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Q1.

①

$2^{x-1} = 4 = 2^{2}$

$x-1=2$

$x=3$

$2^{x-1} = 5$

$\log_2(2^{x}) =$

$f(x) = x+2$

$f^{-1}(x) = x-2$

$f(x)$ $f^{-1}(x)$

$f^{-1}(f(x)) = x$

$f^{-1}(f(x))$

$f^{-1}(x+2) = (x+2)-2 = x$

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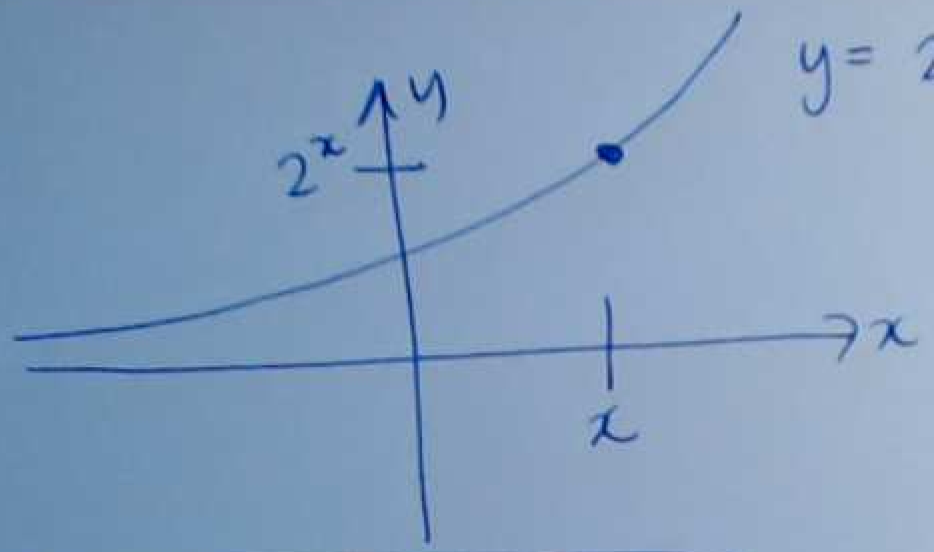
2^x



$y = 2^x$.

inverse function

$\log_2(x)$.

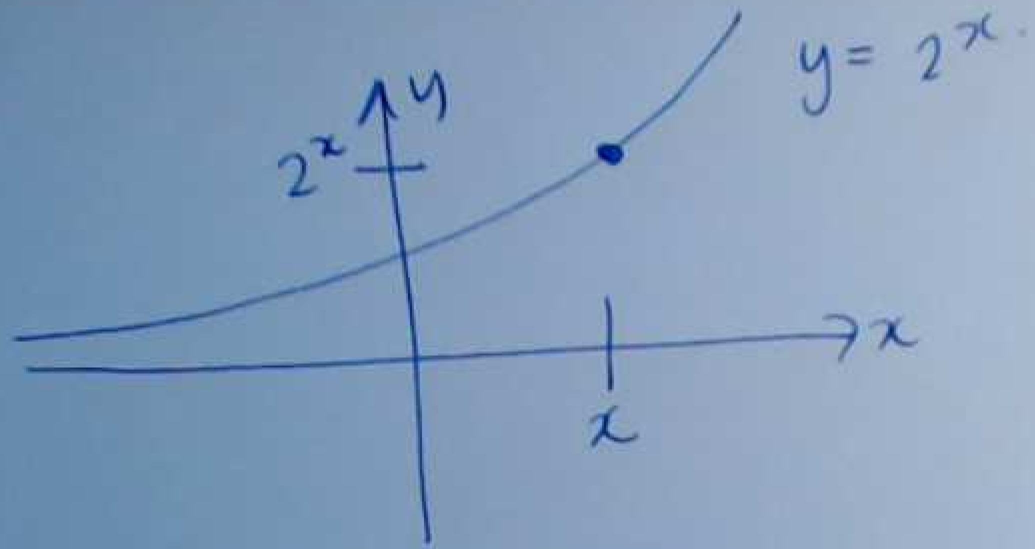
$$\log_2(2^x) = x$$

$$2^{\log_2(x)} = x.$$

$$\log_b(b^x) = x$$

$$b^{\log_b(x)} = x$$

2^x



inverse function

$\log_2(x)$.

$$\boxed{\begin{gathered}\log_2(2^x) = x \\ 2^{\log_2(x)} = x.\end{gathered}}$$

$$\boxed{\begin{gathered}\log_b(b^x) = x \\ b^{\log_b(x)} = x\end{gathered}}$$

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(3)

$$2^{x-1} = 4.$$

$$\log_2\left(2^{x-1}\right) = \log_2(4)$$

$$x - 1 = 2.$$

$$+1 \qquad +1$$

$$x = 3.$$

$\log_{10}(x)$

$\ln(x) = \log_e(x).$

$$2^{x-1} = 5.$$

$$\log_2\left(2^{x-1}\right) = \log_2(5)$$

$$x - 1 = \log_2(5)$$

$$+1 \qquad +1$$

$$x = \log_2(5) + 1.$$

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④

$$x = \log_2(5) + 1$$

$$x = \frac{\ln(5)}{\ln(2)} + 1$$

$2.32\ldots$

$x = 3.32\ldots$

$$\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$$

in particular

$$= \frac{\ln(x)}{\ln(b)}$$

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(5)

Q4.

$$\frac{e^{2x+1}}{2} = \frac{200}{2}$$

$$e = 2.71\ldots$$

$$\ln(e^x) = x$$

$$e^{2x+1} = 200$$

$$e^{\ln(x)} = x$$

$$\ln(x) = \log_e(x)$$

$$\ln(e^{2x+1}) = \ln(200)$$

$$2x+1 = \ln(200)$$

$$-1 \qquad -1$$

$$\frac{2x}{2} = \frac{\ln(200)-1}{2}$$

$$\boxed{x = (\ln(200)-1)/2}$$

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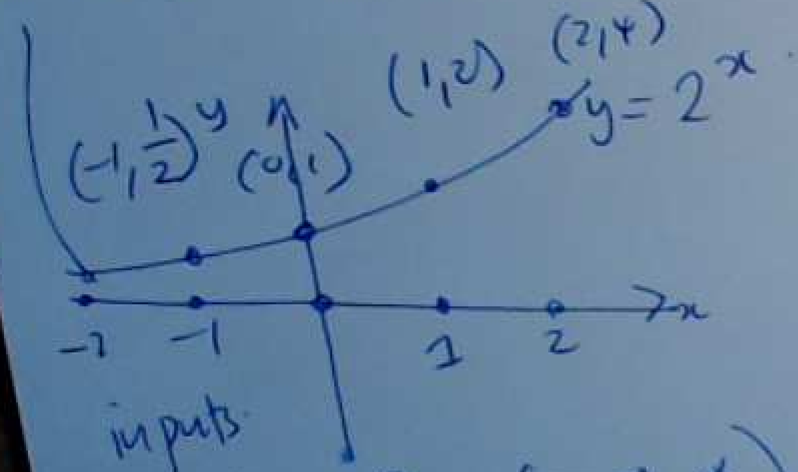
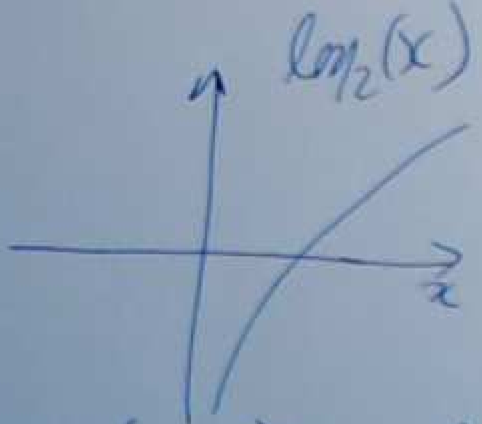
Q5.

$$5^x = 4^{x+1}$$

$$\log_b(5^x) = \log_b(4^{x+1})$$





$1 = 1$

$\ln(1) = \ln(1)$
$= 0$

$-1 = -1$

$\ln(-1) = \ln(-1)$

inputs
domain: $\mathbb{R} = (-\infty, \infty)$ $\quad (0, \infty)$

range
outputs $(0, \infty)$ $\quad \mathbb{R} = (-\infty, \infty)$

$\uparrow$
undefined!

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(7)

$$5^x = 4^{x+1}.$$

$$\log_b(5^x) = \log_b(4^{x+1}).$$

can pick any b! $b = e$.

$$\ln(5^x) = \ln(4^{x+1}).$$

$$\log_{10}(10^x) = x. \qquad \log_{10}(5^x) = ?$$

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(8)

$$x^a \cdot x^b = x^{a+b}.$$

$$(x^a)^b = x^{ab}.$$

$$x^{-a} = \frac{1}{x^a}.$$

$$\log_b(x) = \frac{\log_c(x)}{\log_c(b)}.$$

$$\log_b(b) = 1.$$

$$\log_b(1) = 0$$

$$\log(ab) = \log(a) + \log(b).$$

$$\log(a^b) = b\log(a)$$

$$\log(a+b) = \text{NO RULE}.$$

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$$5^x = 4^{x+1}$$

$\log_2(4) = 2$ 9

$\log_e(4) = -$

$$\ln(5^x) = \ln(4^{x+1})$$

$$\boxed{\ln(a^b) = b\ln(a)}$$

$$x\ln(5) = (x+1)\ln(4)$$

$$x\ln(5) = x\ln(4) + \ln(4)$$

$$-x\ln(4) \qquad -x\ln(4)$$

$$x\ln(5) - x\ln(4) = \ln(4)$$

$$x(\ln(5) - \ln(4)) = \ln(4)$$

$$\boxed{x = \ln(4) / (\ln(5) - \ln(4))}$$

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10

Q6.

$$\log_{10}(x-4) = 2.$$

$$\log_{10}(x-4)+1 = 2+1 = 3.$$

$$\log_{10}\left(\log_{10}(x-4)\right) = \log_{10}(2)$$

$$10^{\log_{10}(x-4)} = 10^2$$

$$x - 4 = 10^2 = 100$$

$+4$ $\quad$ $+4$

$$\boxed{x = 104}$$

$\ln(x)$

$\shortparallel$

$\log_e(x).$

$\log(x).$

$$\boxed{\begin{array}{c} b^{\log_b(x)} = x \\ \log_b(b^x) = x \end{array}}$$

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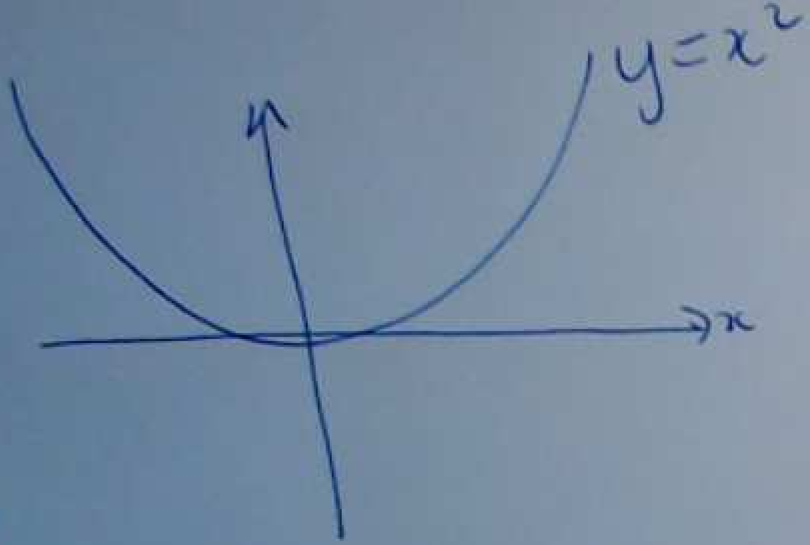
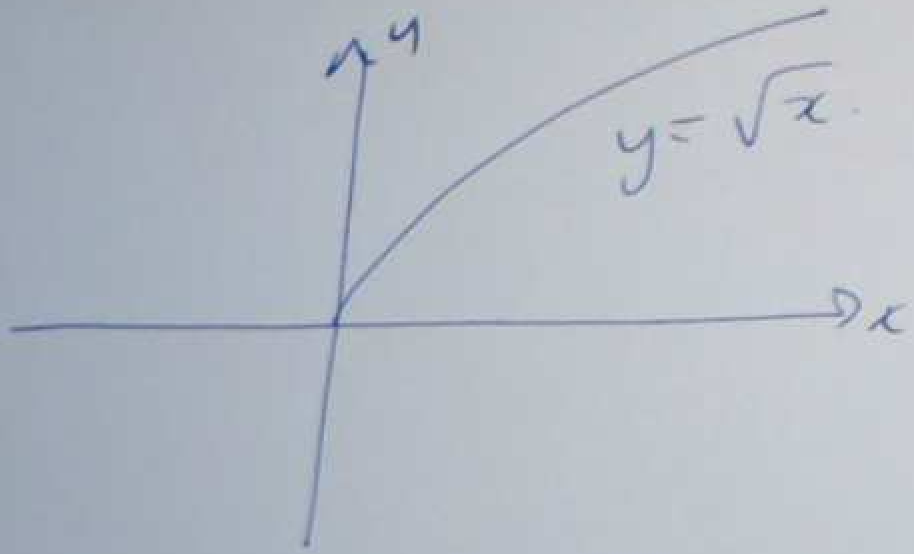
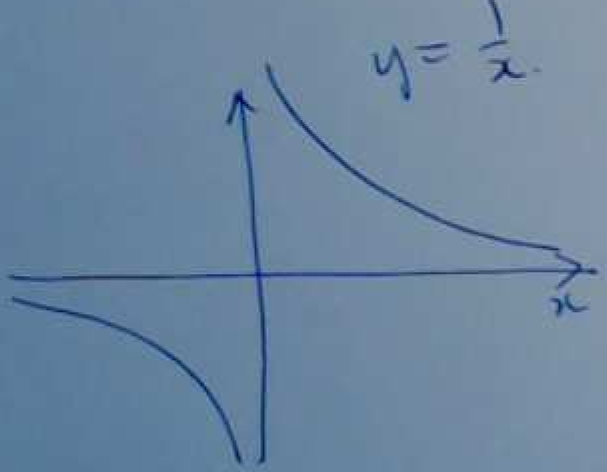
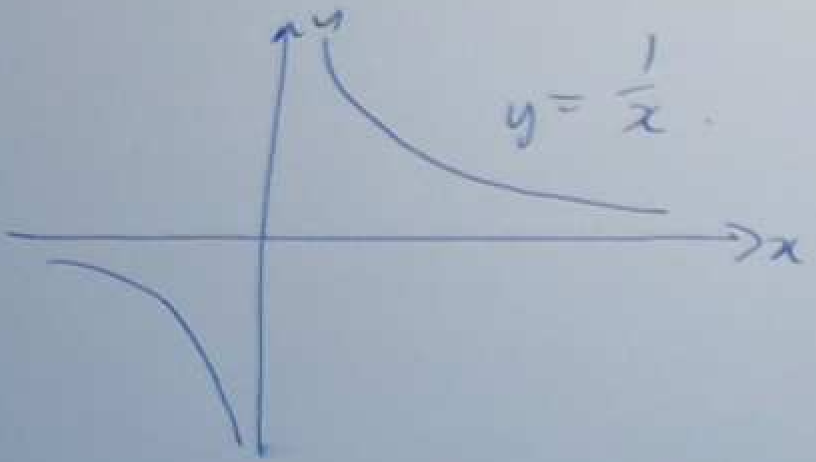

$y = x^2$
x


y
$y = \sqrt{x}$
x


$y = \frac{1}{x}$
x


y
$y = \frac{1}{x}$
x

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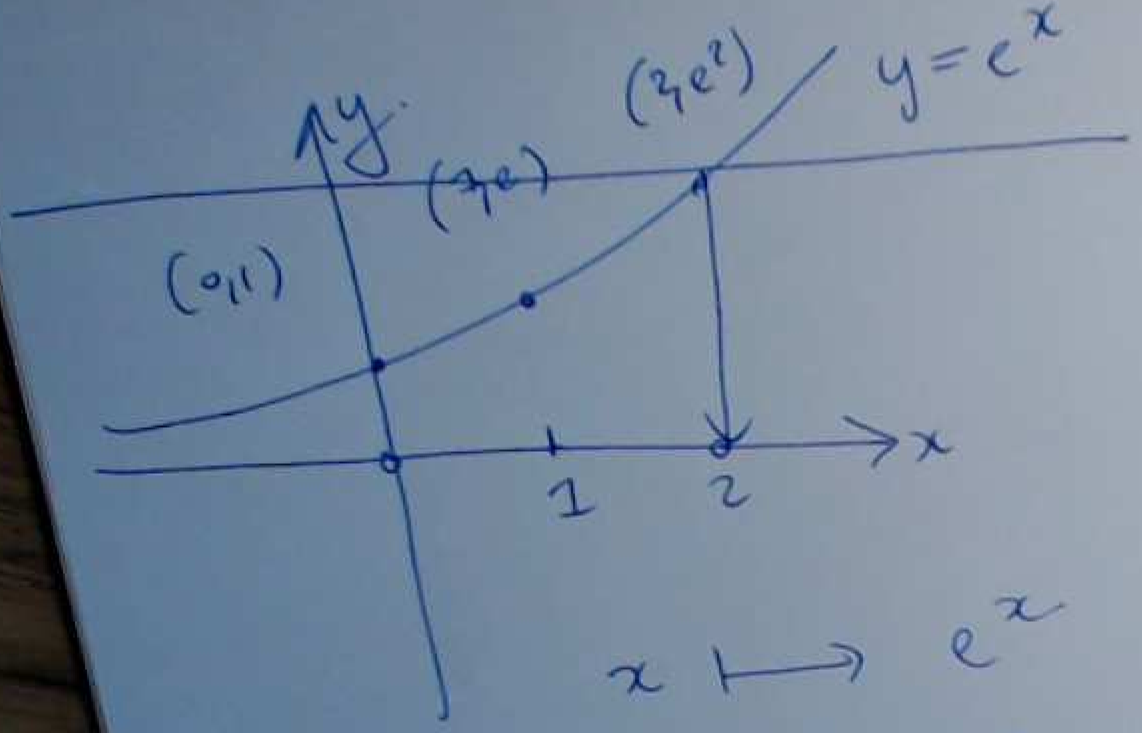
(12)



$\leftarrow$ useful

- interest rates
- half life / radio-active decay
- exponential growth

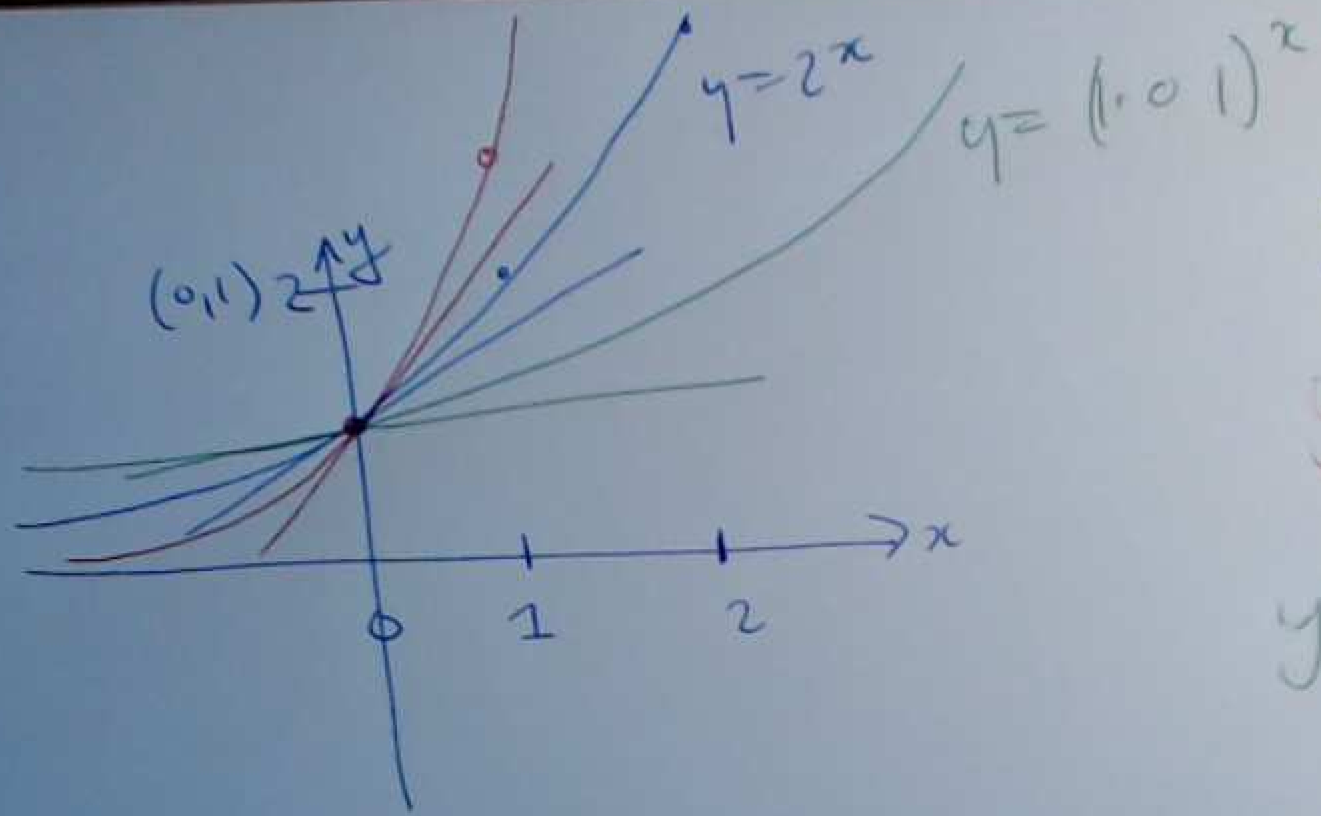
$x \longmapsto e^x$

$1 \longmapsto e^1 = 2.7\ldots$

$2 \longmapsto e^2 = \cancel{8.2\ldots}\ 7.38\ldots$

$\longleftarrow ?$ $\quad e^{x} = 7.38\ldots$

$\log_b(x) \leftarrow$ what number do you raise b to to get x? $\quad b^{\log_b(x)} = x$

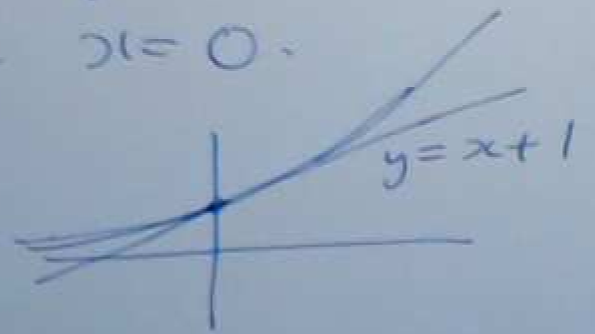


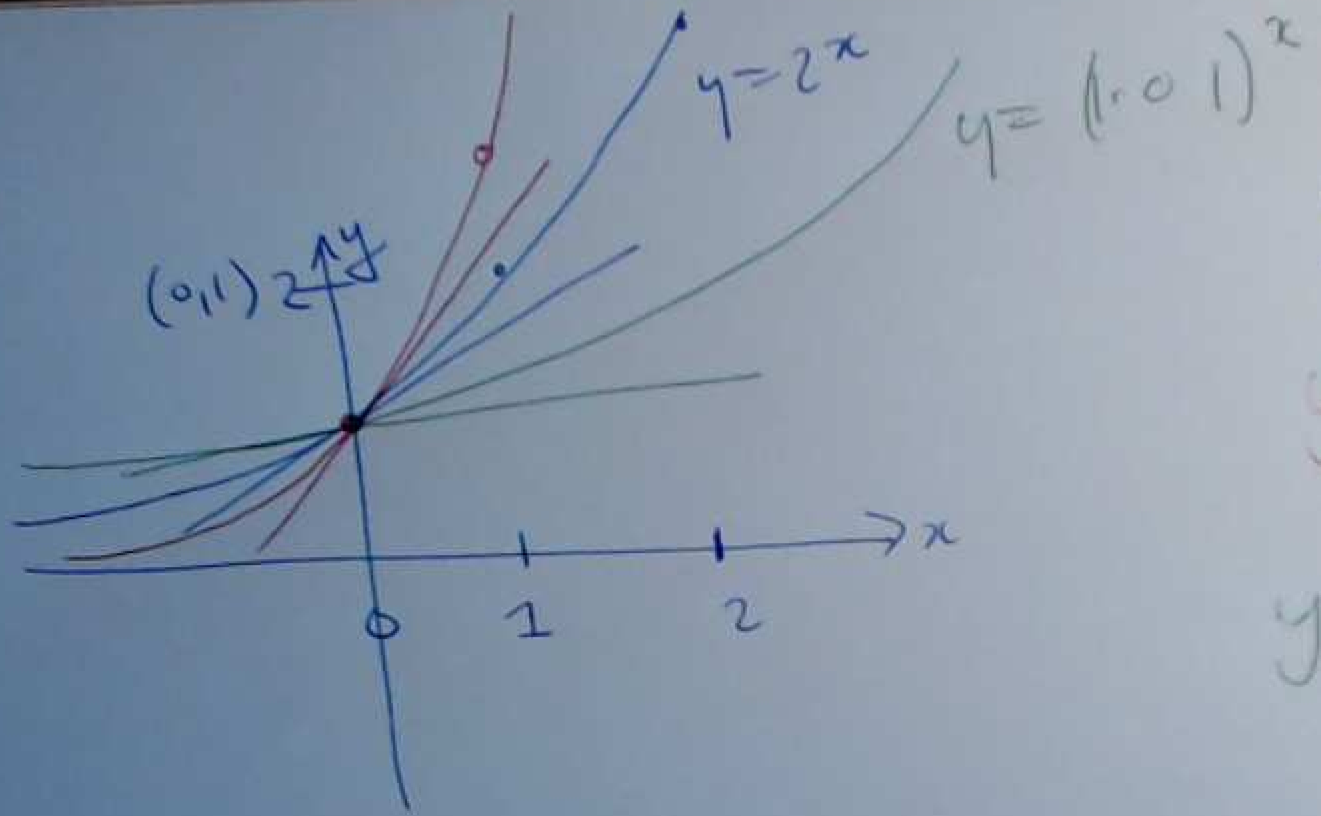
$y = 2^x$

$y = 3^x$

$y = (1.01)^x$

<u>Def$^{\underline{n}}$</u> of e: $y = e^x$ has slope exactly equal to 1 at $x = 0$.





$y = 2^x$

$y = 3^x$

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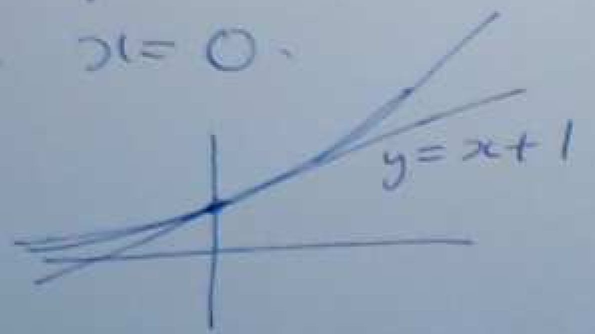
<u>Defn</u> of e: $y = e^x$ has slope exactly equal to 1 at $x = 0$.



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Q7.

$$2^{\log_2(2-x)} = 2^3$$

$$2 - x = 8$$
$$+x \qquad +x$$

$$2 = 8 + x$$
$$-8 \qquad -8$$

$$\boxed{-6 = x}$$

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15

Q8 $\underbrace{2 \log x} = \log 2 + \log(3x-4)$

$\boxed{y \log x = \log(x^y)}$

$\log(ab) = \log(a) + \log(b)$

$\log(x^2) = \log 2 + \log(3x-4)$

$10^{\log(x^2)} = 10^{\log(2(3x-4))}$

$6x-8$

$x^2 = 6x - 8$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2, 4$

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Q8 $\underbrace{2\log x} = \log 2 + \log(3x-4)$

$\boxed{y\log x = \log(x^y)}$ $\log(ab) = \log(a) + \log(b)$

$\log(x^2) = \log 2 + \log(3x-4)$

$\log(x^2) = \log(\underset{6x-8}{2(3x-4)})$

$10^{\log(x^2)}$ $10^{\log(2(3x-4))}$

$x^2 = 6x-8$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2, 4$

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(16)

Q9

$$2^{\left[\log_2(x)\log_2(x-3)\right]} = 2^2 = 4.$$

$$\left(2^{\log_2(x)}\right)^{\log_2(x-3)} = 4. \qquad \cancel{2^{ab} = 2^a 2}$$

$$2^{a+b} = 2^a 2^b$$

$$2^{ab} = (2^a)^b.$$

$$x^{\log_2(x-3)} = 4.$$

typo: $\log_2(x) + \log_2(x-3) = 2$.

↖ can solve this

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Q10

$$5^{\left[\log_5(x+1) - \log_5(x-1)\right]} = 5^2 = 25.$$

$\cancel{x+1} + \cancel{x-1}.$

$2x.$

$(x+1)-(x-1)$

$\cancel{x}+1-\cancel{x}+1$

$2.$

$$5^{a+b} = 5^a 5^b.$$

$$5^{\log_5(x+1)} \cdot 5^{-\log_5(x-1)} = 25$$

$$\boxed{\begin{array}{c}\log(a+b) \\ \neq \log a + \log b.\end{array}}$$

$(x+1)$

$$5^{-a} =$$

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$(x-1)^{-1}$

$$(x+1)\,5^{-\log_5(x-1)} = 25.$$

$$5^{-a} = \frac{1}{5^a}.$$

$$a\log_5(b) = \log_5(b^a)$$

$$(x+1)\,\frac{1}{5^{\log_5(x-1)}} = 25$$

$$(x+1)\,5^{\log_5\left(\frac{1}{x-1}\right)} = 25.$$

$$\frac{x+1}{x-1} = 25.$$

$$\frac{x+1}{x-1} = 25.$$

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(2c)

$$\frac{x+1}{x-1} = 25 \cdot \times (x-1).$$

$(x-1) \times$

$$x+1 = 25(x-1).$$

$$x+1 = 25x - 25.$$

$-x \qquad -x$

$$1 = 24x - 25$$

$+25 \qquad +25$

$$26 = 24x \qquad x = \frac{26}{24} = \frac{13}{12}.$$

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(21)

$x^2 - 2x + 1 = 0 \leftarrow$ quadratic in x!

$x - 2\sqrt{x} + 1 = 0 \leftarrow$ quadratic in $\sqrt{x}$

$\Rightarrow (x-1)^2 = 0 \rightarrow x = 1.$

$\rightarrow (\sqrt{x} - 1)^2 = 0 \rightarrow x = 1.$

$e^{2x} - 2e^x + 1 = 0$

$e^{x+x} = e^x \cdot e^x = (e^x)^2 = e^{2x}$

$(e^x)^2 - 2e^x + 1 = 0$

$(e^x - 1)^2 = 0.$

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$$e^x \times \quad e^x - 2 + e^{-x} = 0 \times e^x$$

$$(e^x)^2 - 2e^x + e^{-x} \cdot e^x = 0.$$

$$e^0.$$

$$= 1.$$

$$(e^x)^2 - 2e^x + 1 = 0$$

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(23)

\$300 3.5% interest rate. 10 years.

annually, semi annually, quarterly, cts/y

300. $t=0$.

$t = 1$ year. $300 + 3.5\%$

$300 \times (1.035)$.

$t = 2$ years. $300 \times (1.035) \times (1.035)$

$300 \times (1.035)^2$.

$t = 3$ years. $300 \times (1.035)^3$.

$\vdots$

$t = 10$ years $300 \times (1.035)^{10}$.

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stuart McLeod entered the waiting room Admit

23

add on.

300 at 3.5%.

$$\frac{300}{100} \times 3.5 = 10.5.$$

300 add on 3.5%.

$$300 + 300 \times \frac{3.5}{100}.$$

$$300\left(1 + \frac{3.5}{100}\right).$$

$$300(1 + 0.035).$$

$$300(1.035).$$

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(23)

t=0. $300

t=1 year. 300 × 1.035. $310.50

t=2 year. 310.50 add on 3.50% of this.

$310.50 \times (1.035)$

$300 \times (1.035) \times (1.035)$

$300 \times (1.035)^2$

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\$300. $t=0$.

1.75%

$t=0.5$. 6 months add on $\frac{3.5\%}{2}$.

$300 \times (1.0175)$. $\times (1.0175)$.

$t=1$. 1 year. add on $\frac{3.5\%}{2} = 1.75\%$

$\times (1.017\cancel{8}5)$.

$300 \times (1.0175)^2$. $=$ 300×1.035306.

after 10 years.

$300 \times (1.\overset{0175}{\cancel{075}})^{20}$.

$t=0.$ $\qquad 300$

$t = 3$ month $\quad$ add a $\frac{3.5\%}{4}$ $\qquad 300 \times \left(1 + \frac{\frac{3.5}{100}}{4}\right)$

$t = 6$ month $\qquad 300 \times \left(1 + \frac{3.5}{400}\right)^2.$

$t = 9$ month $\qquad 300 \times \left(1 + \frac{3.5}{400}\right)^3.$

$t = 1$ year. $\qquad 300 \left(1 + \frac{3.5}{400}\right)^4.$

compound continuously

$$300 \times e^{\left(\frac{3.5}{100}\right) t}.$$

$100 add a 100%.

$200.

0 — 1 year

$e \times 100$

cts/y $\qquad$ $2.71...

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25

$t=0$. 300

$t=3$ month add a $\frac{3.5\%}{4}$. $300 \times \left(1+\frac{\frac{3.5}{100}}{4}\right)$

$t=6$ month $300 \times \left(1+\frac{3.5}{400}\right)^2$.

$t=9$ month $300 \times \left(1+\frac{3.5}{400}\right)^3$.

$t=1$ year $300\left(1+\frac{3.5}{400}\right)^4$.

compound continuously

$$300 \times e^{\left(\frac{3.5}{100}\right)t}$$

$t=10$

\$100 add a 100%.

\$200.

0 1 year

$e \times 100$

\$2.71...

ctsly

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