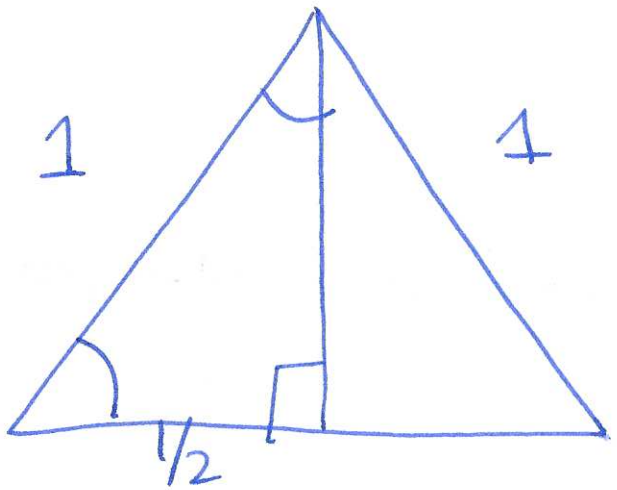
$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{2}}{2}$

135°.

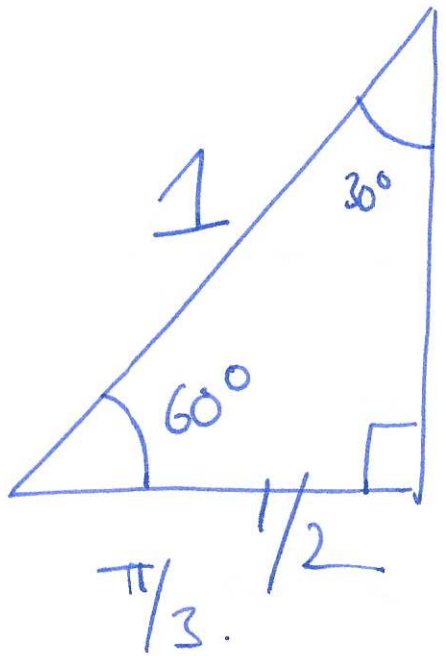
$(x, y) = \left(\cos\left(\frac{3\pi}{4}\right), \sin\left(\frac{3\pi}{4}\right)\right)$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.



$\frac{180^\circ}{3} = 60^\circ$

$\frac{\pi}{3}$.



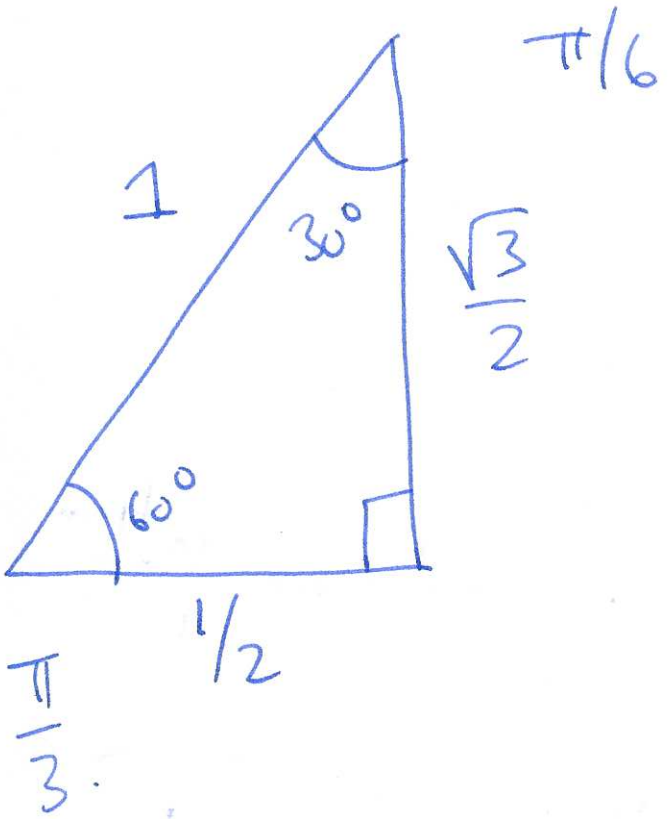
$\frac{\pi}{6}$.

$b \cdot \frac{\sqrt{3}}{2}$.

$b^2 + \left(\frac{1}{2}\right)^2 = 1^2$.

$b^2 + \frac{1}{4} = 1$.

$b^2 = \frac{3}{4}$ $\quad b = \frac{\sqrt{3}}{2}$.

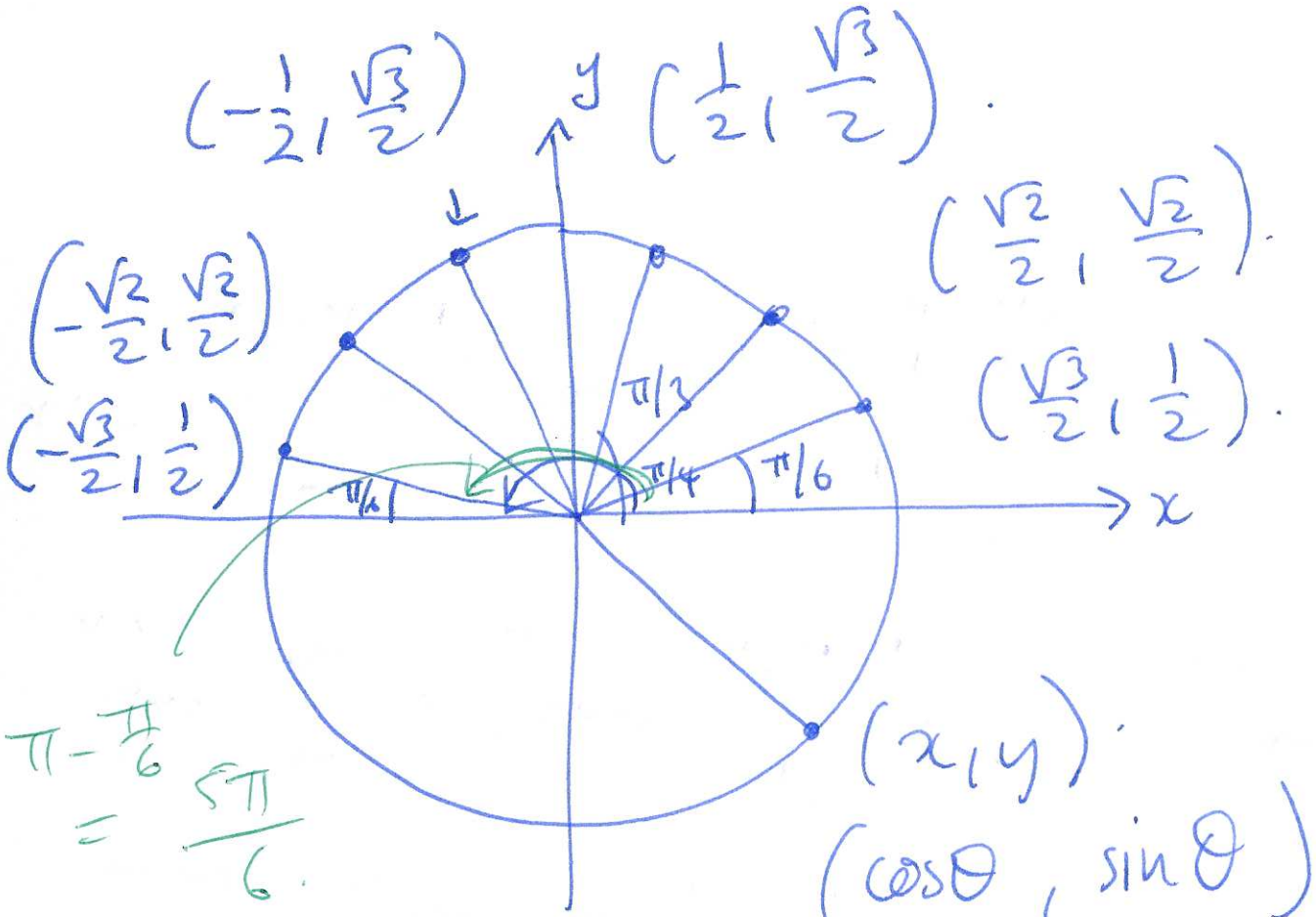


$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}.$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

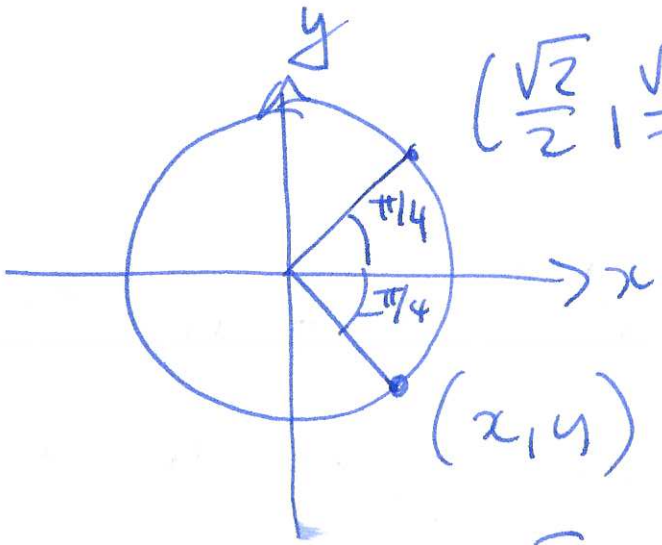
$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\pi/3\right) = \frac{1}{2}.$$


$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$
y
$(\frac{1}{2}, \frac{\sqrt{3}}{2})$.
$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$.
$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$.
$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$
π/3
π/4
π/6
π/6
x
$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.
(x, y).
$(\cos\theta, \sin\theta)$

mnemonic for special values.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0



$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\theta \leftrightarrow t$

a) $\theta = -\frac{\pi}{4}$.

(x, y)

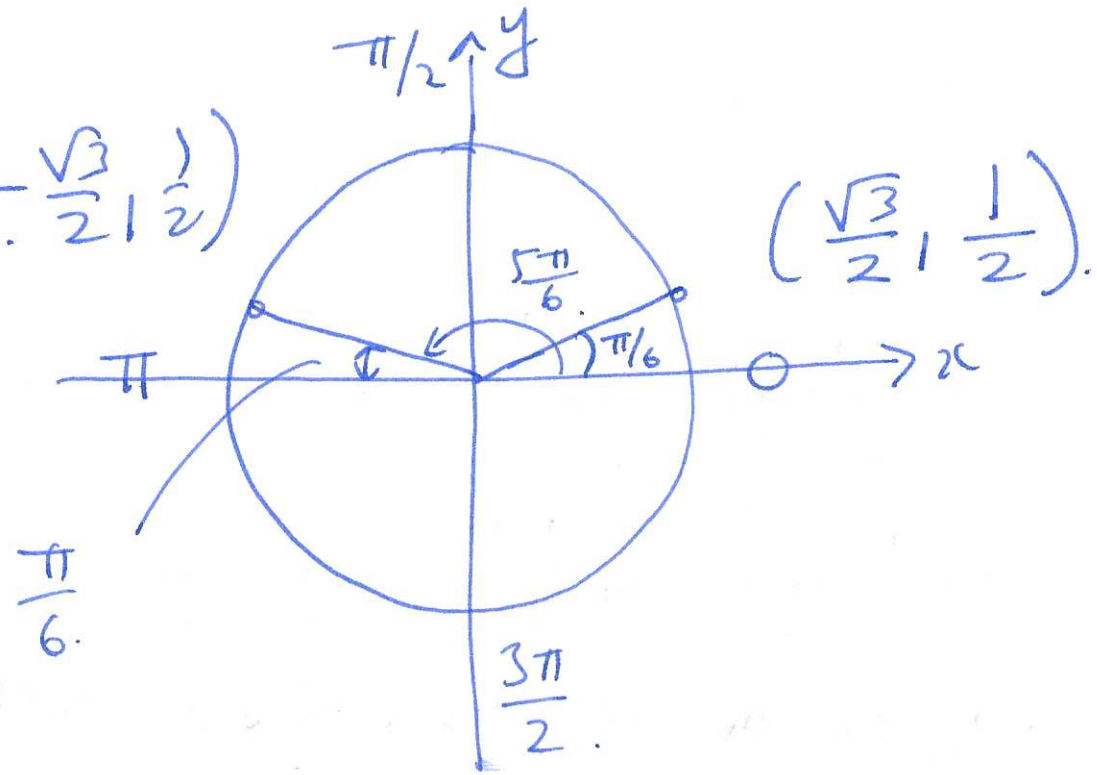
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

$(\cos\theta, \sin\theta)$.

$$\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}.$$

$\frac{5\pi}{6}$



$\frac{1}{2} < \frac{5}{6} < 1$

$\pi - \frac{5}{6}\pi$

$= \pi\left(1 - \frac{5}{6}\right)$

$= \pi \frac{6-5}{6} = \frac{\pi}{6}$.

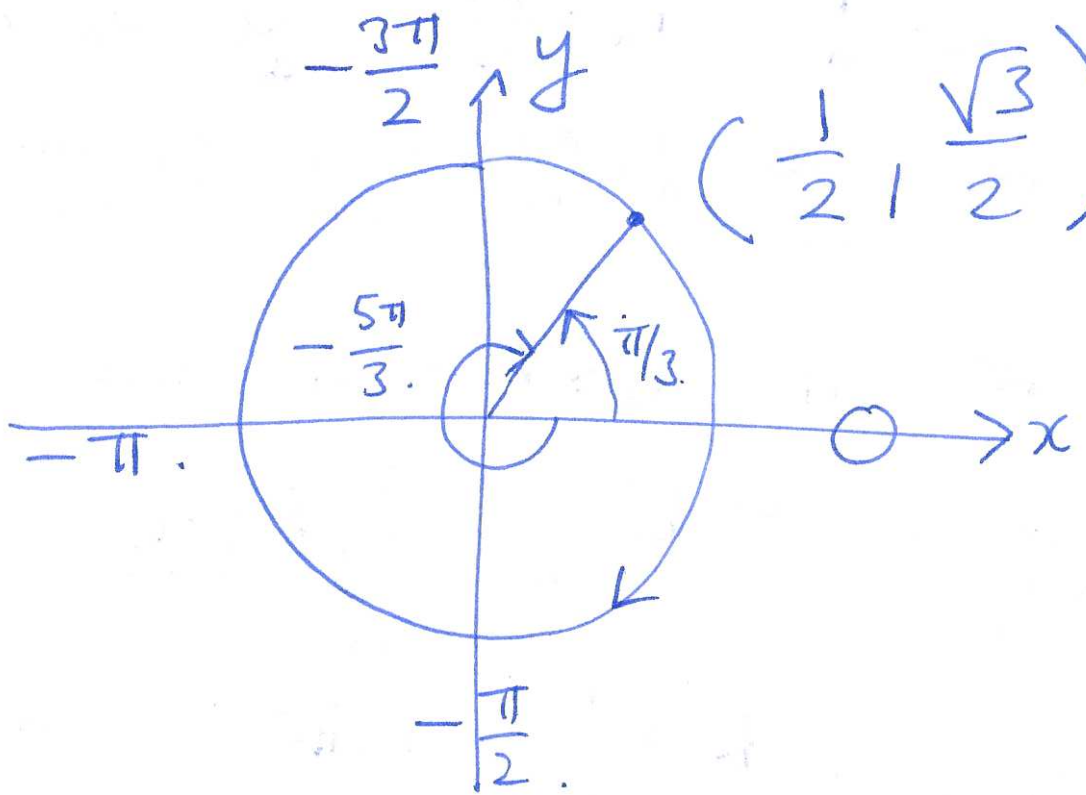
$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$.

$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$.

$$-\frac{5\pi}{3} > -\pi - \frac{2\pi}{3}.$$

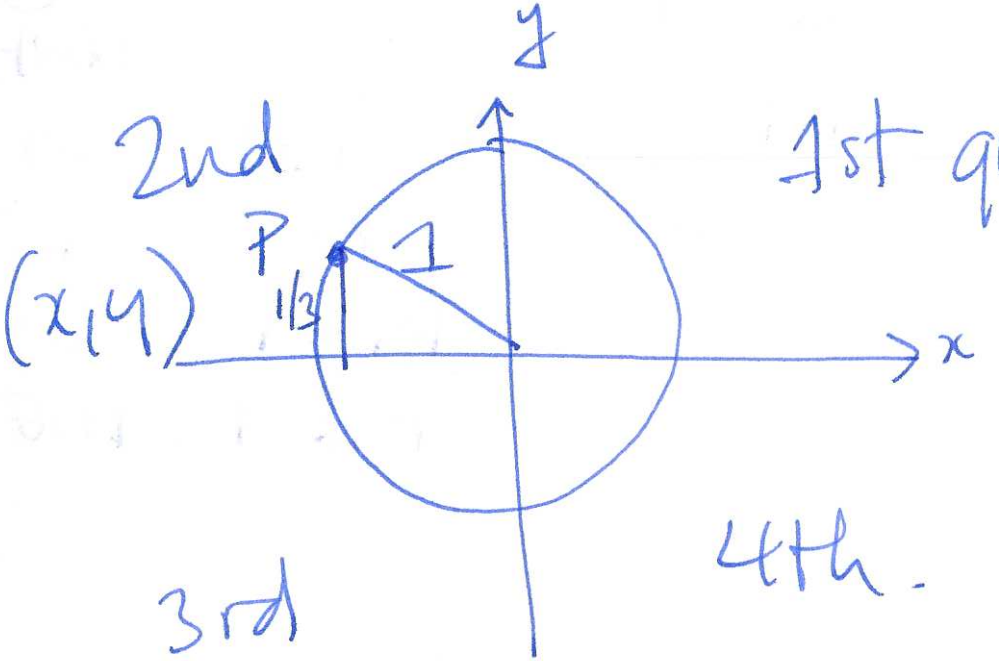
$$2\pi - \frac{5\pi}{3}.$$

$$\frac{6\pi - 5\pi}{3} = \frac{\pi}{3}$$



$$\cos\left(-\frac{5\pi}{3}\right) = \frac{1}{2}.$$

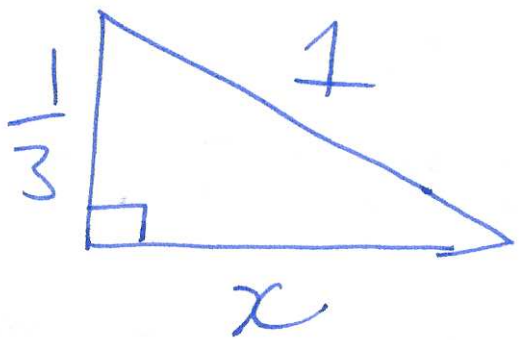
$$\sin\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{2}.$$



find coordinates of P if P is in the second quadrant and y-coord is 1/3.

P has coordinates (x, y).

$y = \frac{1}{3}$.



$x^2 + \left(\frac{1}{3}\right)^2 = 1$.

$x^2 + \frac{1}{9} = 1$.

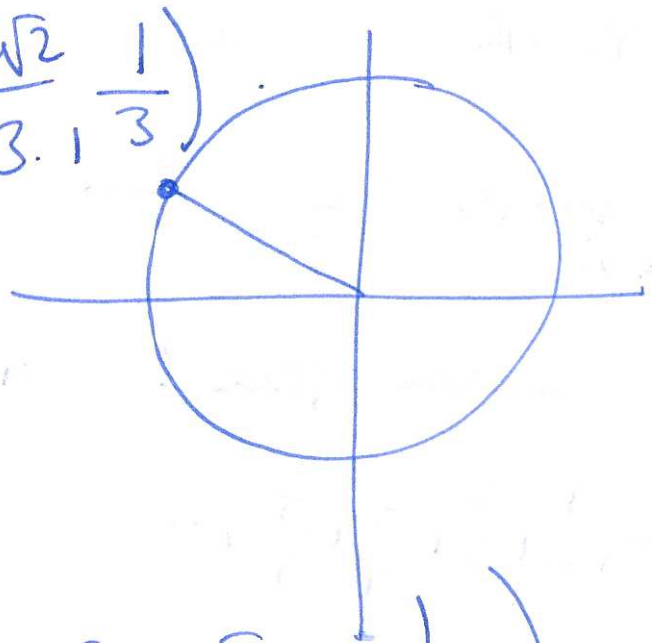
$$x^2 + \frac{1}{9} = 1$$

$$x^2 = 1 - \frac{1}{9} = \frac{9-1}{9} = \frac{8}{9}.$$

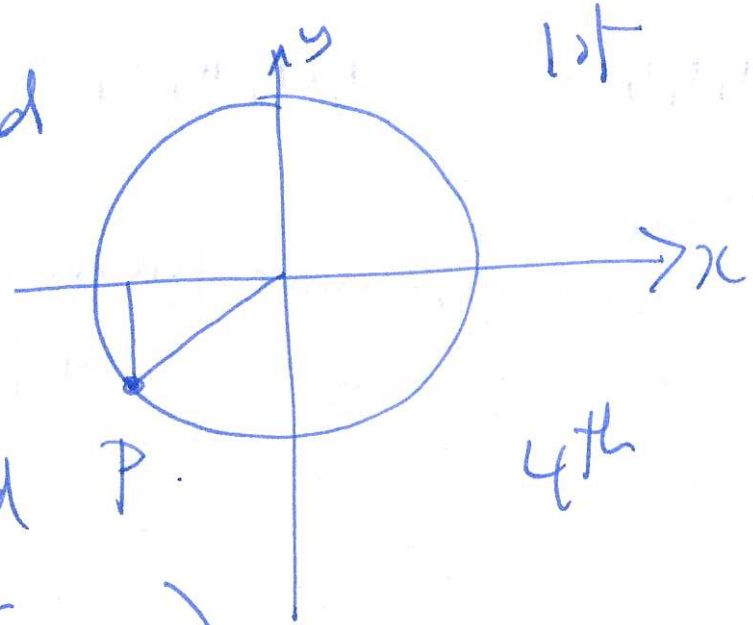
$$x^2 = \frac{8}{9}$$

$$x = \pm\sqrt{\frac{8}{9}} = \pm\frac{\sqrt{8}}{3}.$$

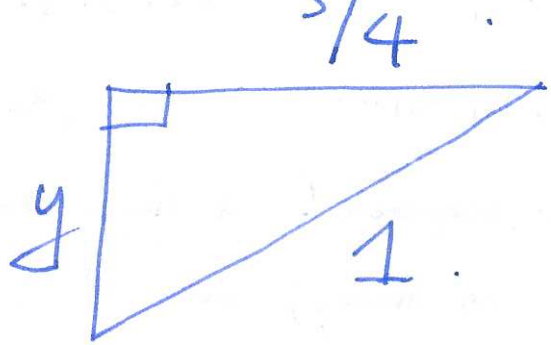
$$= \pm\frac{\sqrt{4}\sqrt{2}}{3} = \pm\frac{2\sqrt{2}}{3}.$$



P is in the 3rd quadrant and $x = -\frac{3}{4}$.



(x, y)



$y^2 + \left(\frac{3}{4}\right)^2 = 1$.

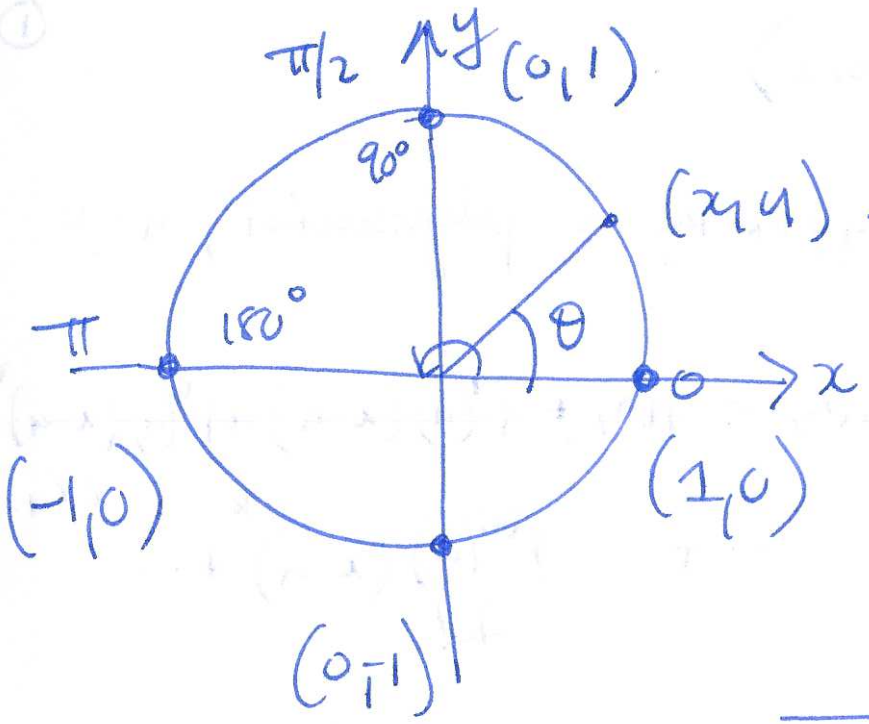
$y^2 + \frac{9}{16} = 1$.

$\left(-\frac{3}{4}, y\right) = \left(-\frac{3}{4}, -\frac{\sqrt{7}}{4}\right)$

$y^2 = 1 - \frac{9}{16} = \frac{16-9}{16} = \frac{7}{16}$.

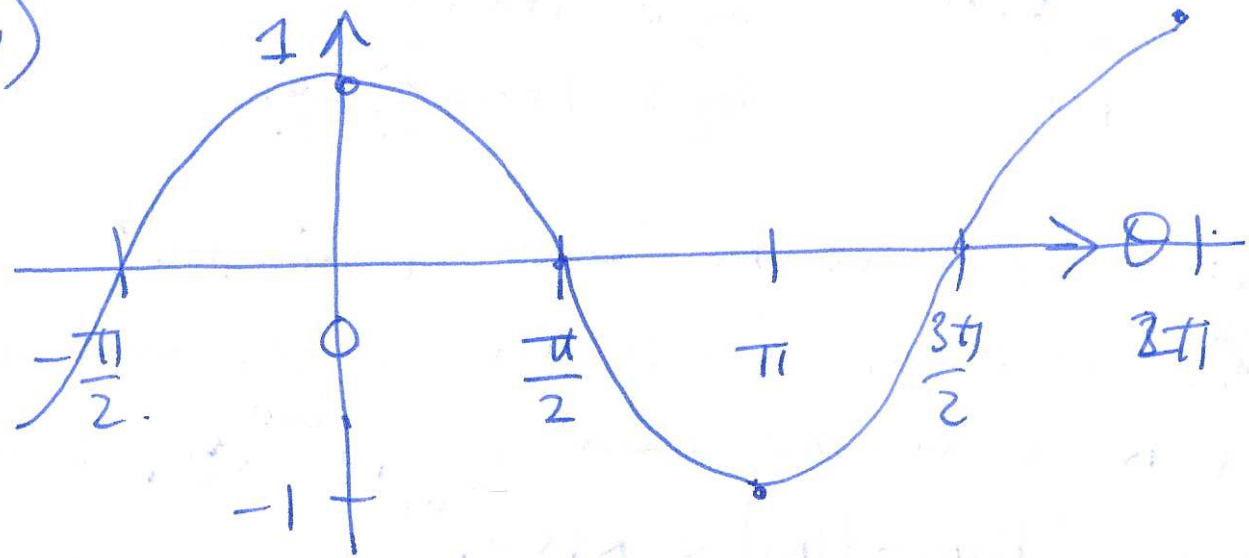
$y = \pm \frac{\sqrt{7}}{4}$.

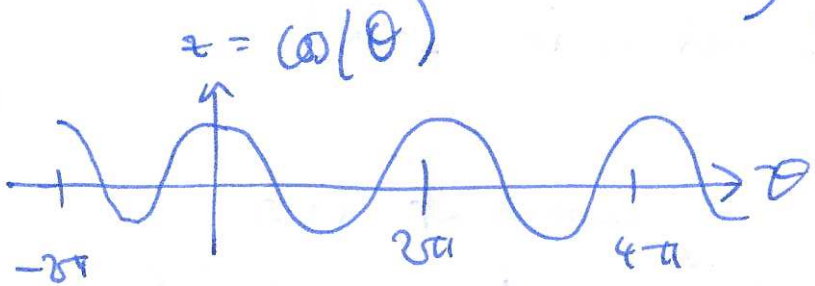
$y = -\frac{\sqrt{7}}{4}$.

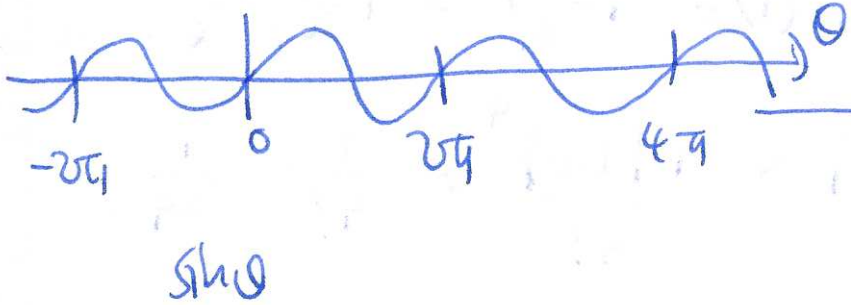


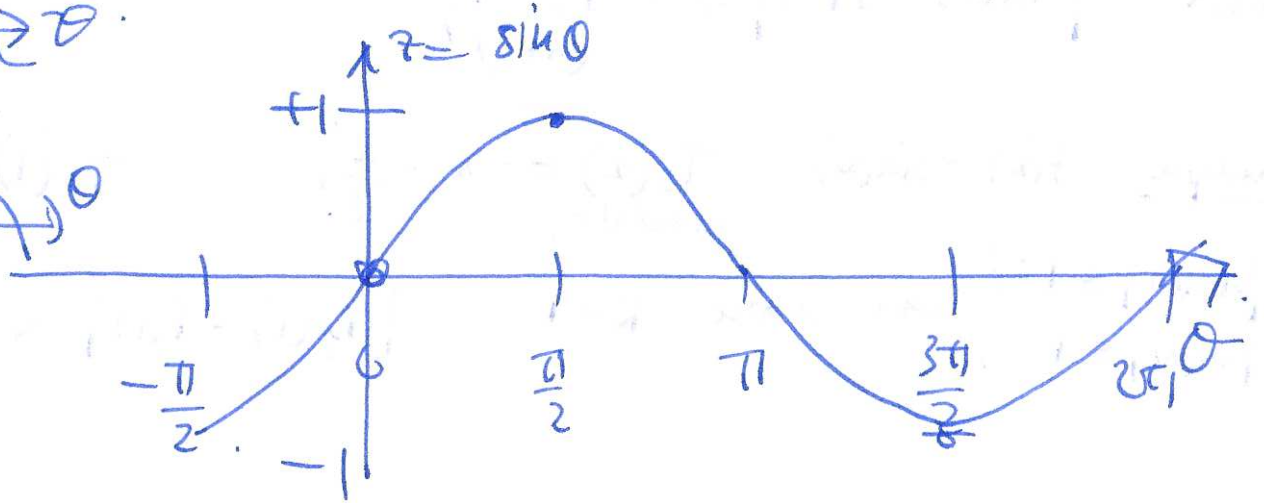
$(x,y) \leftrightarrow (\cos\theta, \sin\theta)$

$z = \cos(\theta)$

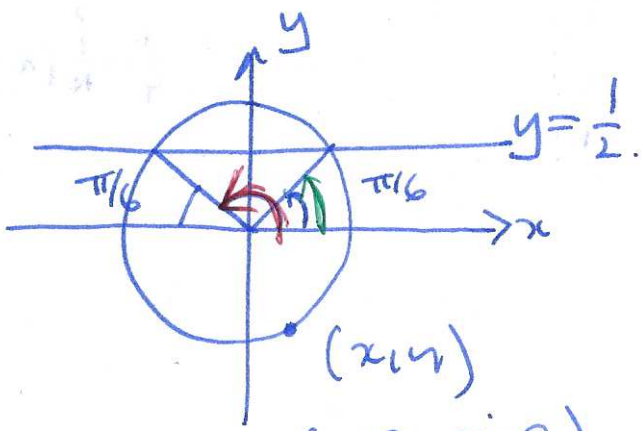


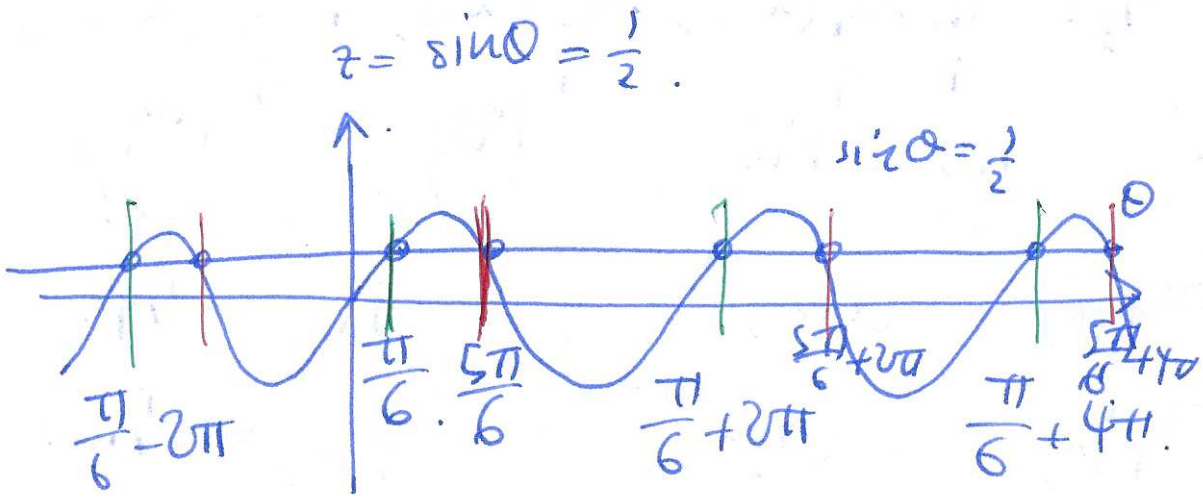






find all solutions of $\sin\theta = \frac{1}{2}$.





θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$.
$\sin\theta$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$.

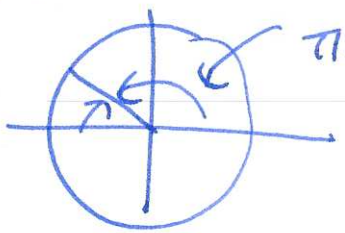
$\frac{1}{2}$.

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$.

$\updownarrow$

$y\text{-coord} = \frac{1}{2}$.



$\pi - \frac{\pi}{6} = \pi\frac{6-1}{6} = \frac{5\pi}{6}$.

all solutions

$$\frac{\pi}{6} + 2\pi n, \quad n \in \mathbb{Z}$$

$$\frac{5\pi}{6} + 2\pi n.$$