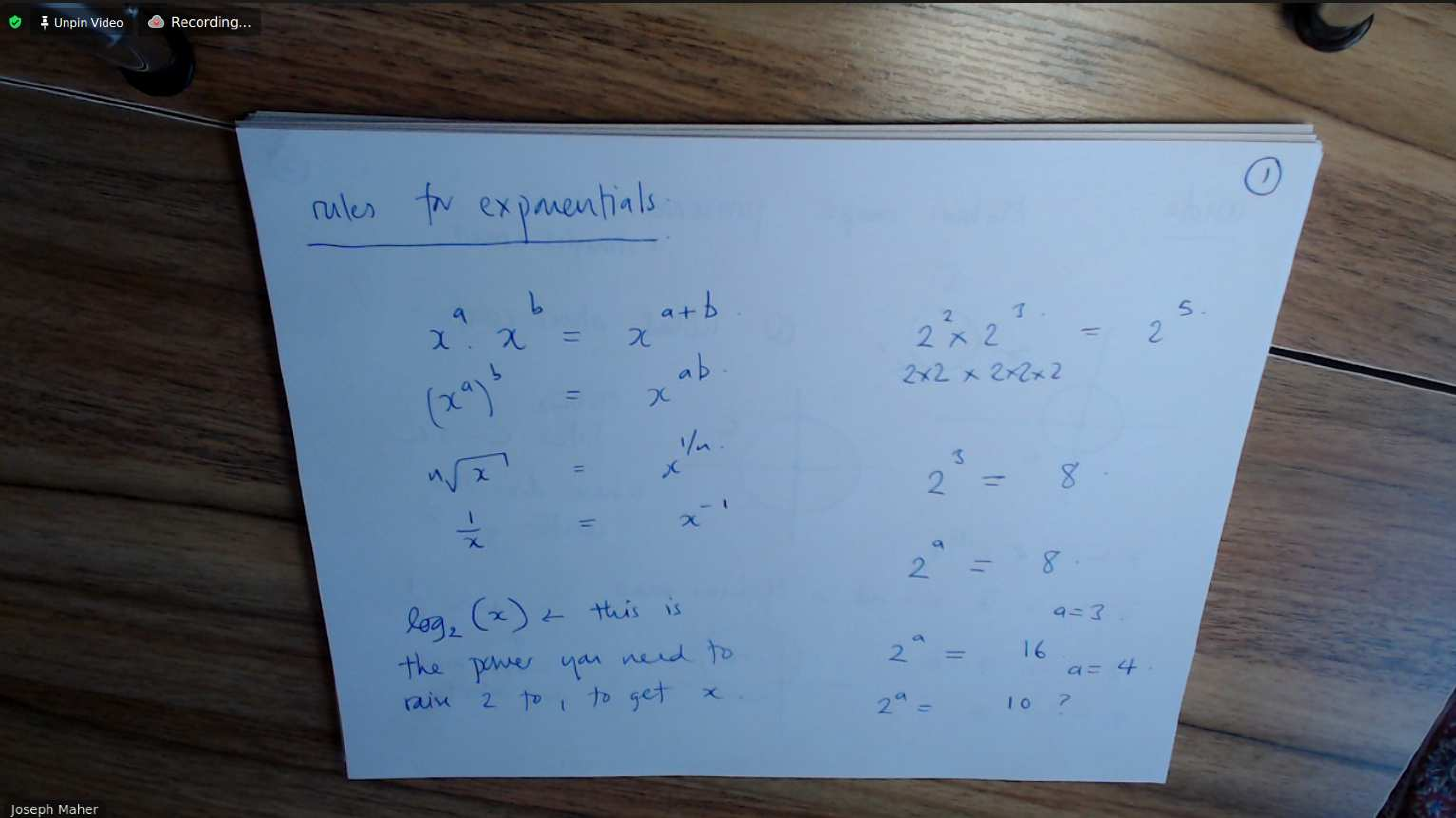


rules for exponentials

$x^a . x^b = x^{a+b}$.

$(x^a)^b = x^{ab}$.

$\sqrt[n]{x} = x^{1/n}$.

$\frac{1}{x} = x^{-1}$

$\log_2(x)$ ← this is the power you need to raise 2 to, to get x.

$2^2 \times 2^3 = 2^5$.

$2\times2 \times 2\times2\times2$

$2^3 = 8$.

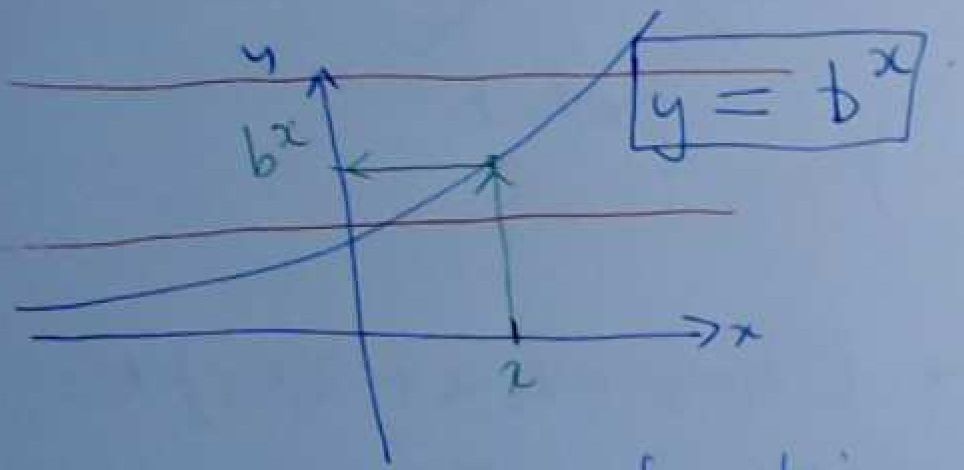
$2^a = 8$.

$a=3$.

$2^a = 16$ $a=4$.

$2^a = 10$?

$\log_b(x)$ ← this is the number you need to raise ~~3~~ b to the power of to get x

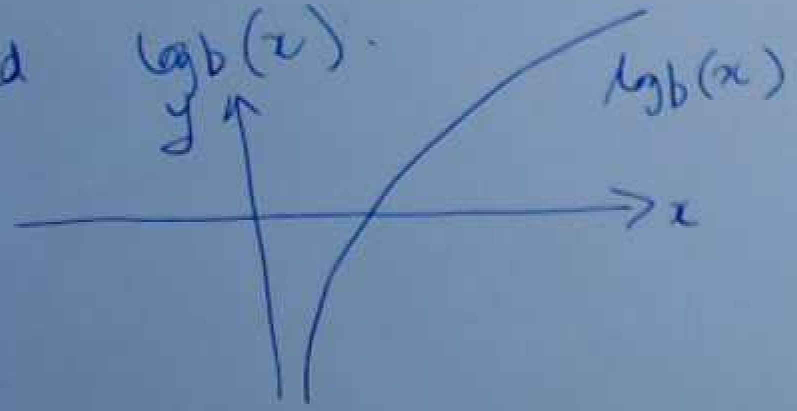


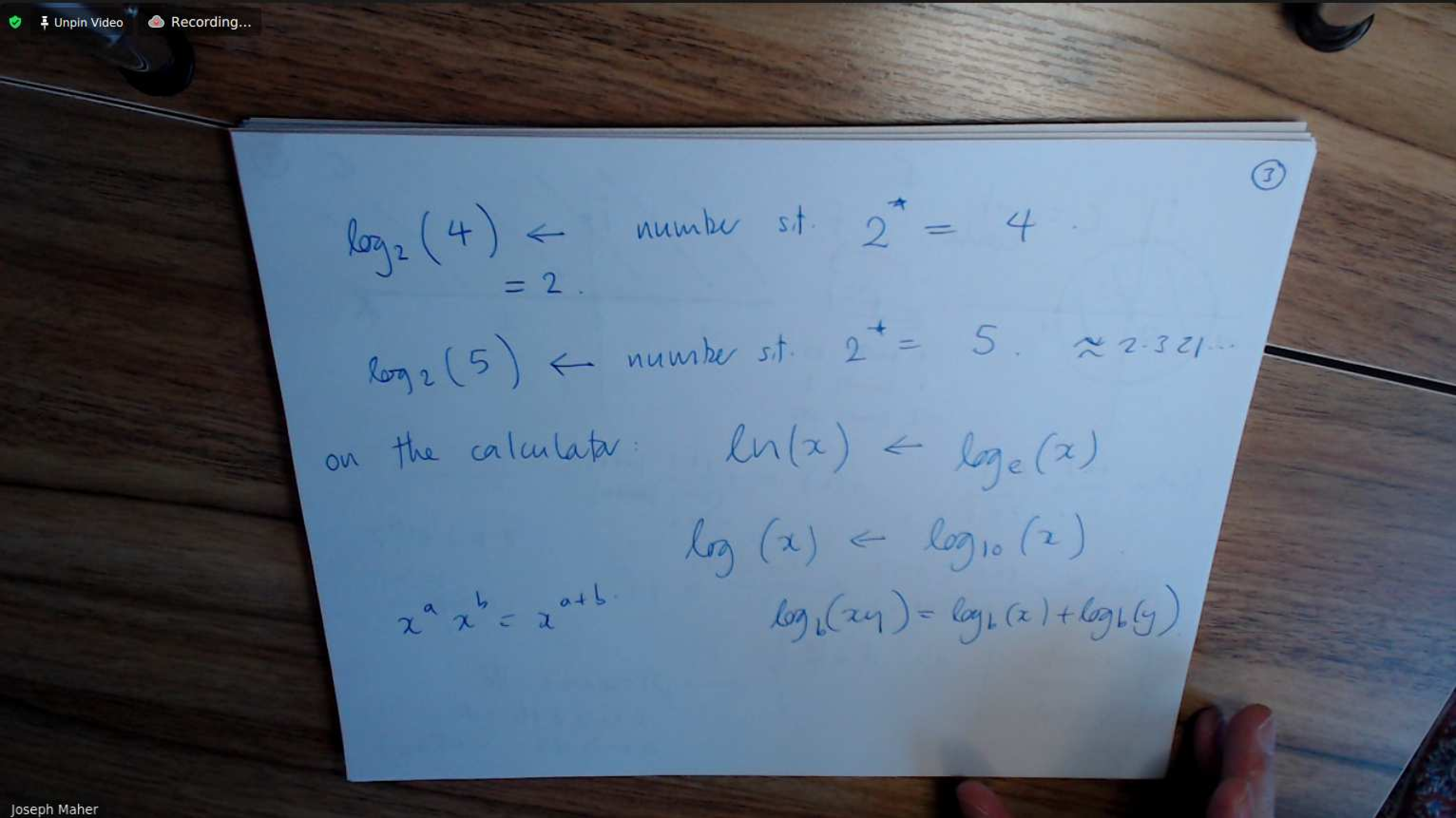
Q: when does a function have an inverse?

A: it passes the horizontal line test.

i.e. every horizontal line hits the graph of the function exactly once

there is an inverse function called $\log_b(x)$.





$\log_2(4) \leftarrow$ number s.t. $2^{*} = 4$.

$= 2$.

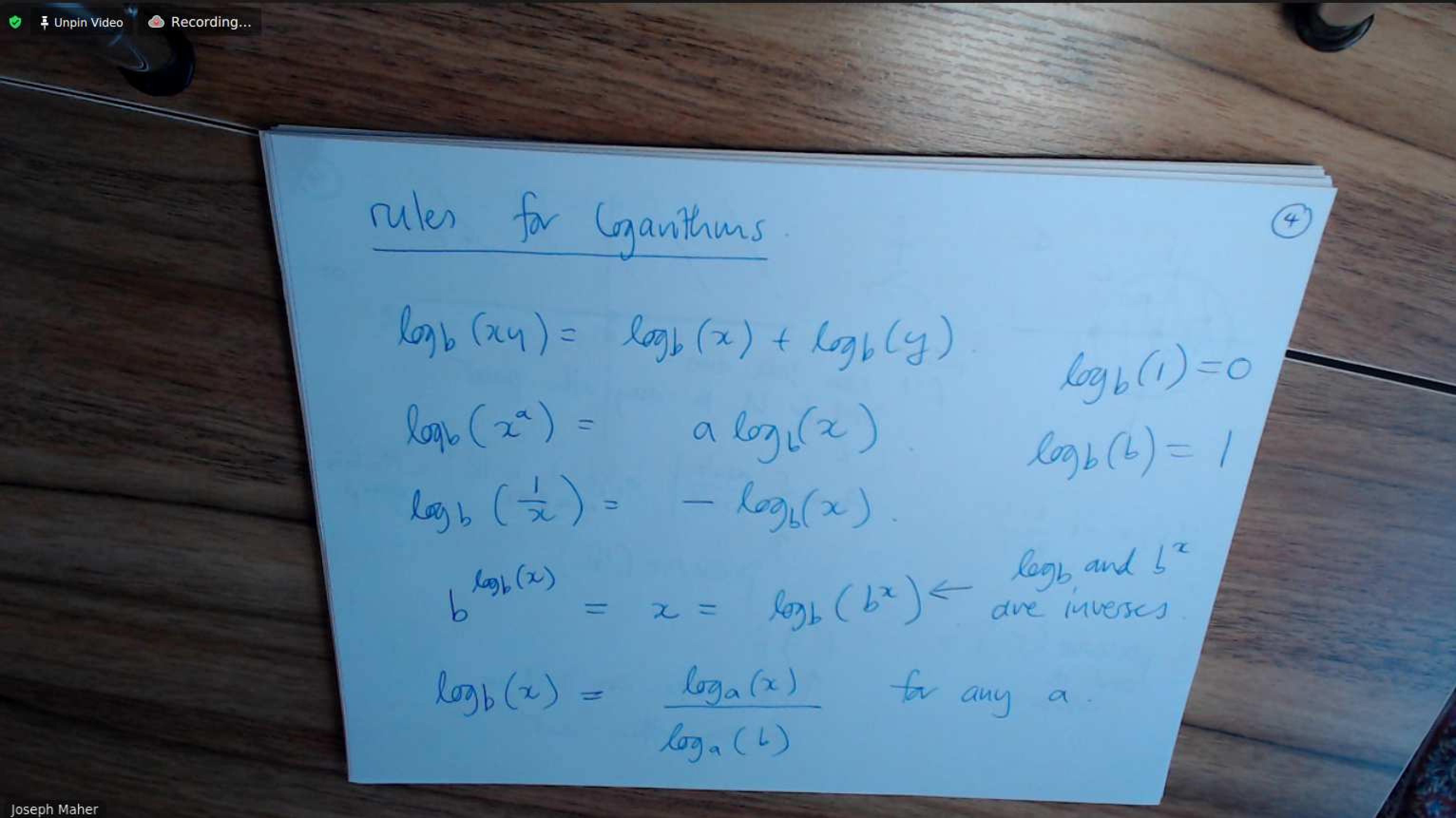
$\log_2(5) \leftarrow$ number s.t. $2^{*} = 5$. $\approx 2.321\ldots$

on the calculator: $\ln(x) \leftarrow \log_e(x)$

$\log(x) \leftarrow \log_{10}(x)$

$x^a x^b = x^{a+b}$.

$\log_b(xy) = \log_b(x) + \log_b(y)$



rules for logarithms

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b(1) = 0$$

$$\log_b(x^a) = a\log_b(x)$$

$$\log_b(b) = 1$$

$$\log_b\left(\frac{1}{x}\right) = -\log_b(x).$$

$$b^{\log_b(x)} = x = \log_b(b^x) \leftarrow$$ $\log_b$ and b^x are inverses.

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$ for any a.

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$\ln(x) = \log_e(x)$ ⑤

$$\log_2(5) = \frac{\ln(5)}{\ln(2)} = \frac{\log_{10}(5)}{\log_{10}(2)} \approx 2.31298\ldots$$

$2.313.$

$\log_2\left(\frac{1}{2}\right) = -1 \qquad 2^x = \frac{1}{2}.$

$\log_2\left(\frac{1}{4}\right) = -2 \qquad 2^2 = 4.$

$\log_2(\sqrt{2}) = \frac{1}{2} \qquad 2^x = \sqrt{2} \approx 1.4\ldots$

$\log_2\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}. \qquad 2^{\frac{1}{2}} = \sqrt[2]{2}.$

Joseph Maher

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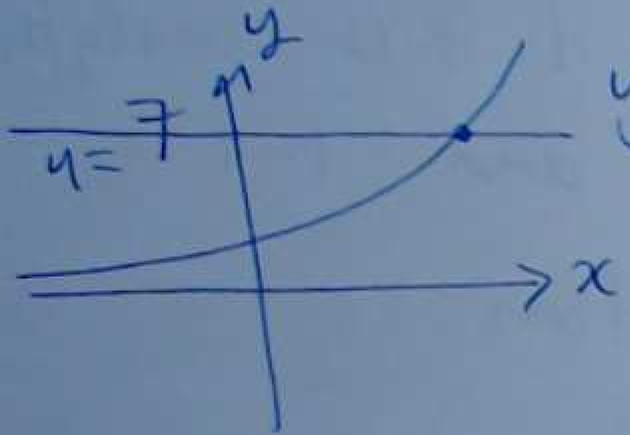


$y = e^x$

find x s.t.

$$e^x = 7$$

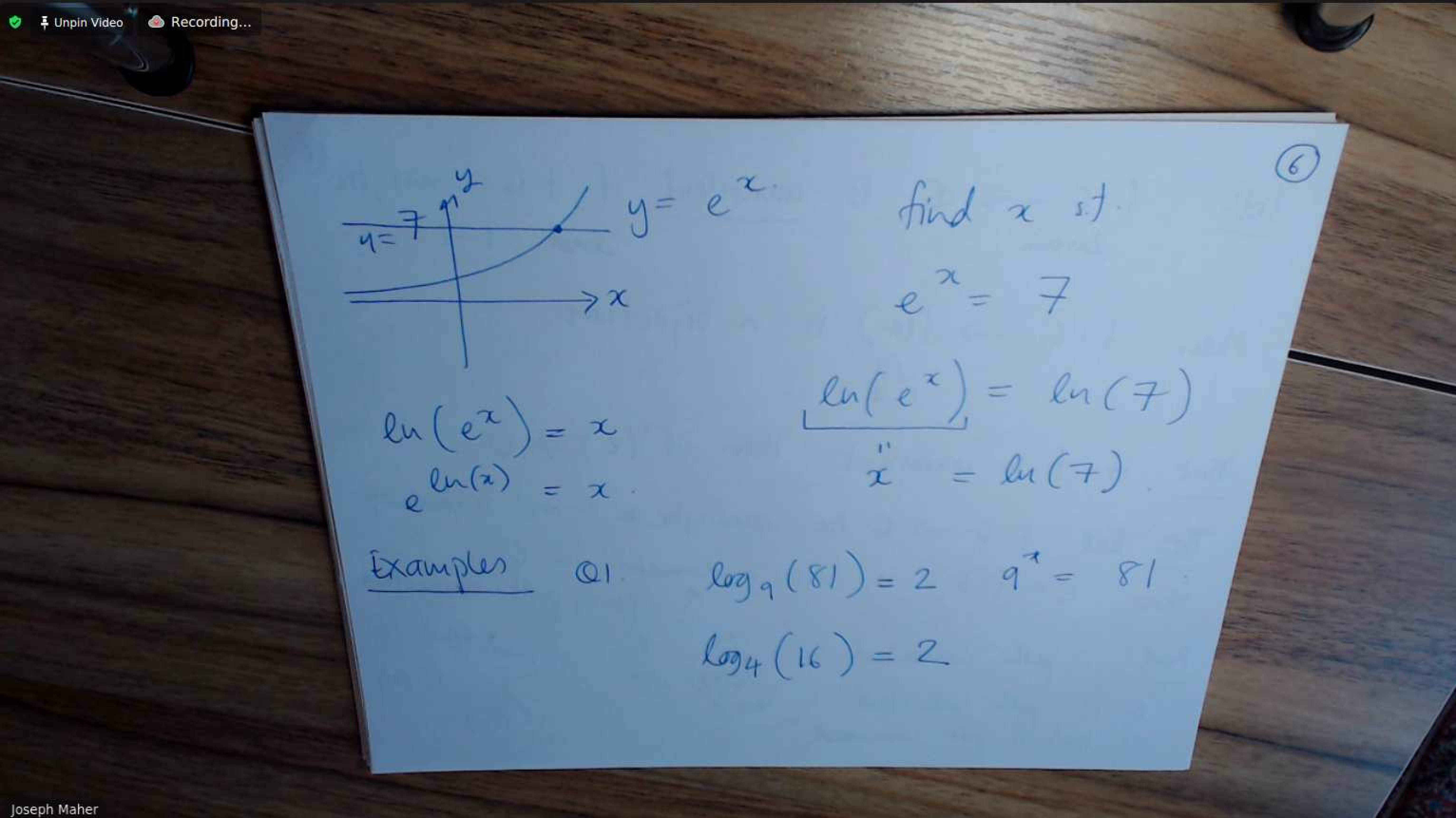
$$\underbrace{\ln(e^x)}_{} = \ln(7)$$

$$\overset{''}{x} = \ln(7)$$

$$\ln(e^x) = x$$

$$e^{\ln(x)} = x$$

Joseph Maher



find x s.t.

$$e^x = 7$$

$$\underbrace{\ln(e^x)}_{\overset{\shortparallel}{x}} = \ln(7)$$

$$x = \ln(7)$$

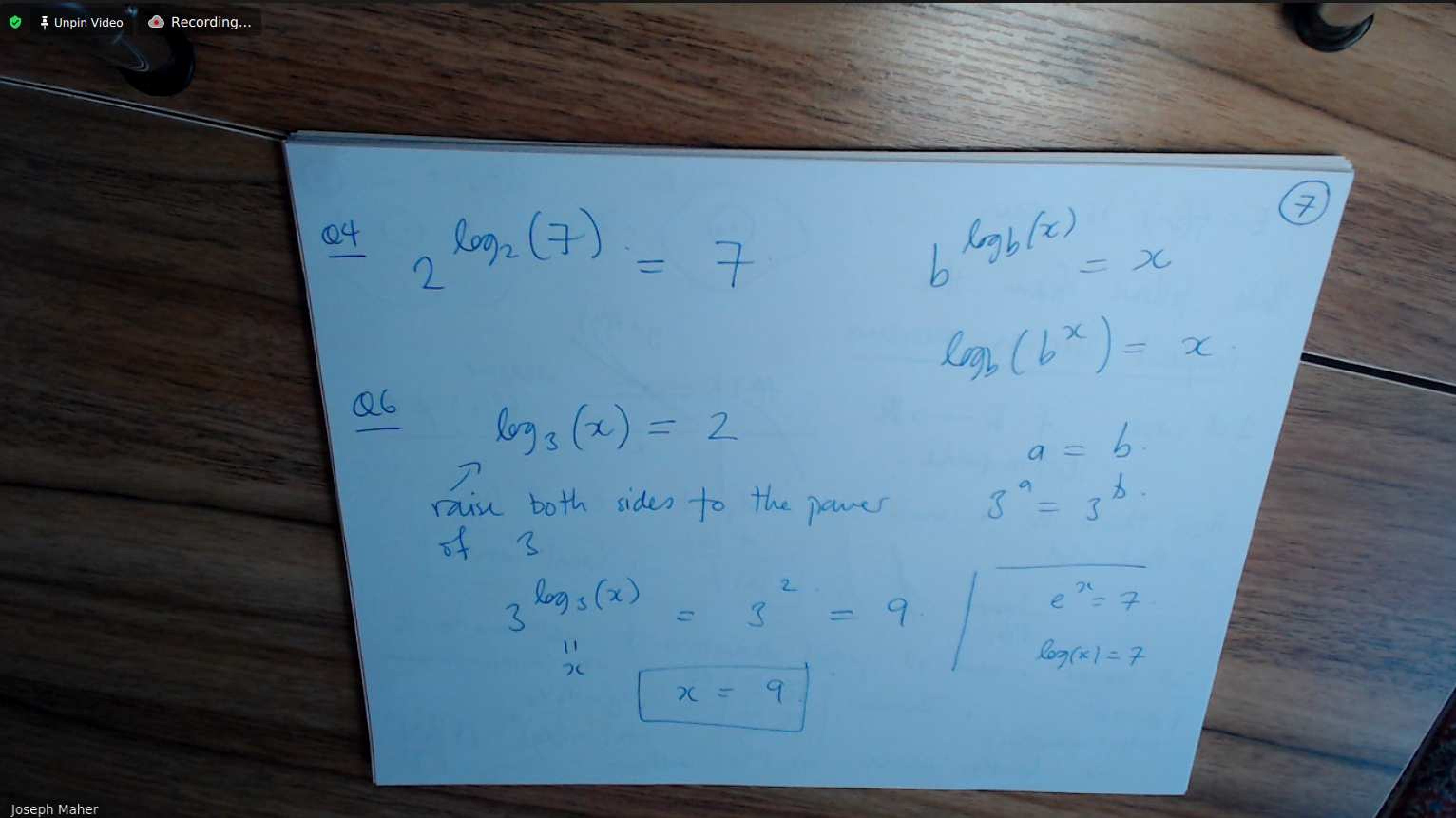
$$\ln(e^x) = x$$

$$e^{\ln(x)} = x.$$

Examples Q1

$\log_9(81) = 2$ $\quad 9^x = 81$

$\log_4(16) = 2$



Q4 $2^{\log_2(7)} = 7$

$b^{\log_b(x)} = x$

$\log_b(b^x) = x$

Q6 $\log_3(x) = 2$

$\nearrow$ raise both sides to the power of 3

$a = b$

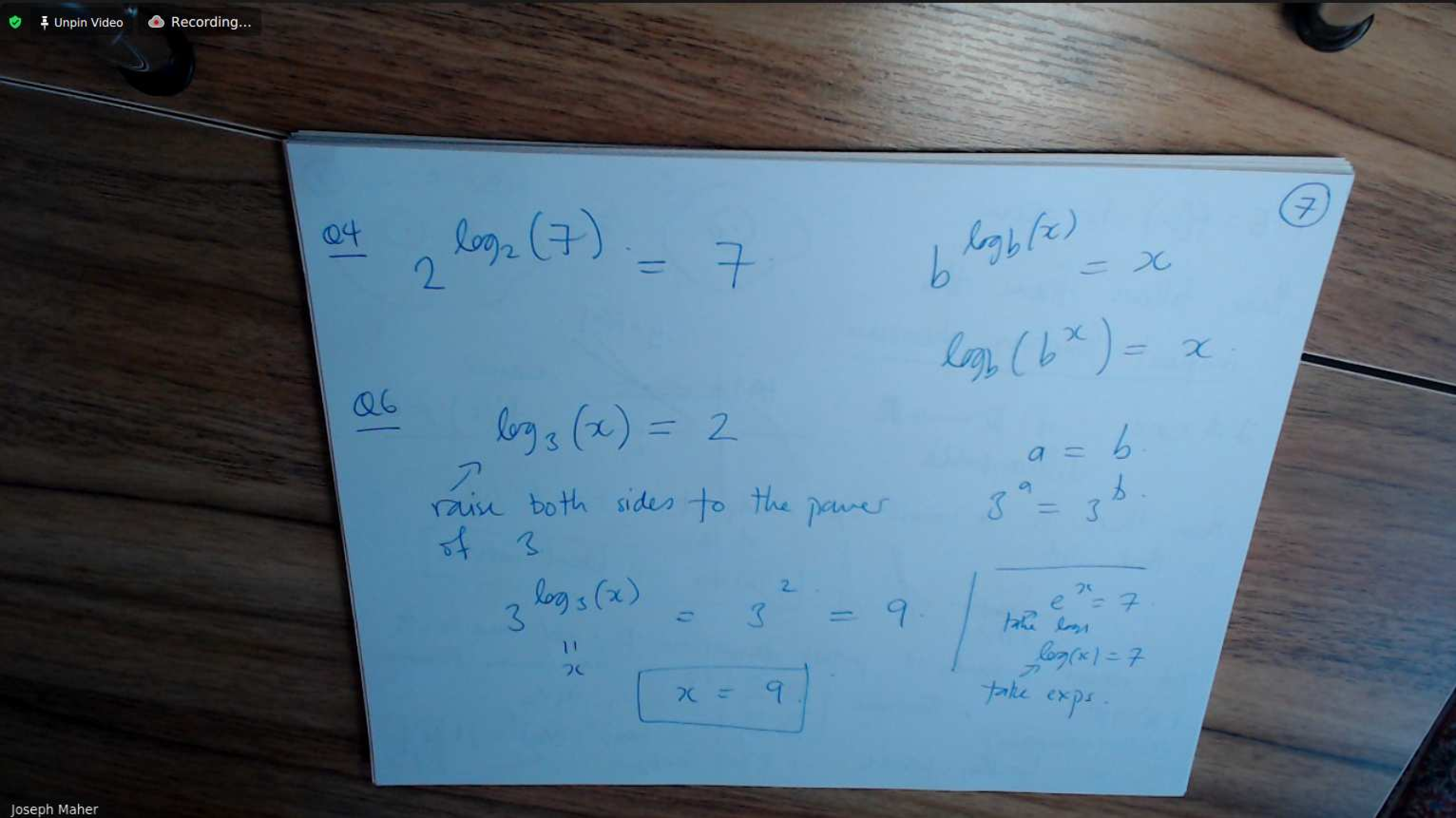
$3^a = 3^b$

$\underset{x}{\underset{\shortparallel}{3^{\log_3(x)}}} = 3^2 = 9$

$\boxed{x = 9}$

$e^x = 7$

$\log(x) = 7$



Q4 $2^{\log_2(7)} = 7$

$b^{\log_b(x)} = x$

$\log_b(b^x) = x$

Q6 $\log_3(x) = 2$

↗ raise both sides to the power of 3

$$\underset{x}{\underset{\parallel}{3^{\log_3(x)}}} = 3^2 = 9$$

$$\boxed{x = 9}$$

$a = b$

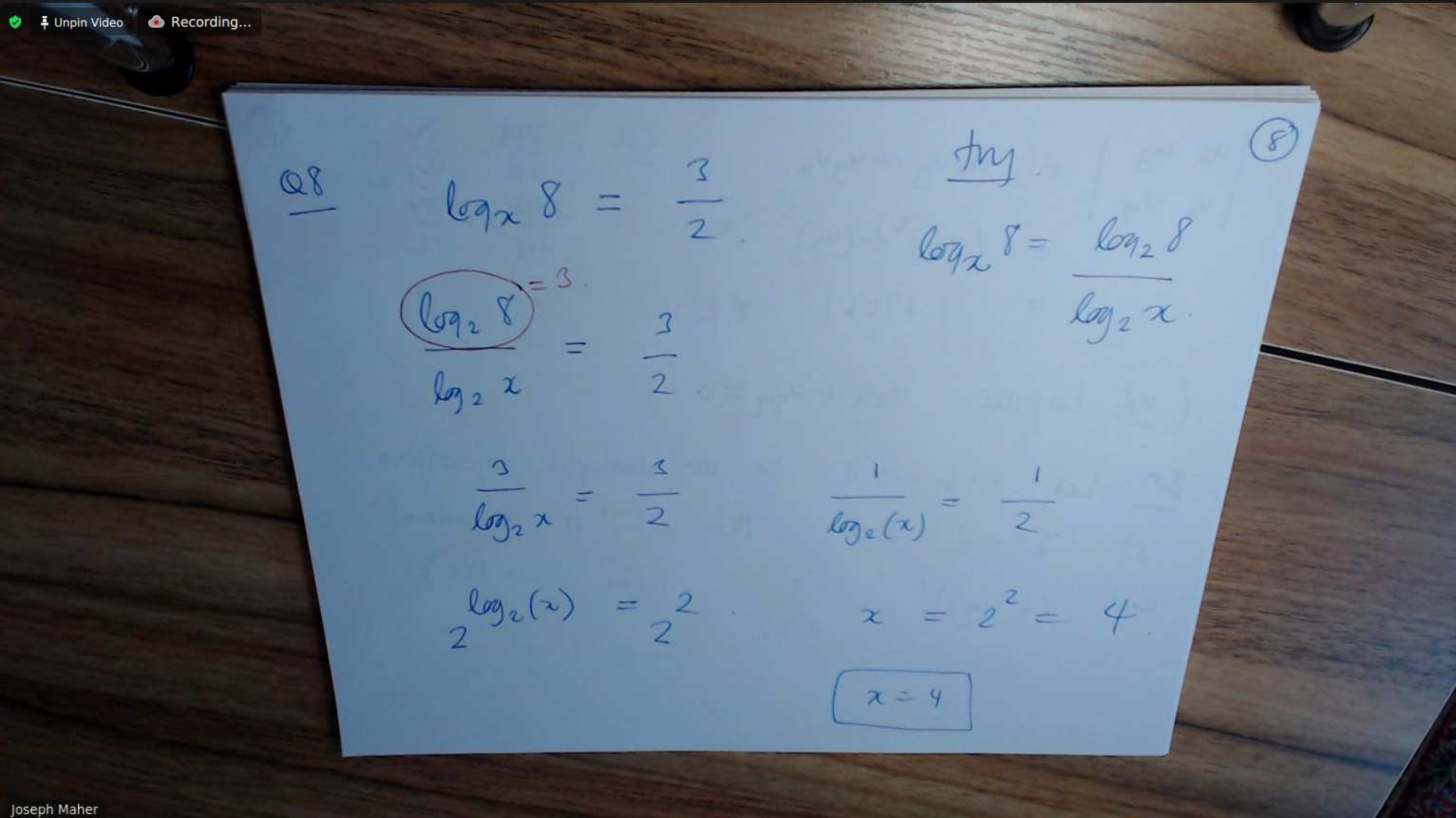
$3^a = 3^b$

$e^x = 7$

take logs

$\log(x) = 7$

↗ take exps.



⑧

Q8

$$\log_x 8 = \frac{3}{2}.$$

try.

$$\log_x 8 = \frac{\log_2 8}{\log_2 x}.$$

$$\frac{\boxed{\log_2 8} = 3.}{\log_2 x} = \frac{3}{2}.$$

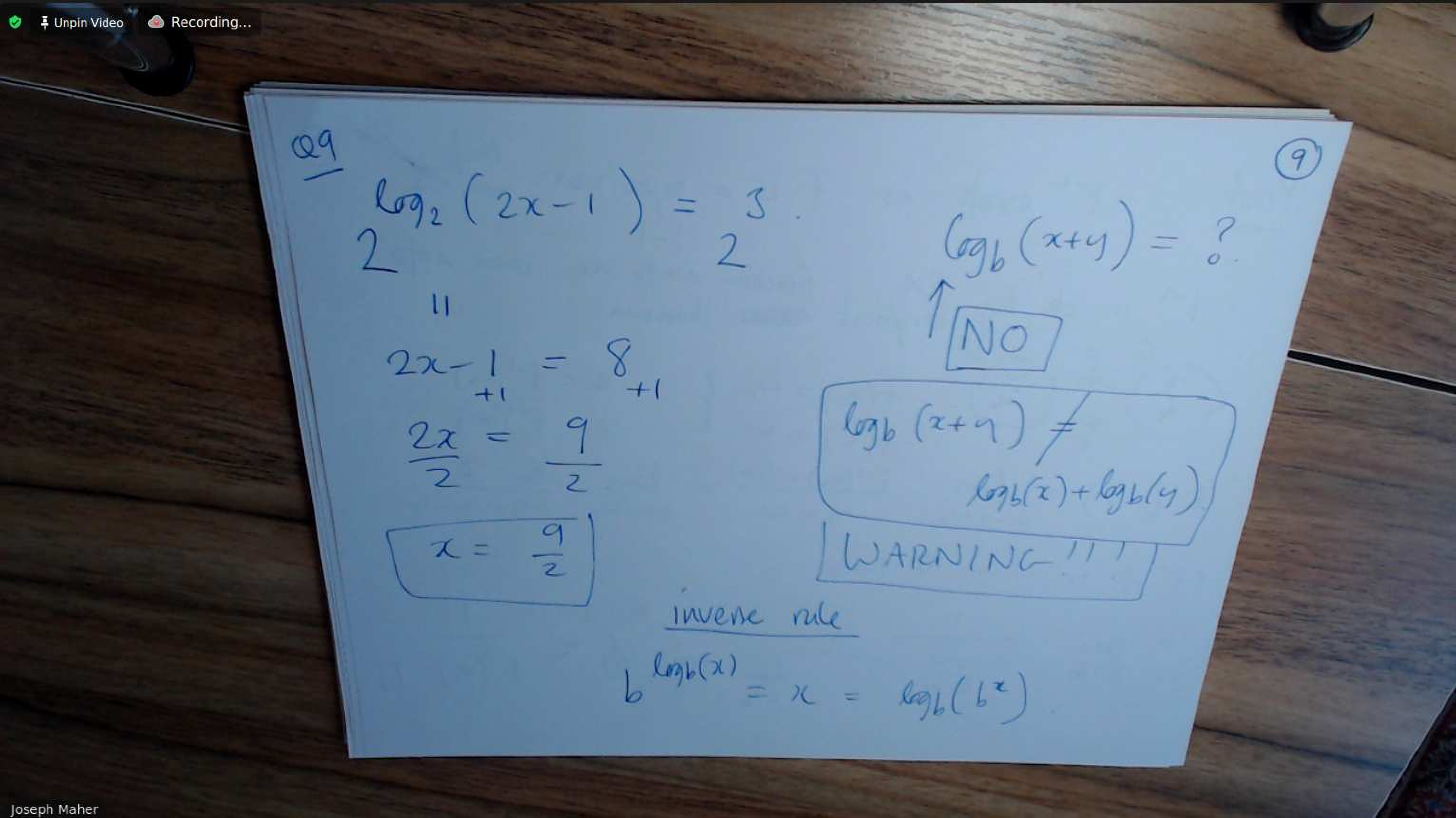
$$\frac{3}{\log_2 x} = \frac{3}{2}$$

$$\frac{1}{\log_2(x)} = \frac{1}{2}.$$

$$2^{\log_2(x)} = 2^2.$$

$$x = 2^2 = 4.$$

$$\boxed{x = 4}$$



Q9

$$\log_2(2x-1) = 3$$

$$2^{\log_2(2x-1)} = 2^{3}$$

$$\|$$

$$\underset{+1}{2x-1} = \underset{+1}{8}$$

$$\frac{2x}{2} = \frac{9}{2}$$

$$\boxed{x = \frac{9}{2}}$$

$$\log_b(x+y) = ?$$

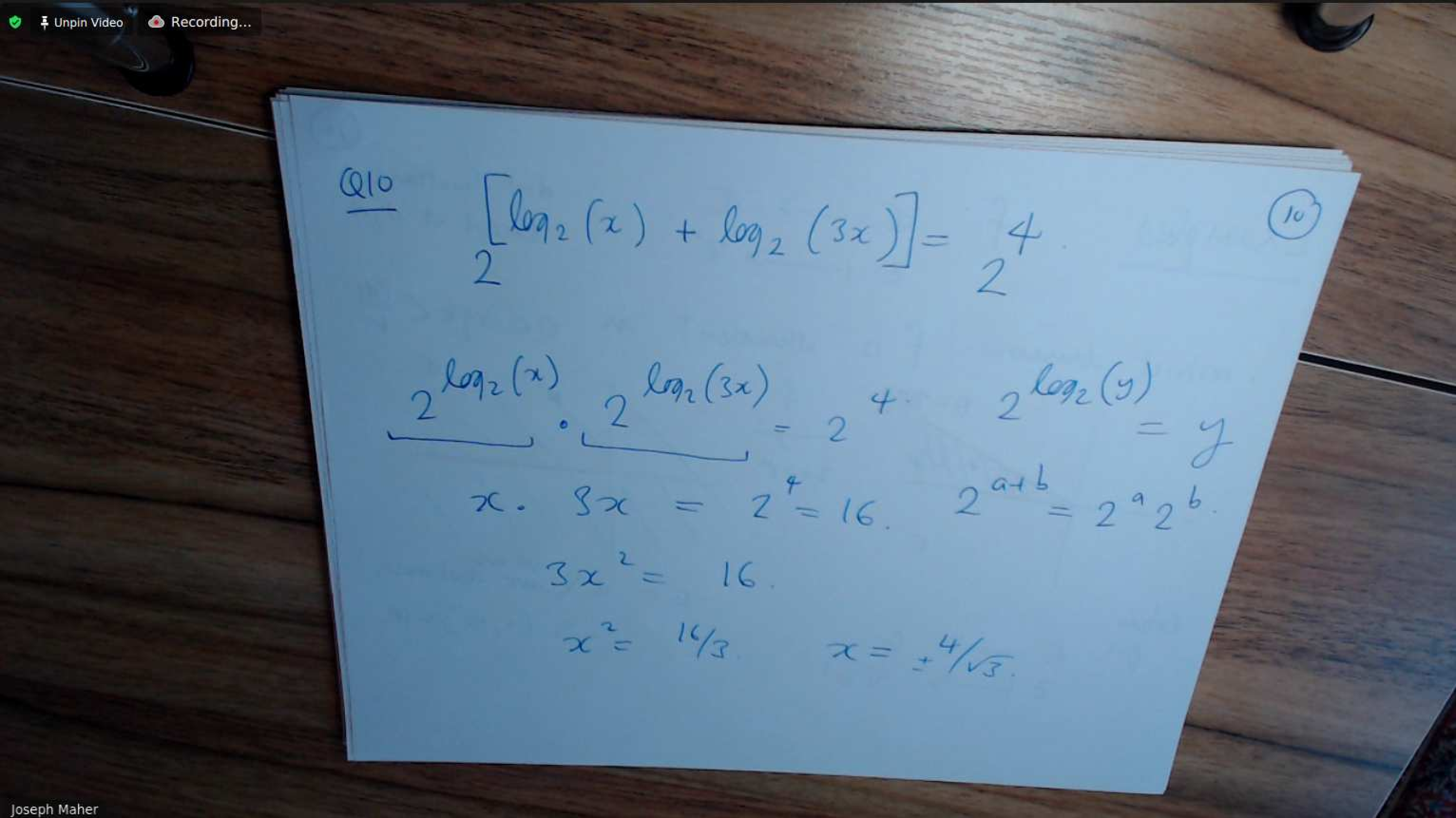
↑ NO

$$\boxed{\log_b(x+y) \neq \log_b(x)+\log_b(y)}$$

WARNING!!!

inverse rule

$$b^{\log_b(x)} = x = \log_b(b^{x})$$



Q10

⑩

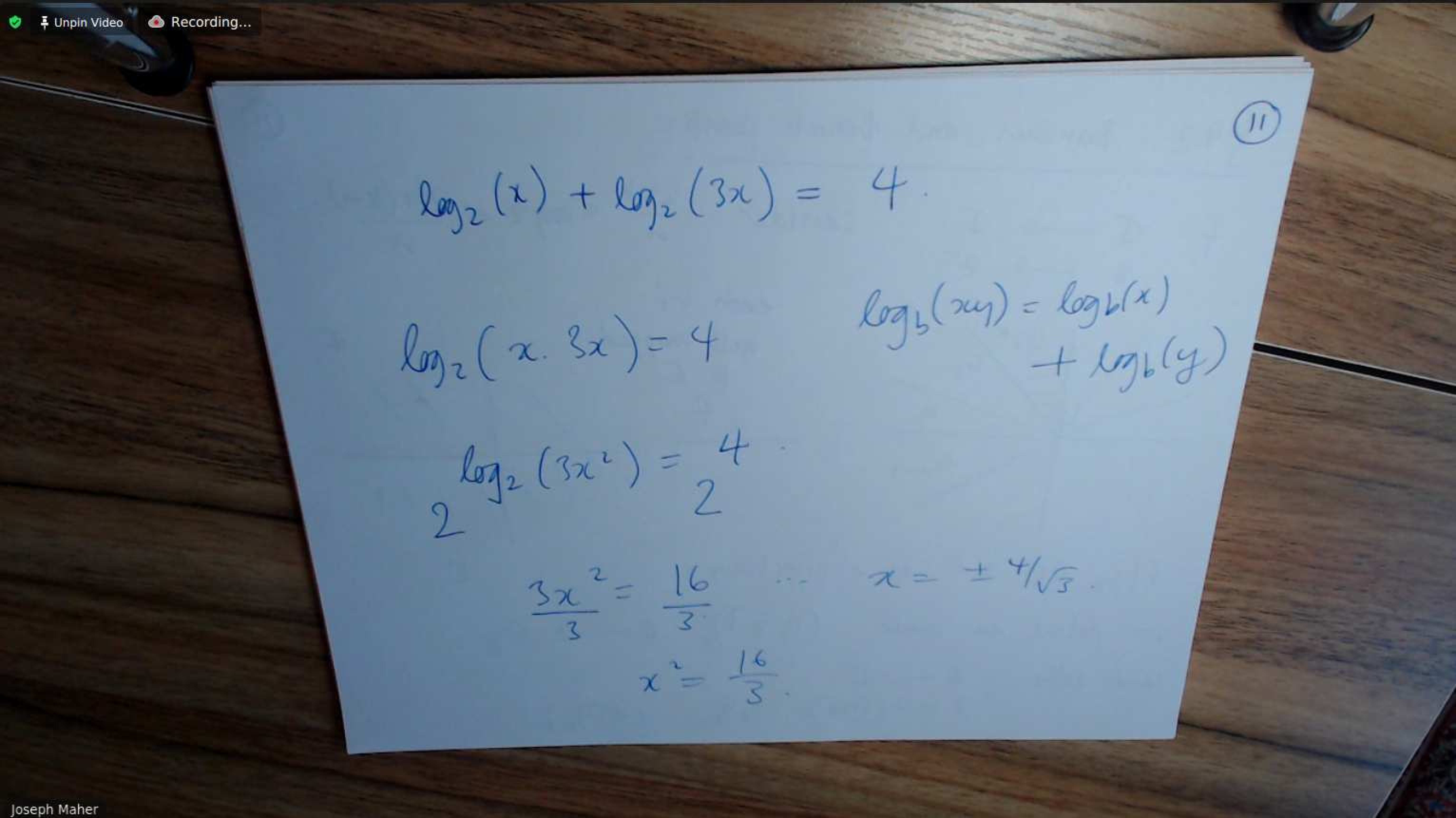
$$2^{\left[\log_2(x) + \log_2(3x)\right]} = 2^4$$

$$\underbrace{2^{\log_2(x)}} \cdot \underbrace{2^{\log_2(3x)}} = 2^4 \qquad 2^{\log_2(y)} = y$$

$$x \cdot 3x = 2^4 = 16 \qquad 2^{a+b} = 2^a 2^b$$

$$3x^2 = 16$$

$$x^2 = 16/3 \qquad x = \pm 4/\sqrt{3}$$



⑪

$$\log_2(x) + \log_2(3x) = 4.$$

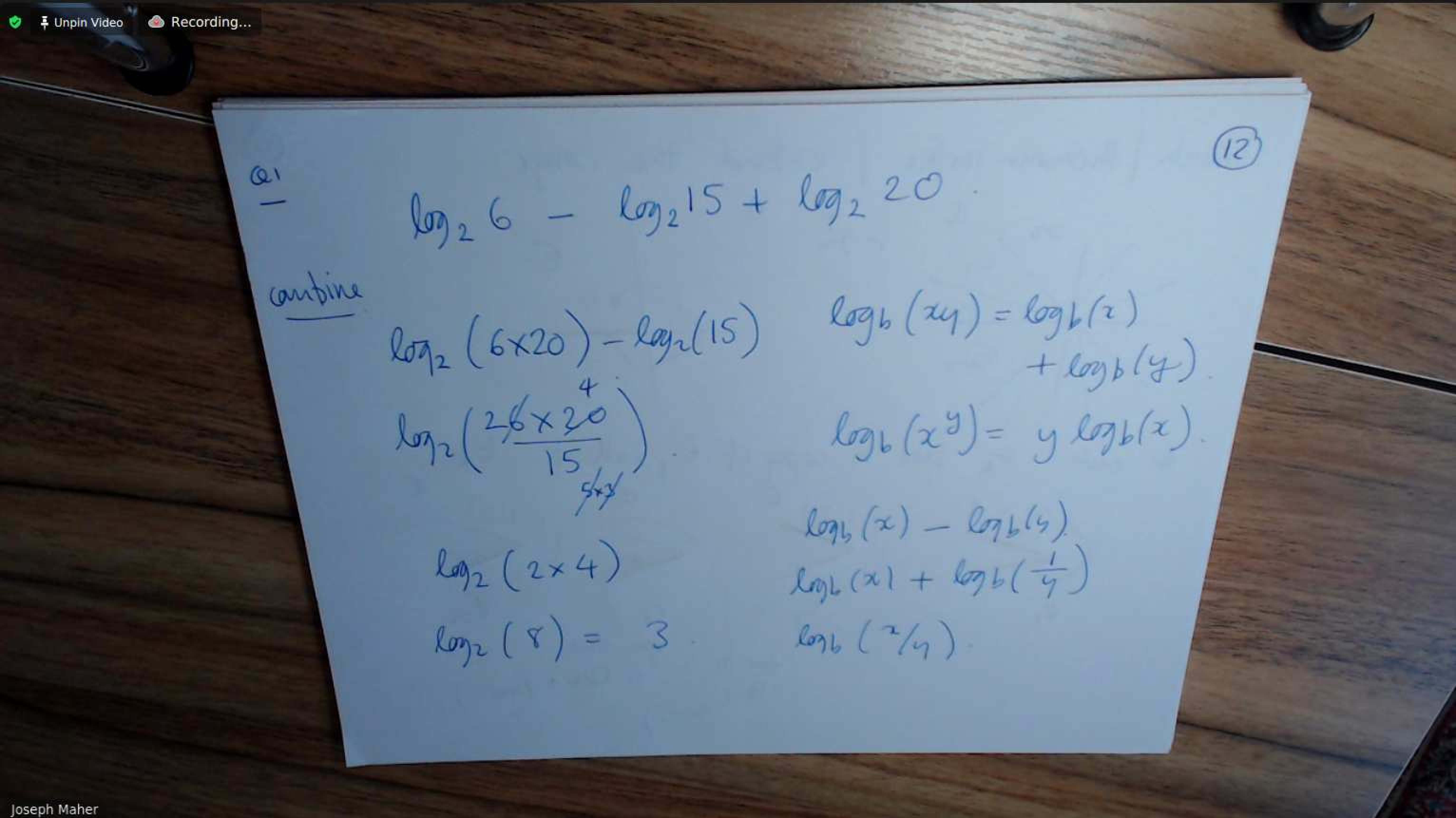
$$\log_2(x \cdot 3x) = 4$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$2^{\log_2(3x^2)} = 2^{4}.$$

$$\frac{3x^2}{3} = \frac{16}{3} \quad \ldots \quad x = \pm 4/\sqrt{3}.$$

$$x^2 = \frac{16}{3}.$$



Q1

$$\log_2 6 - \log_2 15 + \log_2 20$$

combine

$$\log_2(6\times20) - \log_2(15)$$

$$\log_2\left(\frac{2\cancel{6}\times\overset{4}{\cancel{20}}}{\underset{\cancel{5}\times\cancel{3}}{\cancel{15}}}\right)$$

$$\log_2(2\times4)$$

$$\log_2(8) = 3$$

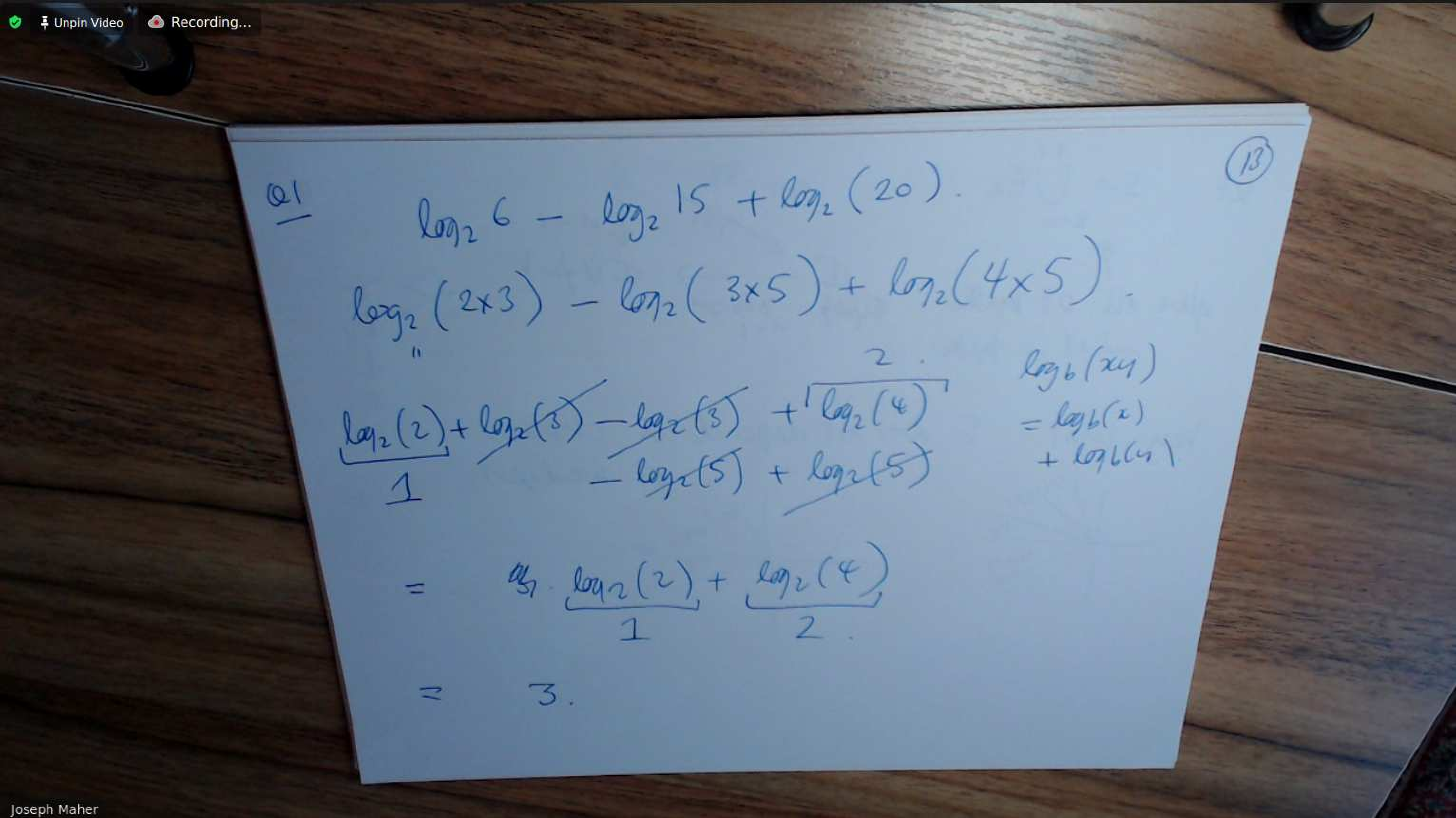
$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b(x^y) = y\log_b(x)$$

$$\log_b(x) - \log_b(y)$$

$$\log_b(x) + \log_b\left(\frac{1}{y}\right)$$

$$\log_b(x/y)$$



Q1

$$\log_2 6 - \log_2 15 + \log_2(20).$$

$$\log_2(2\times3) - \log_2(3\times5) + \log_2(4\times5).$$

"

$$\underbrace{\log_2(2)}_{1} + \cancel{\log_2(3)} - \cancel{\log_2(3)} + \overbrace{\log_2(4)}^{2}$$

$$- \cancel{\log_2(5)} + \cancel{\log_2(5)}$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$= \underbrace{\log_2(2)}_{1} + \underbrace{\log_2(4)}_{2}.$$

$$= 3.$$

ANSWER ☐

Classwork 12

Intermediate Algebra MTH 35

Topic: Logarithms

Name: ____

Evaluate the expression using Laws of Logarithms.

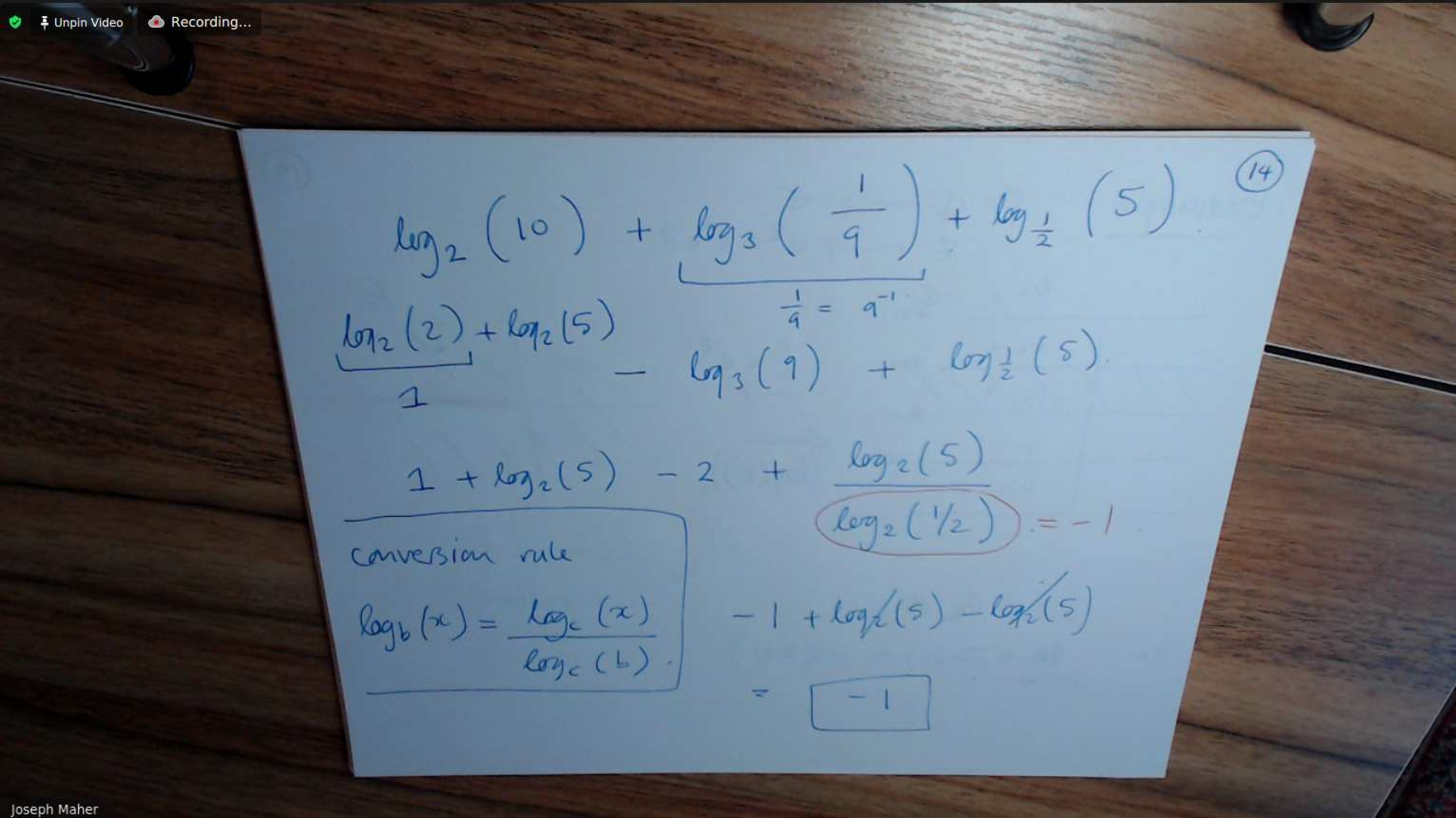
1. $\log_2 6 - \log_3 15 + \log_2 20$

2. $\log_4 16^{100} + \log_4 64$

3. $\ln(\ln e^{e^2})$

Use Laws of Logarithms to expand or combine the expressions.

4. (Expand) $\log_a x\sqrt{y}z^2$



⑭

$$\log_2(10) + \underbrace{\log_3\left(\frac{1}{9}\right)}_{\frac{1}{9} = 9^{-1}} + \log_{\frac{1}{2}}(5)$$

$$\underbrace{\log_2(2)}_{1} + \log_2(5) - \log_3(9) + \log_{\frac{1}{2}}(5)$$

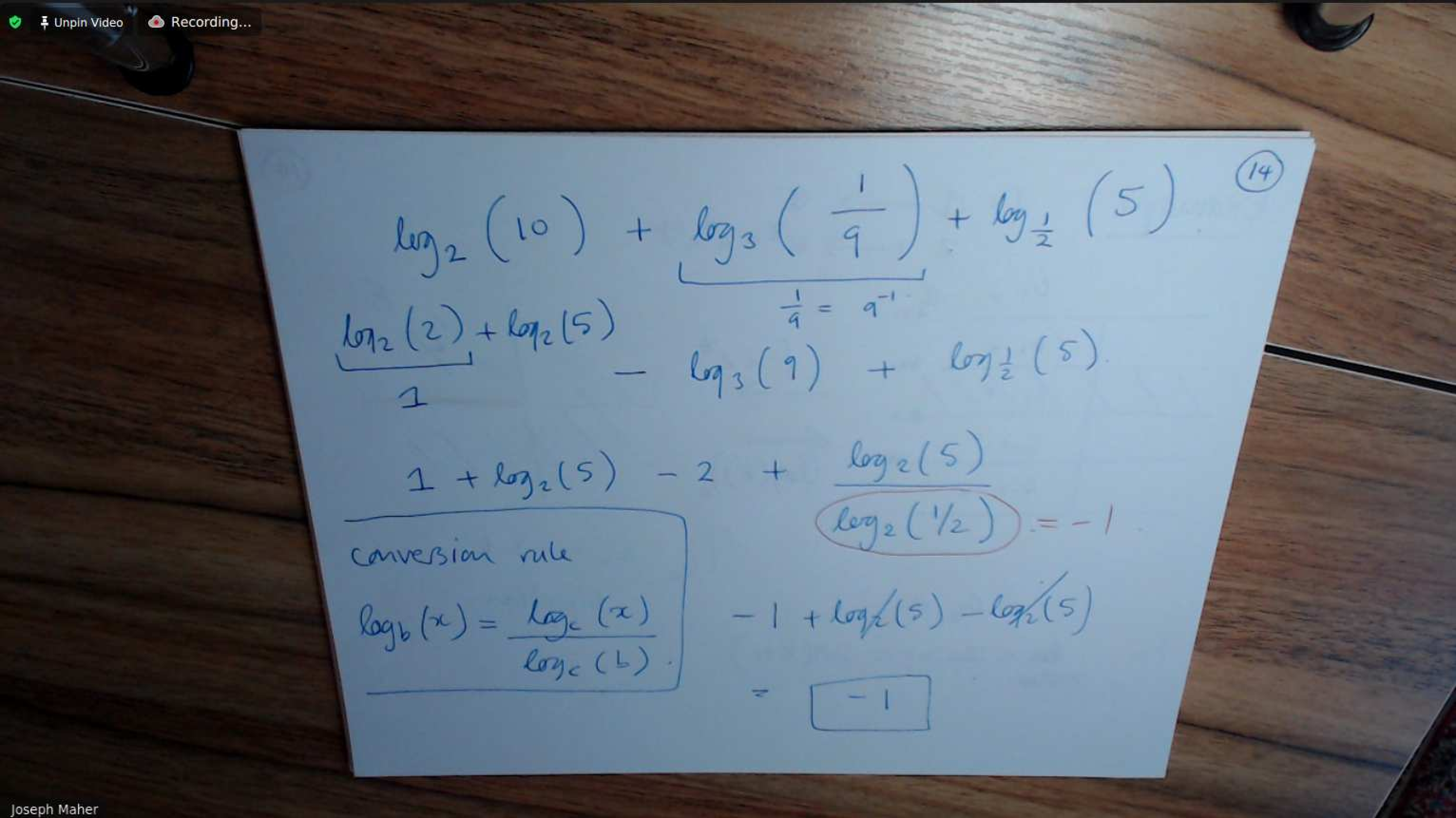
$$1 + \log_2(5) - 2 + \frac{\log_2(5)}{\boxed{\log_2(1/2)} = -1}$$

conversion rule

$$\boxed{\log_b(x) = \frac{\log_c(x)}{\log_c(b)}}$$

$$-1 + \cancel{\log_2(5)} - \cancel{\log_2(5)}$$

$$= \boxed{-1}$$



⑭

$$\log_2(10) + \underbrace{\log_3\left(\frac{1}{9}\right)}_{\frac{1}{9} = 9^{-1}} + \log_{\frac{1}{2}}(5)$$

$$\underbrace{\log_2(2)}_{1} + \log_2(5) - \log_3(9) + \log_{\frac{1}{2}}(5)$$

$$1 + \log_2(5) - 2 + \frac{\log_2(5)}{\log_2(1/2)} \quad = -1$$

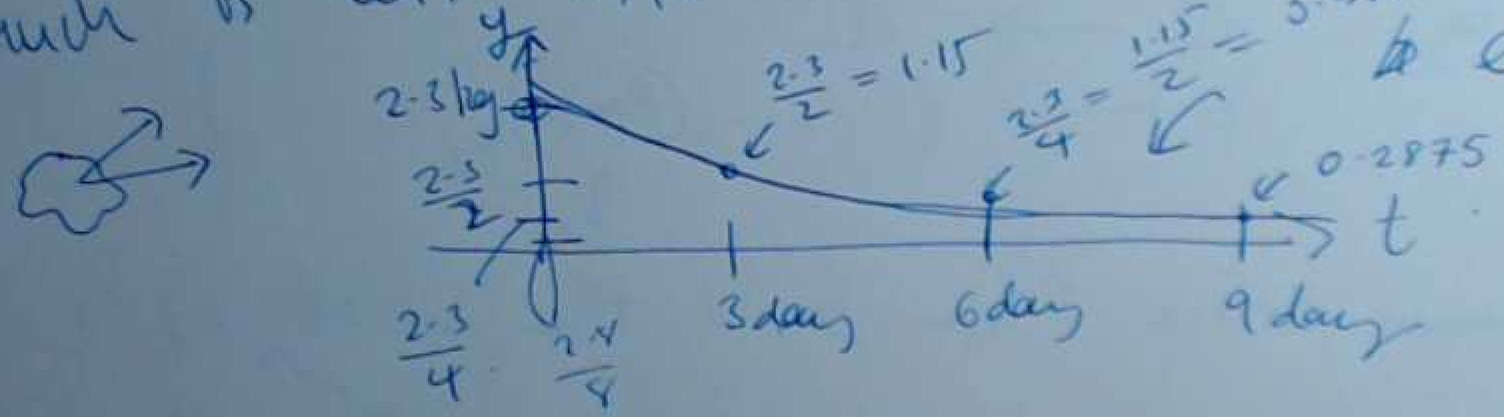
conversion rule

$$\log_b(x) = \frac{\log_c(x)}{\log_c(b)}.$$

$$-1 + \cancel{\log_2(5)} - \cancel{\log_2(5)}$$

$$= \boxed{-1}$$

You have 2.3 kg of radioactive material with a half life of 3 days. ~~How long~~ How much is left after a week?



$~~b~~ \; e^{\lambda x}$

b^x

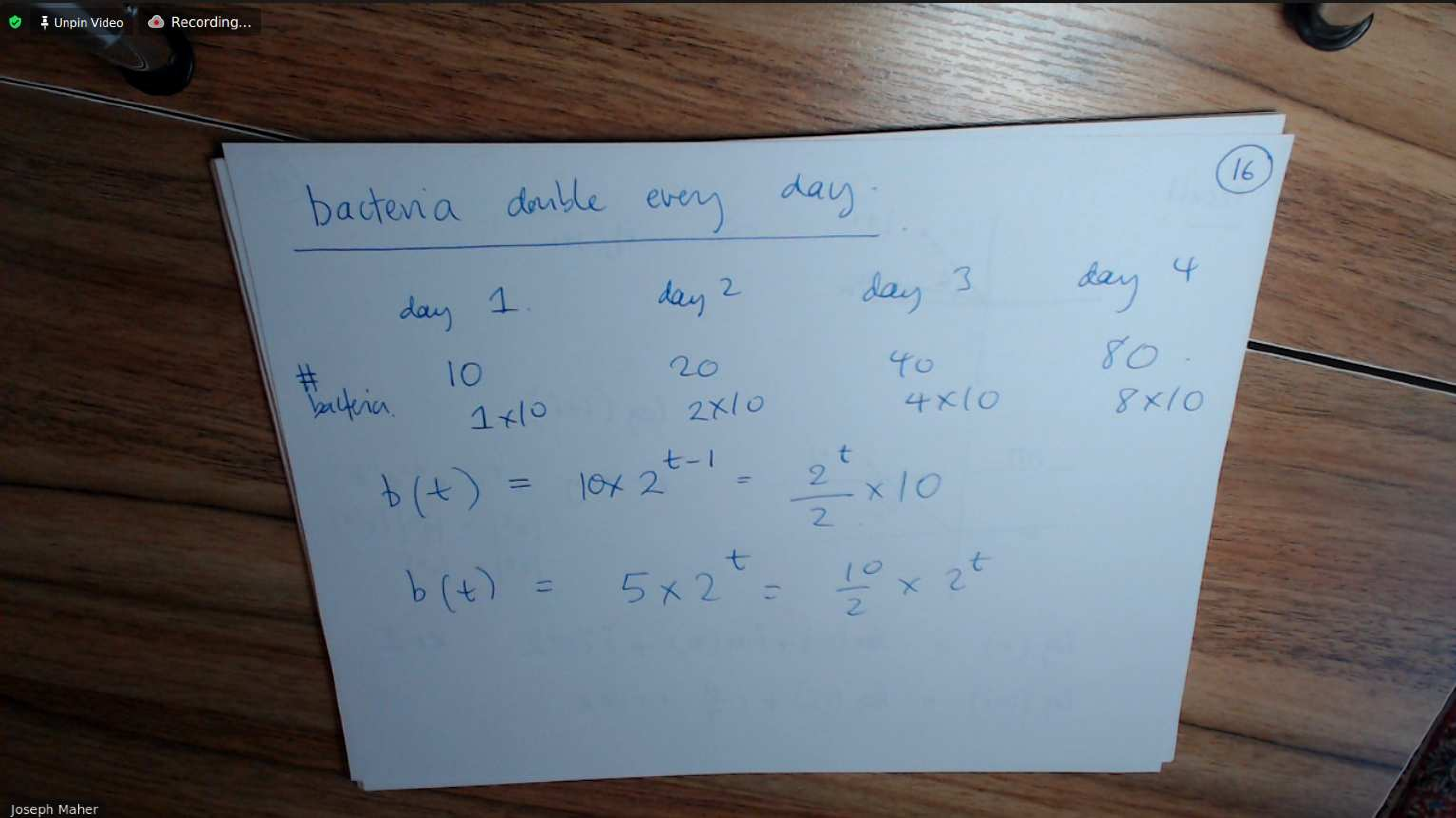
$y(0) = 2.3$

$y(3) = \frac{2.3}{2} = 1.15$

$y(6) = \frac{2.3}{4}$

$y(t) = \frac{2.3}{2^{t/3}}$

$$\boxed{y(t) = 2.3 \times 2^{-t/3}}$$

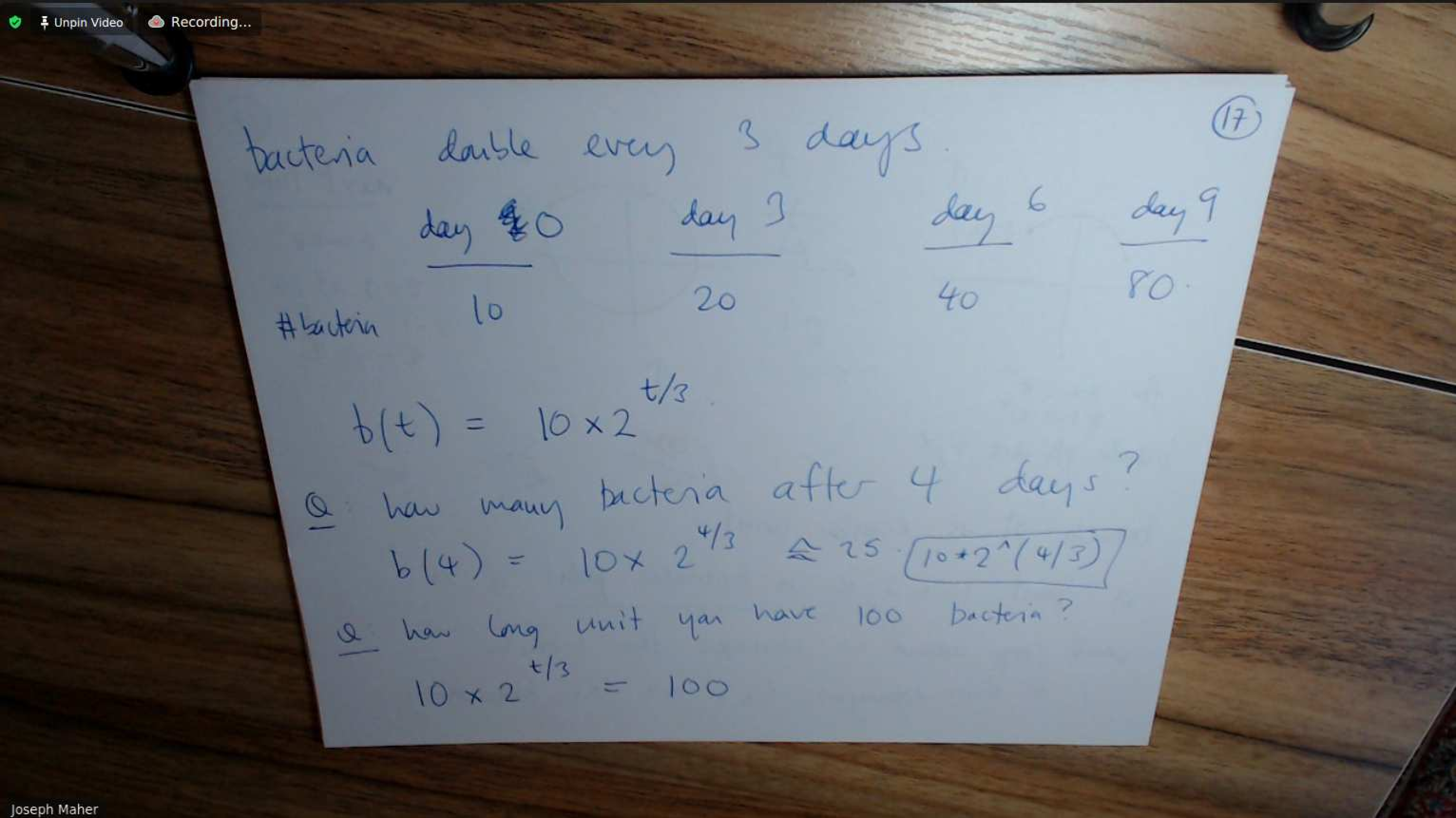


bacteria double every day

	day 1	day 2	day 3	day 4
# bacteria	10	20	40	80
	1×10	2×10	4×10	8×10

$$b(t) = 10 \times 2^{t-1} = \frac{2^t}{2} \times 10$$

$$b(t) = 5 \times 2^t = \frac{10}{2} \times 2^t$$



⑰

bacteria double every 3 days.

	day 0	day 3	day 6	day 9
#bacteria	10	20	40	80.

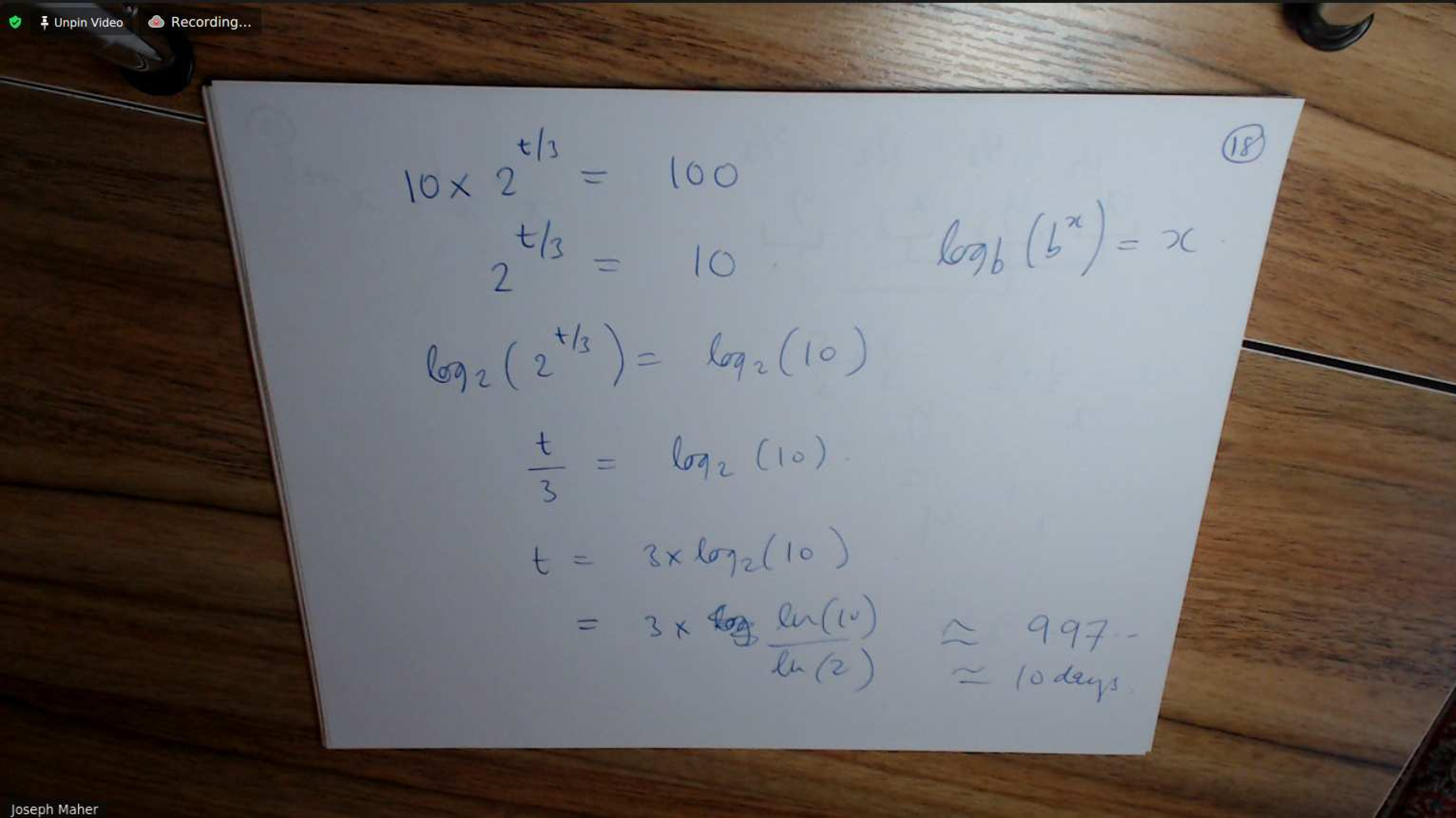
$$b(t) = 10 \times 2^{t/3}.$$

Q how many bacteria after 4 days?

$$b(4) = 10 \times 2^{4/3} \approx 25. \quad \boxed{10*2^\wedge(4/3)}$$

Q how long unit you have 100 bacteria?

$$10 \times 2^{t/3} = 100$$



$$10 \times 2^{t/3} = 100$$

$$2^{t/3} = 10$$

$$\log_b(b^x) = x$$

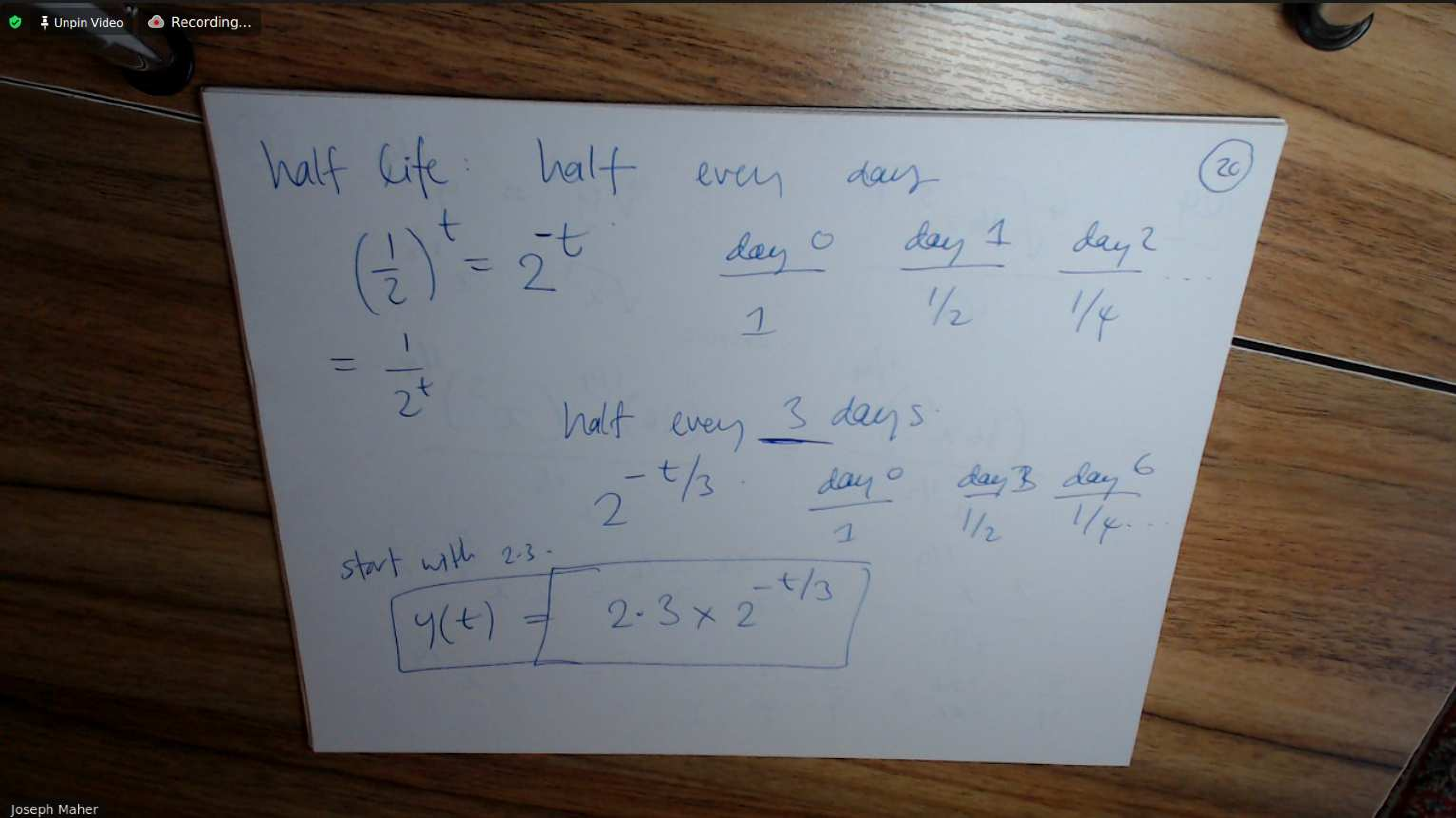
$$\log_2(2^{t/3}) = \log_2(10)$$

$$\frac{t}{3} = \log_2(10)$$

$$t = 3 \times \log_2(10)$$

$$= 3 \times \cancel{\log} \frac{\ln(10)}{\ln(2)} \simeq 997\ldots$$

$$\simeq 10 \text{ days}$$



⑳

half life: half every day

$$\left(\frac{1}{2}\right)^{t} = 2^{-t}$$

$$= \frac{1}{2^{t}}$$

day 0	day 1	day 2 ...
1	1/2	1/4

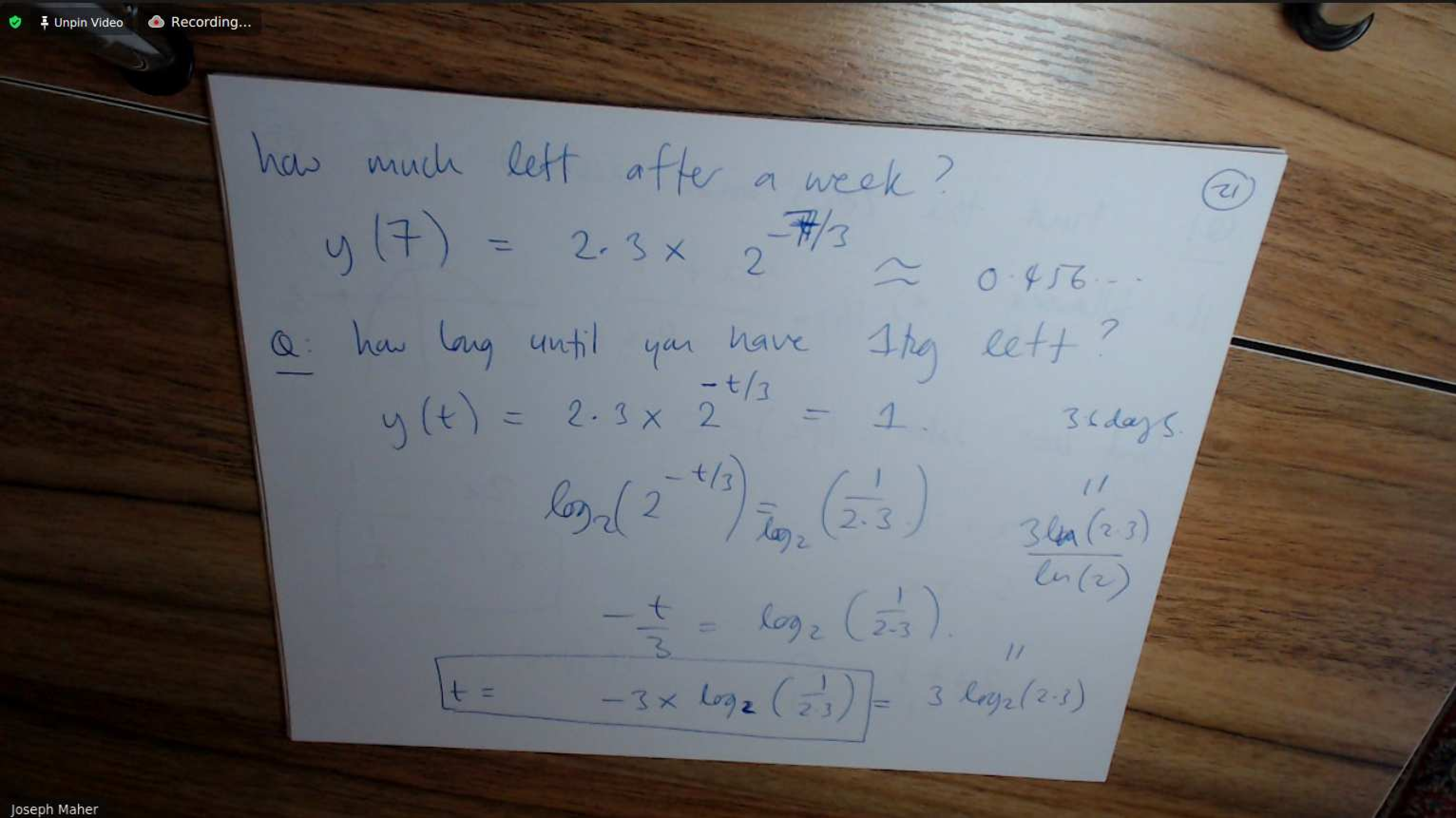
half every 3 days.

$$2^{-t/3}.$$

day 0	day 3	day 6
1	1/2	1/4 ...

start with 2.3.

$$\boxed{y(t) = 2.3 \times 2^{-t/3}}$$



how much left after a week? ㉑

$$y(7) = 2.3 \times 2^{-7/3} \approx 0.456\ldots$$

Q: how long until you have 1kg left?

$$y(t) = 2.3 \times 2^{-t/3} = 1.$$

3.6 days.

$$\log_2\left(2^{-t/3}\right) = \log_2\left(\frac{1}{2.3}\right)$$

$$\shortparallel$$

$$\frac{3\ln(2.3)}{\ln(2)}$$

$$-\frac{t}{3} = \log_2\left(\frac{1}{2.3}\right).$$

$$\shortparallel$$

$$\boxed{t = -3 \times \log_2\left(\frac{1}{2.3}\right)} = 3\log_2(2.3)$$