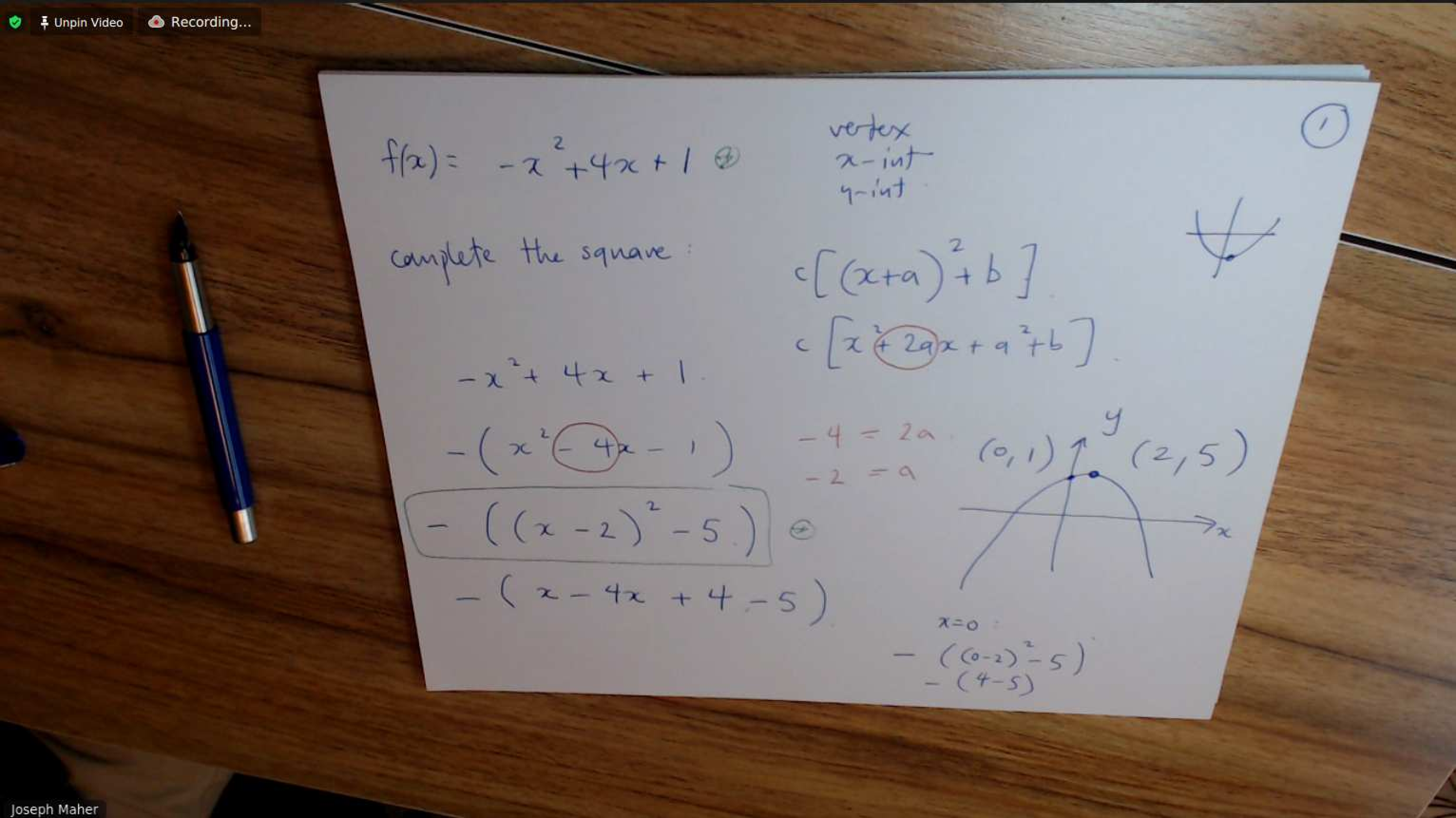


①

$f(x) = -x^2 + 4x + 1$ $\oplus$

vertex
x-int
y-int

complete the square:

$c[(x+a)^2 + b]$

$c[x^2 \boxed{+2a}x + a^2 + b]$

$-x^2 + 4x + 1$

$-(x^2 \boxed{-4}x - 1)$

$-4 = 2a$

$-2 = a$

$\boxed{-((x-2)^2 - 5)}$ $\oplus$

$-(x - 4x + 4 - 5)$

$x=0$:

$-((0-2)^2 - 5)$

$-(4-5)$

$f(x) = -x^2 + 4x + 1$ $\qquad -((x-2)^2 - 5)$.

x-intercepts.

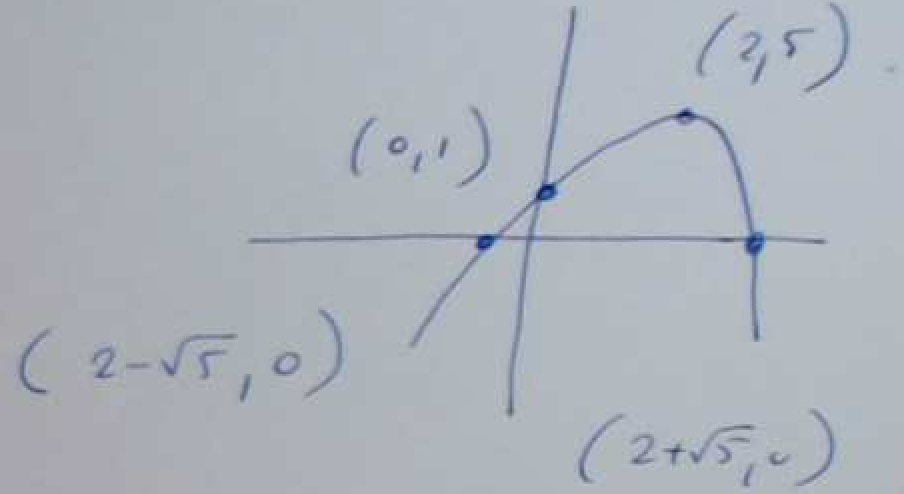
$$-((x-2)^2 - 5) = 0$$

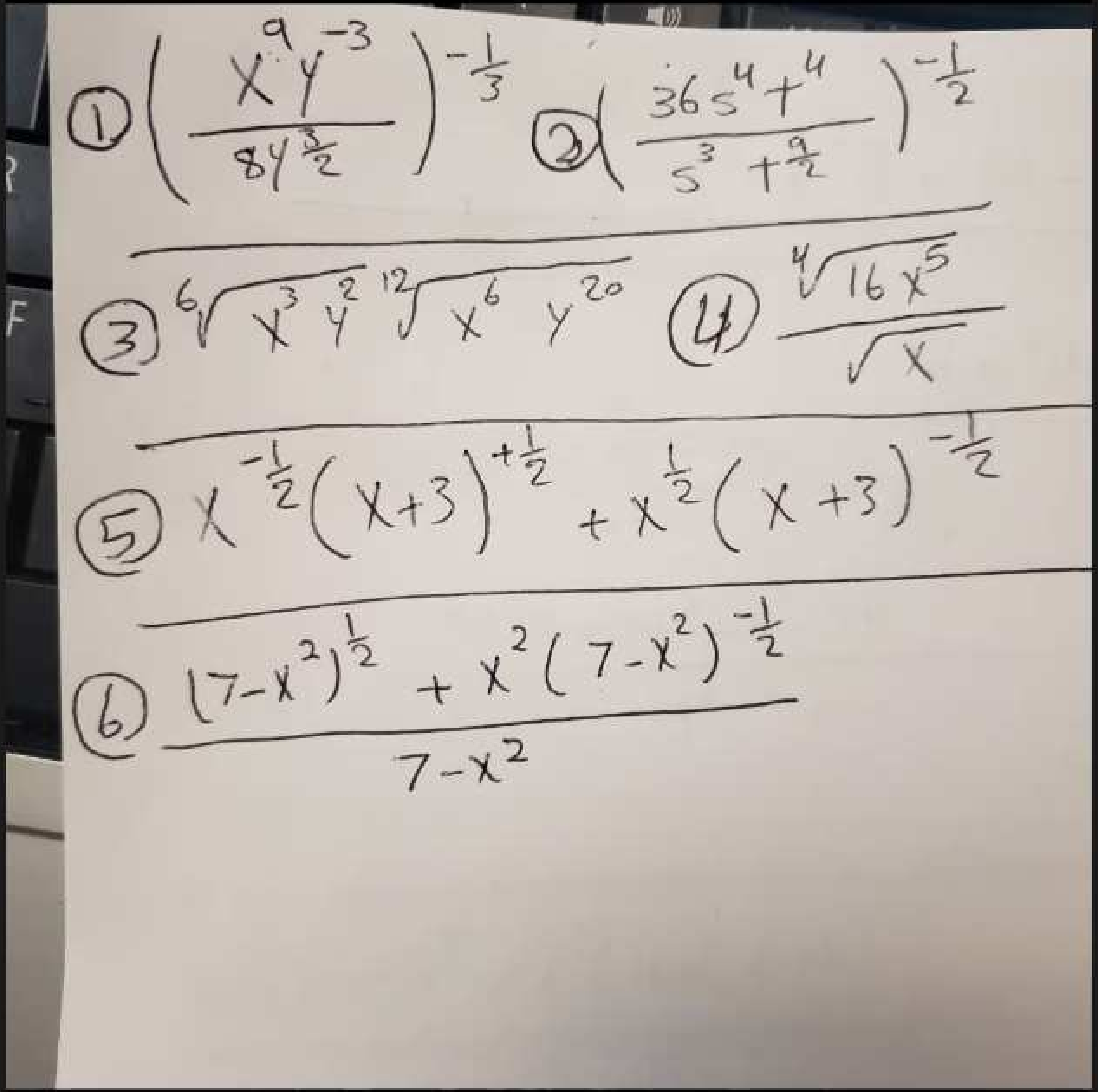
$$(x-2)^2 - 5 = 0$$

$$(x-2)^2 = 5$$

$$x - 2 = \pm\sqrt{5}$$

$$x = 2 \pm \sqrt{5}$$



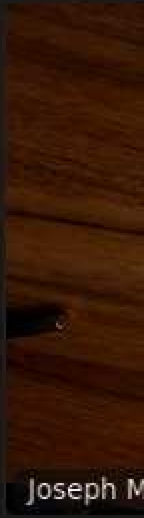


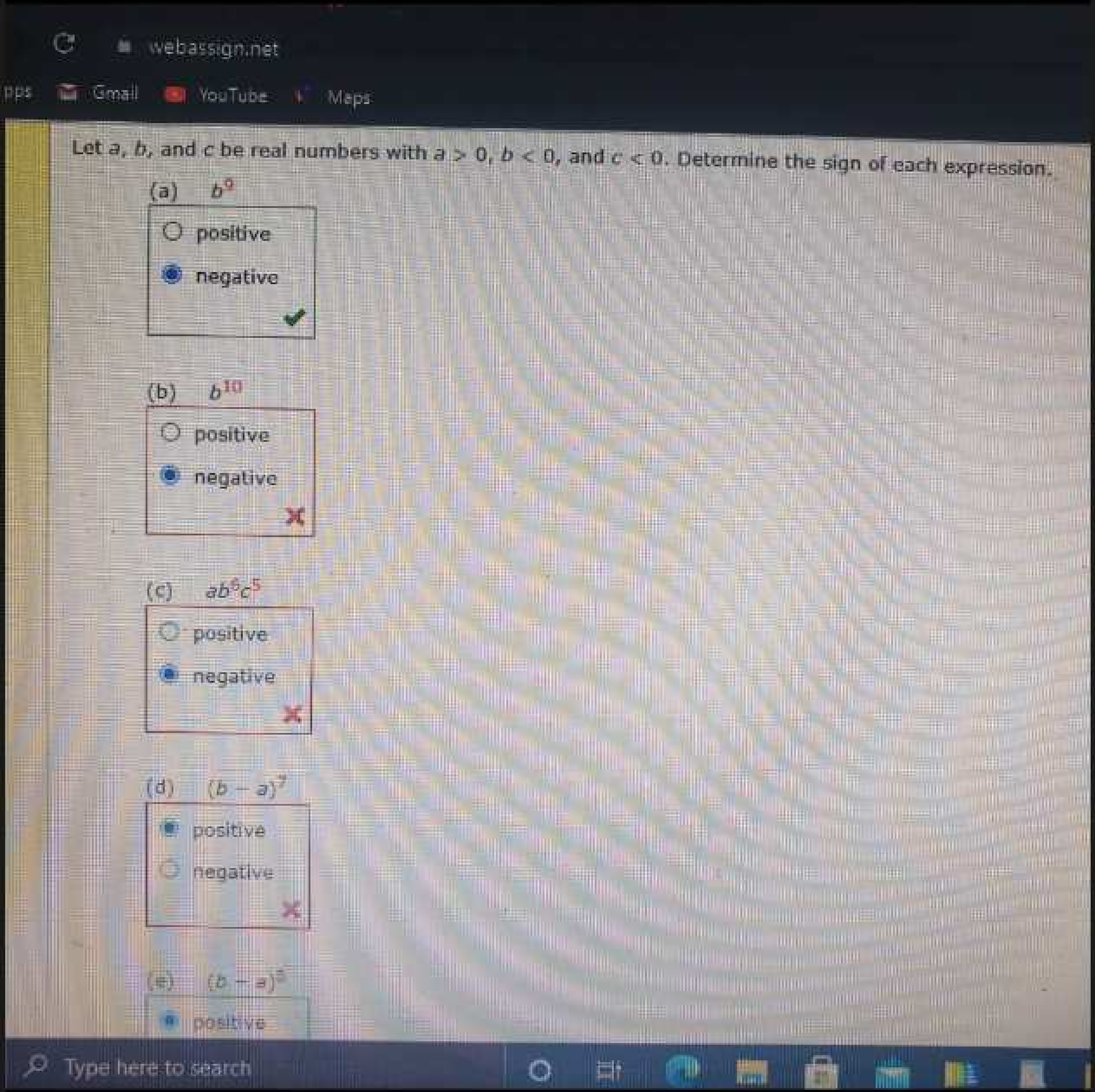
① $\left(\frac{x^{9}y^{-3}}{8y^{\frac{3}{2}}}\right)^{-\frac{1}{3}}$ ② $\left(\frac{36s^{4}t^{4}}{s^{3}t^{\frac{9}{2}}}\right)^{-\frac{1}{2}}$

③ $\sqrt[6]{x^{3}y^{2}}\,\sqrt[12]{x^{6}y^{20}}$ ④ $\frac{\sqrt[4]{16x^{5}}}{\sqrt{x}}$

⑤ $x^{-\frac{1}{2}}(x+3)^{+\frac{1}{2}}+x^{\frac{1}{2}}(x+3)^{-\frac{1}{2}}$

⑥ $\frac{(7-x^{2})^{\frac{1}{2}}+x^{2}(7-x^{2})^{-\frac{1}{2}}}{7-x^{2}}$





Let a, b, and c be real numbers with $a > 0$, $b < 0$, and $c < 0$. Determine the sign of each expression.

(a) b^9

○ positive
● negative
✓

(b) b^{10}

○ positive
● negative
✗

(c) ab^6c^5

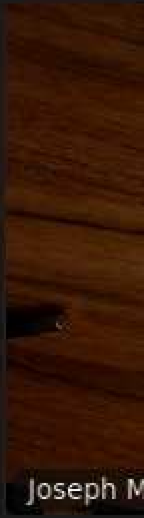
○ positive
● negative
✗

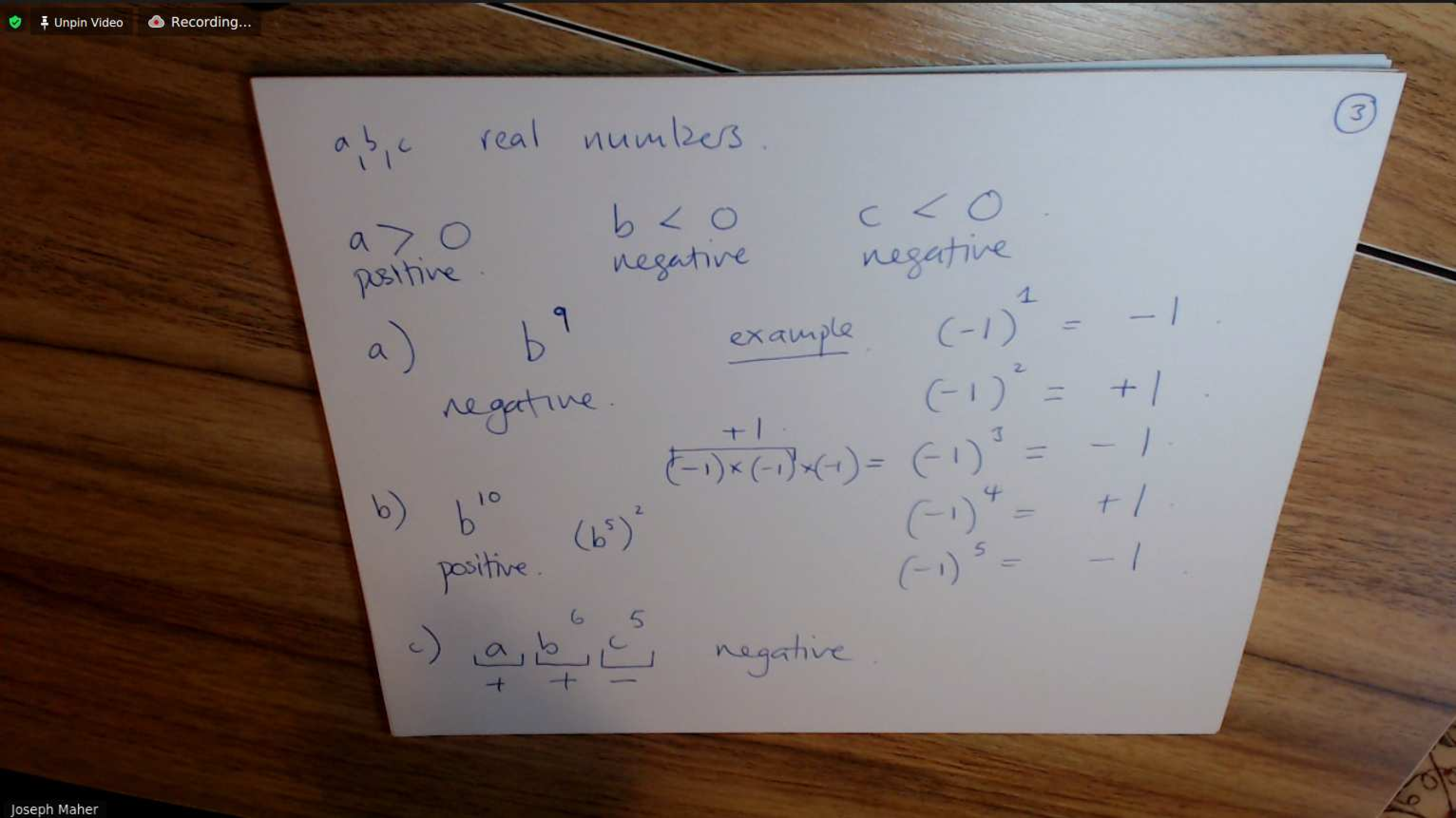
(d) $(b - a)^7$

● positive
○ negative
✗

(e) $(b - a)^8$

● positive





③

a, b, c real numbers.

$a > 0$ positive.

$b < 0$ negative

$c < 0$. negative

a) b^9

negative.

example.

$(-1)^1 = -1$.

$(-1)^2 = +1$.

$\overbrace{(-1)\times(-1)}^{+1}\times(-1) = (-1)^3 = -1$.

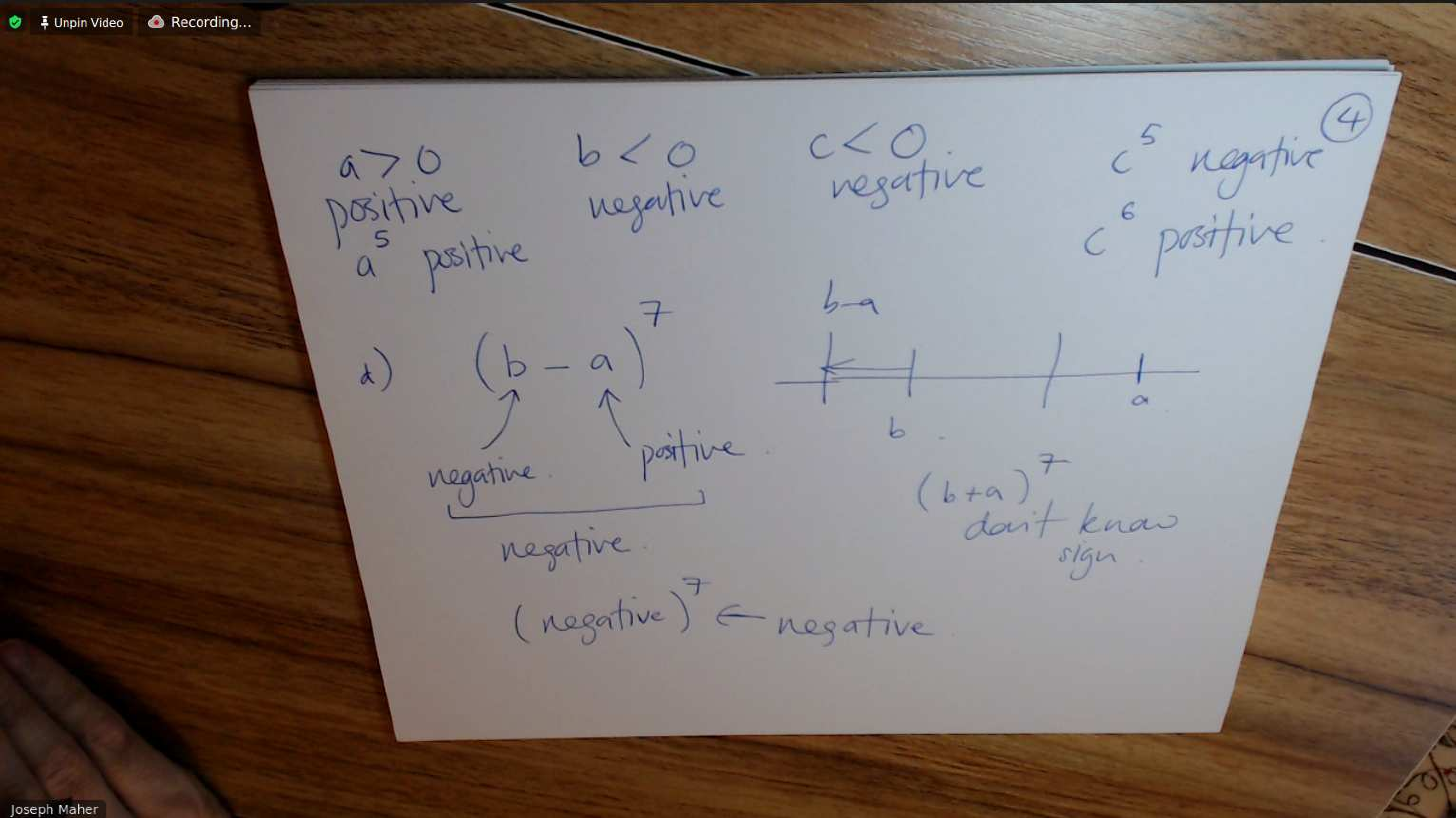
$(-1)^4 = +1$.

$(-1)^5 = -1$.

b) b^{10} $(b^5)^2$

positive.

c) $\underbrace{a}_{+}\underbrace{b^6}_{+}\underbrace{c^5}_{-}$ negative.



④

$a > 0$ positive

a^5 positive

$b < 0$ negative

$c < 0$ negative

c^5 negative

c^6 positive

d) $(b - a)^7$

b ← negative

a ← positive

$\underbrace{\text{negative} \quad \text{positive}}_{\text{negative}}$

$(\text{negative})^7 \leftarrow$ negative

$(b+a)^7$

don't know sign

⑤

Q1

$$\left(\frac{x^{9}y^{-3}}{8y^{3/2}}\right)^{-1/3}$$

$$=\left(\frac{8y^{3/2}}{x^{9}y^{-3}}\right)^{+1/3}$$

$$=\frac{8^{1/3}\left(y^{3/2}\right)^{1/3}}{\left(x^{9}\right)^{1/3}\left(y^{-3}\right)^{1/3}}=\frac{y^{\frac{3}{2}\times\frac{1}{3}}}{x^{9\times\frac{1}{3}}\;y^{-3\times\frac{1}{3}}}$$

$$\left(\frac{a}{b}\right)^{-1}=\left(\frac{b}{a}\right)^{1}=\frac{b}{a}$$

$$(ab)^{1/3}=a^{1/3}b^{1/3}$$

$$\left((a)^{n}\right)^{m}=a^{nm}$$

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⑥

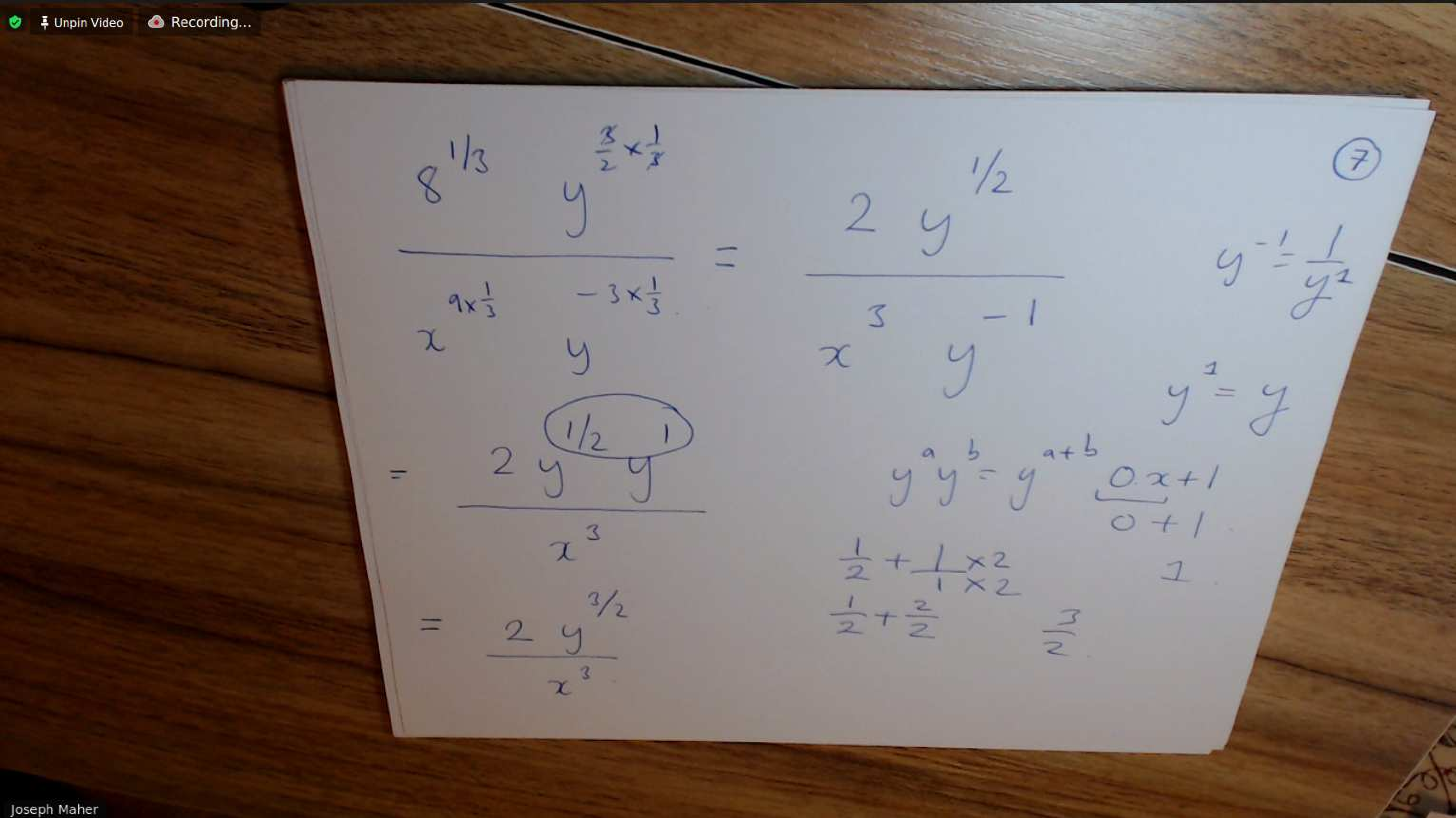
$8^{1/3}$ = cube root of 8.

$$(8^{1/3})^3 = 8^{\frac{1}{3}\times 3} = 8^1 = 8.$$

$$2\times 2\times 2 = 2^3 = 8.$$

$$8^{1/3} = (2^3)^{1/3} = 2^{3\times\frac{1}{3}} = 2^1 = 2.$$

Joseph Maher



⑦

$$\frac{8^{1/3}\; y^{\frac{\cancel{3}}{2}\times\frac{1}{\cancel{3}}}}{x^{9\times\frac{1}{3}}\; y^{-3\times\frac{1}{3}}} = \frac{2\; y^{1/2}}{x^{3}\; y^{-1}}$$

$$y^{-1} = \frac{1}{y^{1}}$$

$$y^{1} = y$$

$$= \frac{2\; \boxed{y^{1/2}\; y^{1}}}{x^{3}}$$

$$y^{a}y^{b} = y^{a+b}$$

$$\underbrace{0.x}+1$$

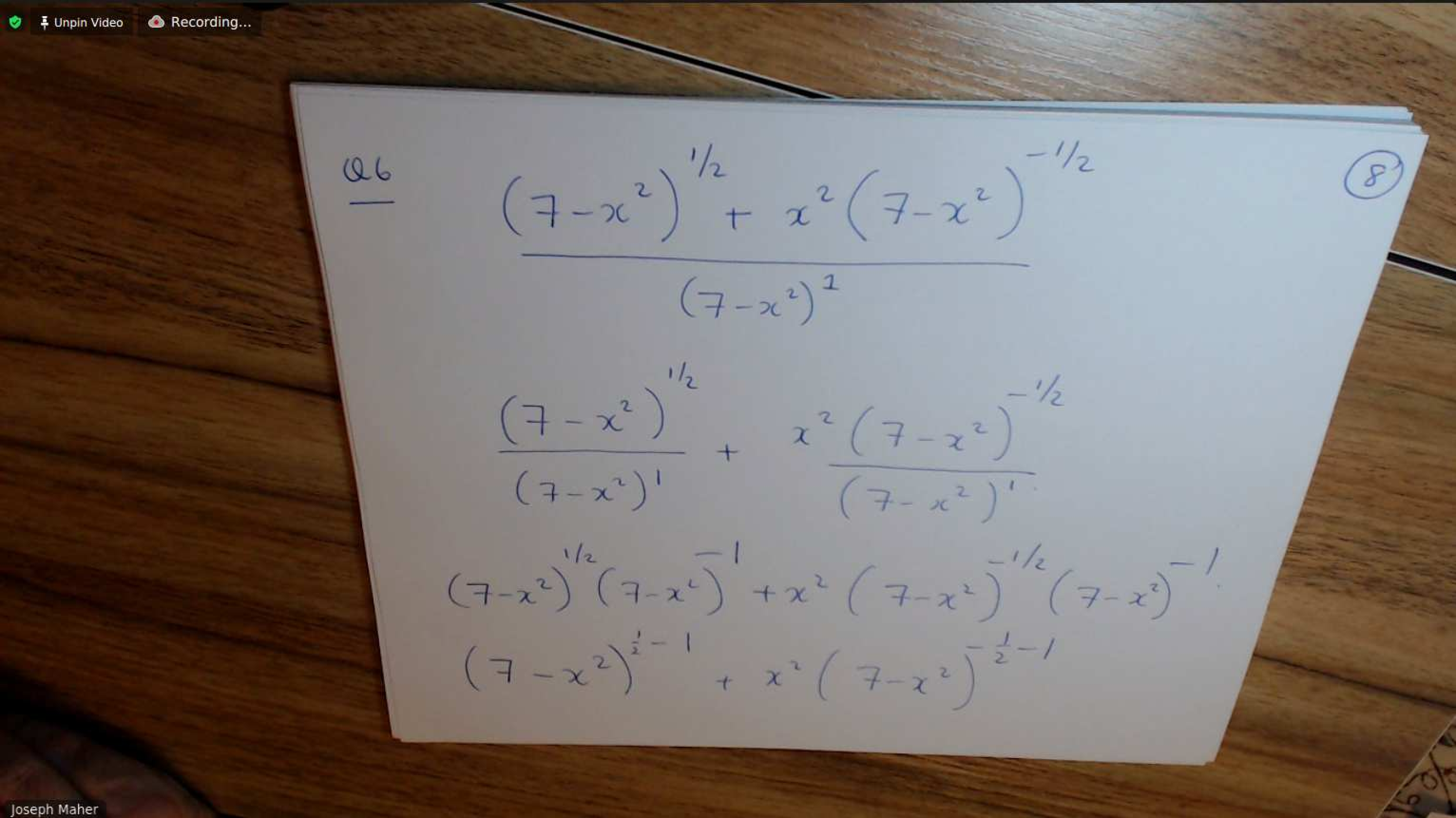
$$0+1$$

$$1$$

$$\frac{1}{2} + \frac{1\times 2}{1\times 2}$$

$$\frac{1}{2} + \frac{2}{2} \qquad \frac{3}{2}$$

$$= \frac{2\; y^{3/2}}{x^{3}}$$



<u>Q6</u>

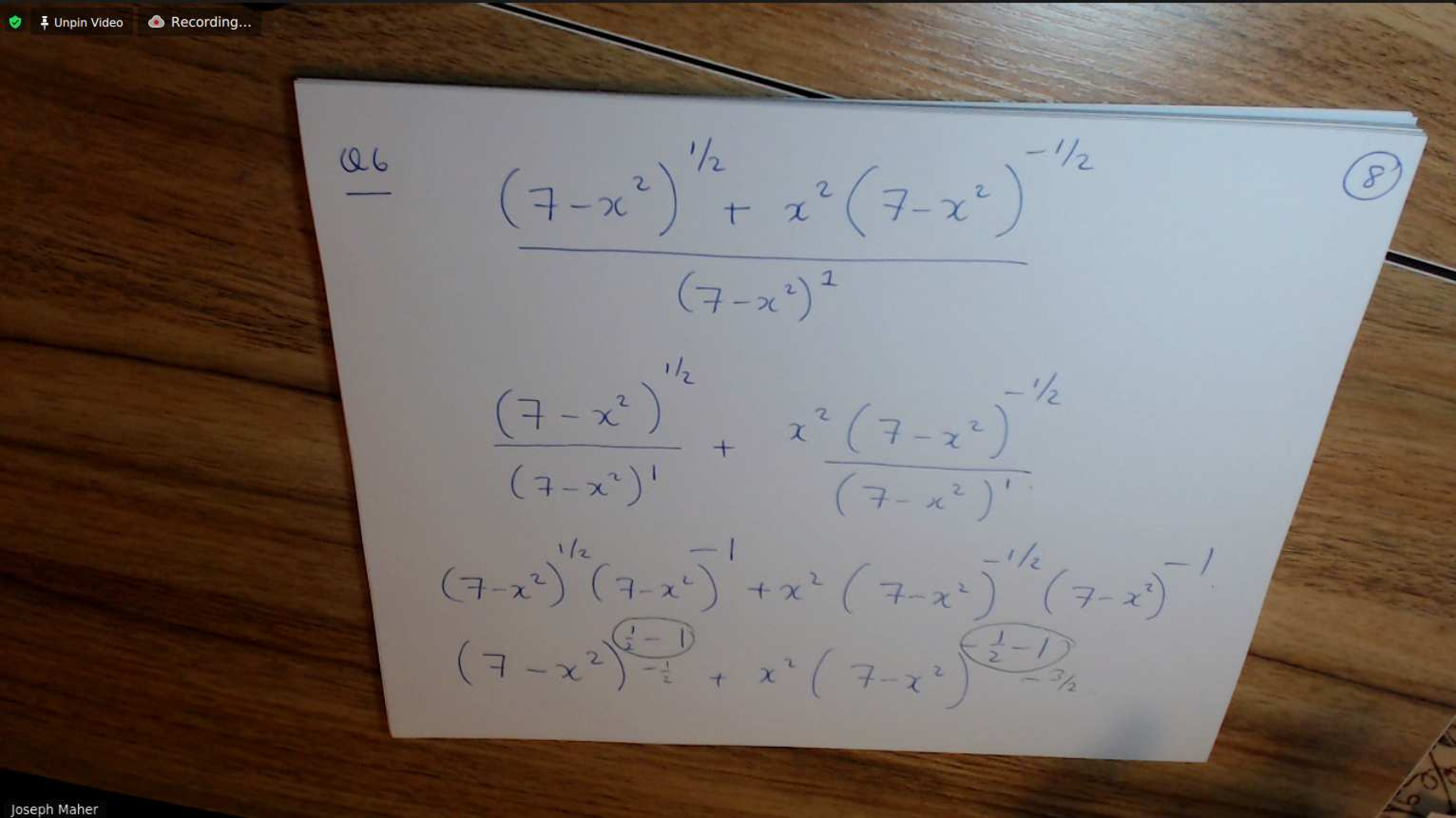
⑧

$$\frac{(7-x^2)^{1/2} + x^2(7-x^2)^{-1/2}}{(7-x^2)^{1}}$$

$$\frac{(7-x^2)^{1/2}}{(7-x^2)^{1}} + \frac{x^2(7-x^2)^{-1/2}}{(7-x^2)^{1}}$$

$$(7-x^2)^{1/2}(7-x^2)^{-1} + x^2(7-x^2)^{-1/2}(7-x^2)^{-1}$$

$$(7-x^2)^{\frac{1}{2}-1} + x^2(7-x^2)^{-\frac{1}{2}-1}$$



Q6

⑧

$$\frac{(7-x^2)^{1/2} + x^2(7-x^2)^{-1/2}}{(7-x^2)^{1}}$$

$$\frac{(7-x^2)^{1/2}}{(7-x^2)^{1}} + \frac{x^2(7-x^2)^{-1/2}}{(7-x^2)^{1}}$$

$$(7-x^2)^{1/2}(7-x^2)^{-1} + x^2(7-x^2)^{-1/2}(7-x^2)^{-1}$$

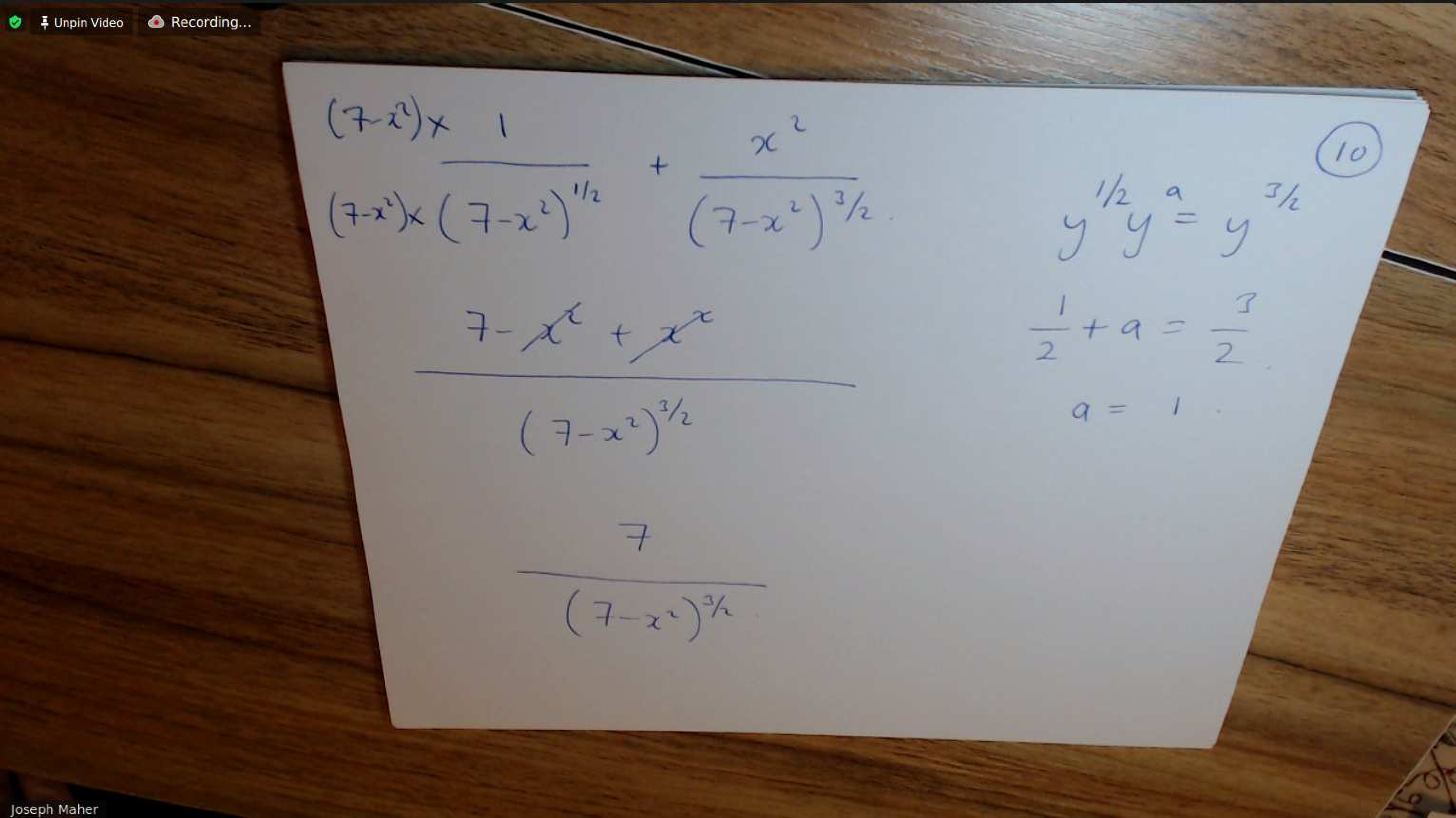
$$(7-x^2)^{\underset{-\frac{1}{2}}{\boxed{\frac{1}{2}-1}}} + x^2(7-x^2)^{\underset{-3/2}{\boxed{-\frac{1}{2}-1}}}$$

⑨

$$(7-x^2)^{-1/2} + x^2(7-x^2)^{-3/2}.$$

$$\frac{(7-x^2)\times 1}{(7-x^2)\times(7-x^2)^{1/2}} + \frac{x^2}{(7-x^2)^{3/2}}.$$

$$\frac{(7-\cancel{x^2}) + \cancel{x^2}}{(7-x^2)^{3/2}}$$



$$\frac{(7-x^2)\times 1}{(7-x^2)\times(7-x^2)^{1/2}} + \frac{x^2}{(7-x^2)^{3/2}}$$

$$\frac{7-\cancel{x^2}+\cancel{x^2}}{(7-x^2)^{3/2}}$$

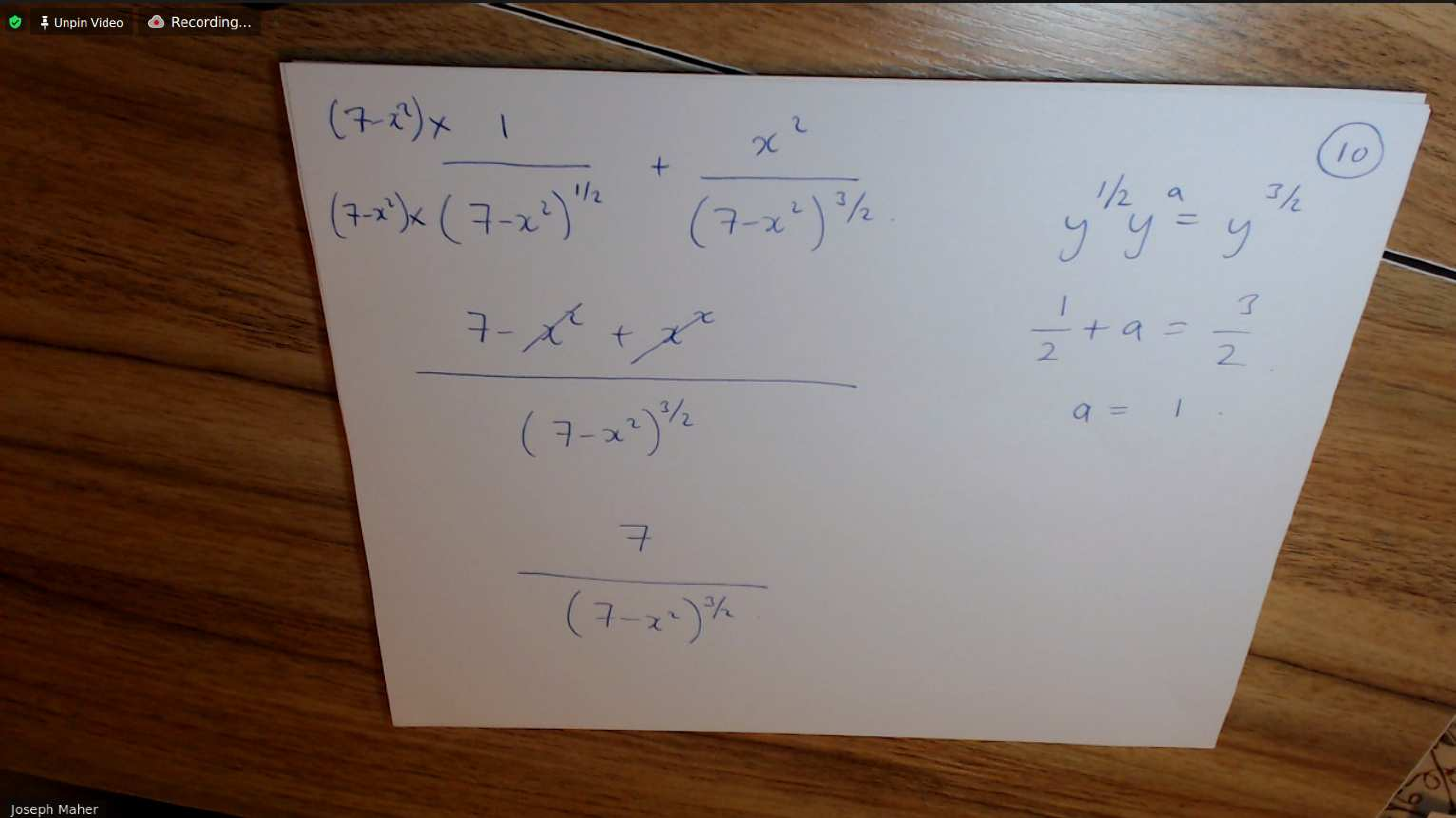
$$\frac{7}{(7-x^2)^{3/2}}$$

⑩

$$y^{1/2}y^{a} = y^{3/2}$$

$$\frac{1}{2}+a=\frac{3}{2}$$

$$a = 1$$



$$\frac{(7-x^2)\times 1}{(7-x^2)\times(7-x^2)^{1/2}} + \frac{x^2}{(7-x^2)^{3/2}}$$

$$\frac{7-\cancel{x^2}+\cancel{x^2}}{(7-x^2)^{3/2}}$$

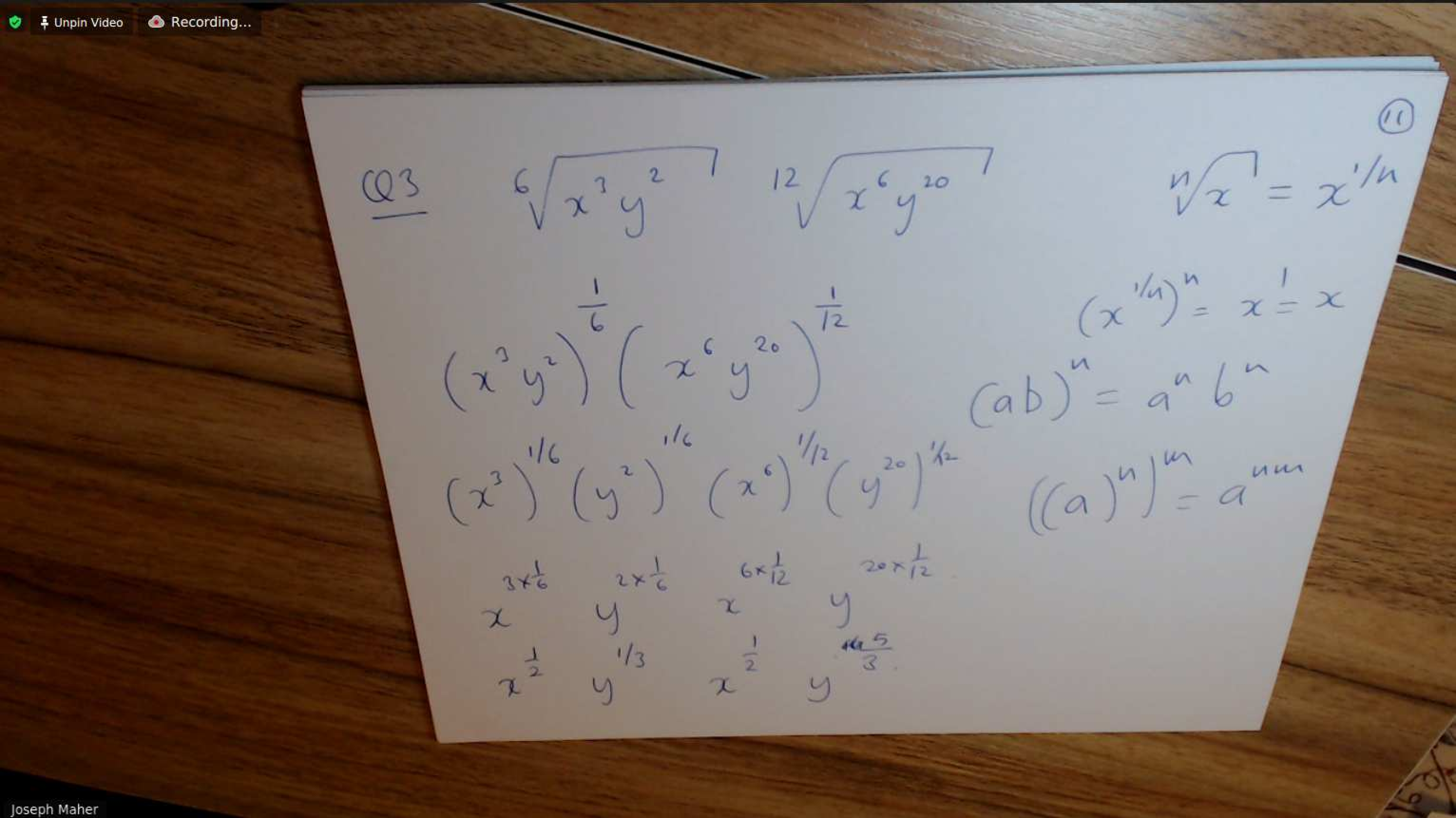
$$\frac{7}{(7-x^2)^{3/2}}$$

⑩

$$y^{1/2}y^{a} = y^{3/2}$$

$$\frac{1}{2}+a=\frac{3}{2}$$

$$a = 1$$



⑪

Q3

$$\sqrt[6]{x^3 y^2} \qquad \sqrt[12]{x^6 y^{20}} \qquad \sqrt[n]{x} = x^{1/n}$$

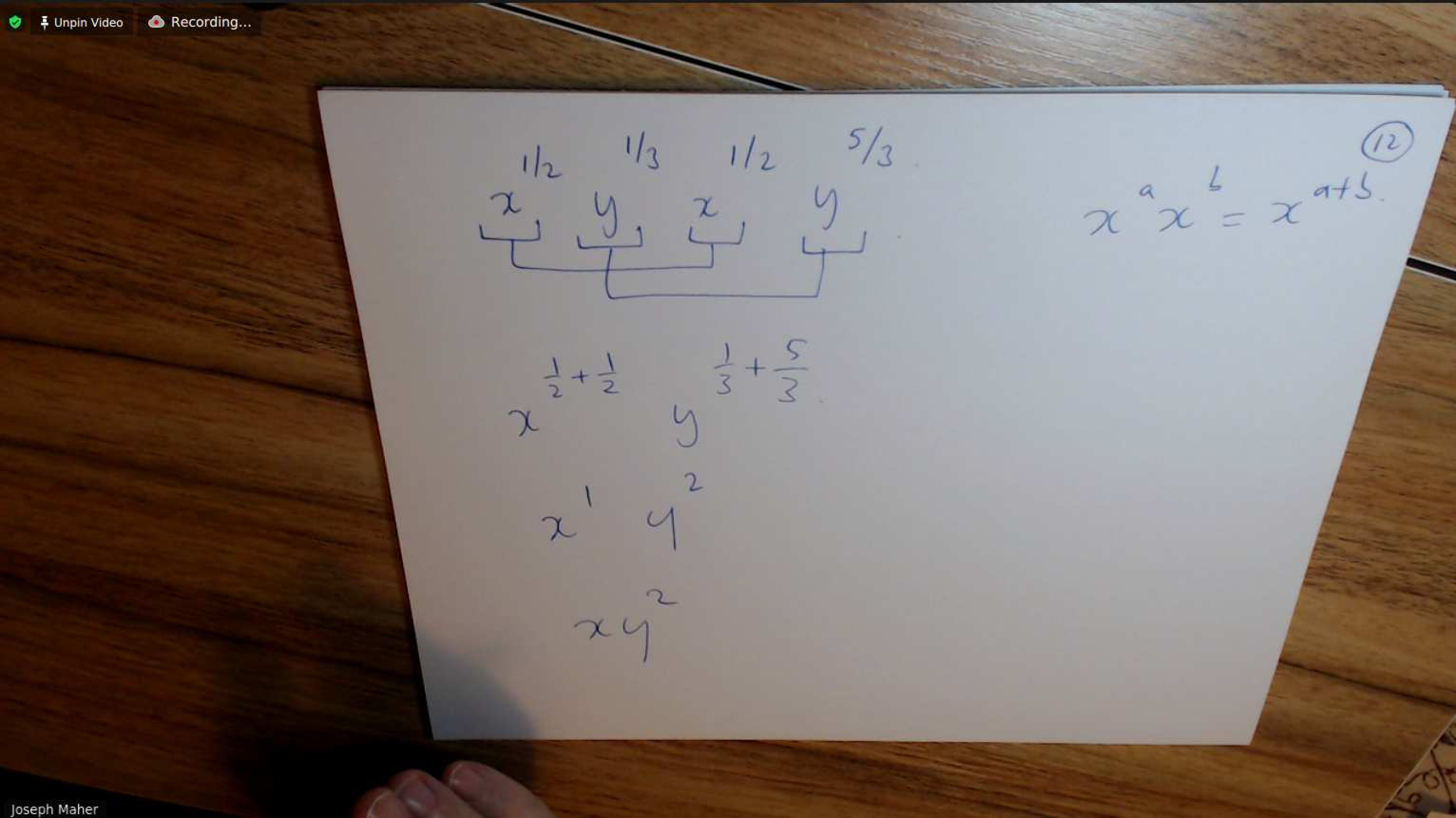
$$(x^3y^2)^{\frac{1}{6}} \left(x^6 y^{20} \right)^{\frac{1}{12}} \qquad (x^{1/n})^n = x^{\frac{1}{}} = x$$

$$(ab)^n = a^n b^n$$

$$(x^3)^{1/6} (y^2)^{1/6} (x^6)^{1/12} (y^{20})^{1/12} \qquad ((a)^n)^m = a^{nm}$$

$$x^{3 \times \frac{1}{6}} \quad y^{2 \times \frac{1}{6}} \quad x^{6 \times \frac{1}{12}} \quad y^{20 \times \frac{1}{12}}$$

$$x^{\frac{1}{2}} \quad y^{1/3} \quad x^{\frac{1}{2}} \quad y^{\frac{5}{3}}$$



⑫

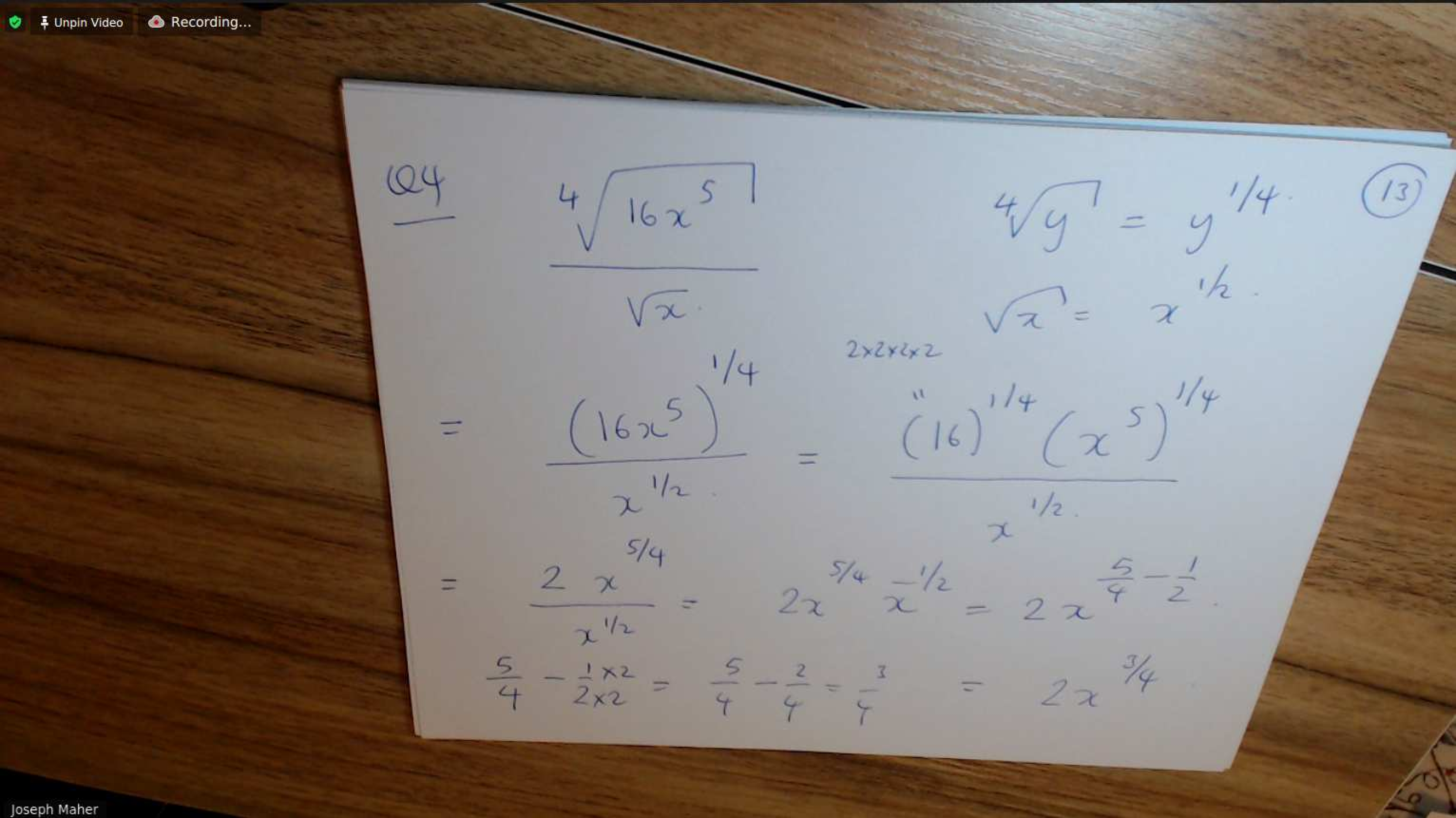
$$x^{1/2}\,y^{1/3}\,x^{1/2}\,y^{5/3}$$

$$x^a x^b = x^{a+b}$$

$$x^{\frac{1}{2}+\frac{1}{2}}\quad y^{\frac{1}{3}+\frac{5}{3}}$$

$$x^1\ y^2$$

$$xy^2$$



⑬

$\underline{\text{Q4}}$

$$\frac{\sqrt[4]{16x^5}}{\sqrt{x}}. \qquad \sqrt[4]{y} = y^{1/4}. \qquad \sqrt{x} = x^{1/2}.$$

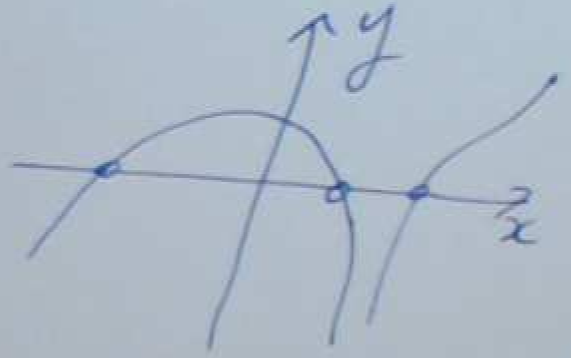
$$= \frac{\left(16x^5\right)^{1/4}}{x^{1/2}}. = \frac{\overset{\overset{2\times2\times2\times2}{''}}{(16)}^{1/4}\left(x^5\right)^{1/4}}{x^{1/2}}.$$

$$= \frac{2\;x^{5/4}}{x^{1/2}} = 2x^{5/4}\,x^{-1/2} = 2x^{\frac{5}{4}-\frac{1}{2}}.$$

$$\frac{5}{4} - \frac{1\times2}{2\times2} = \frac{5}{4} - \frac{2}{4} = \frac{3}{4} \qquad = 2x^{3/4}$$

Q1 Find the zeros and domain of the following:

a) $f(x) = \dfrac{2x+1}{5-4x}$



find zeros: solve $f(x) = 0$

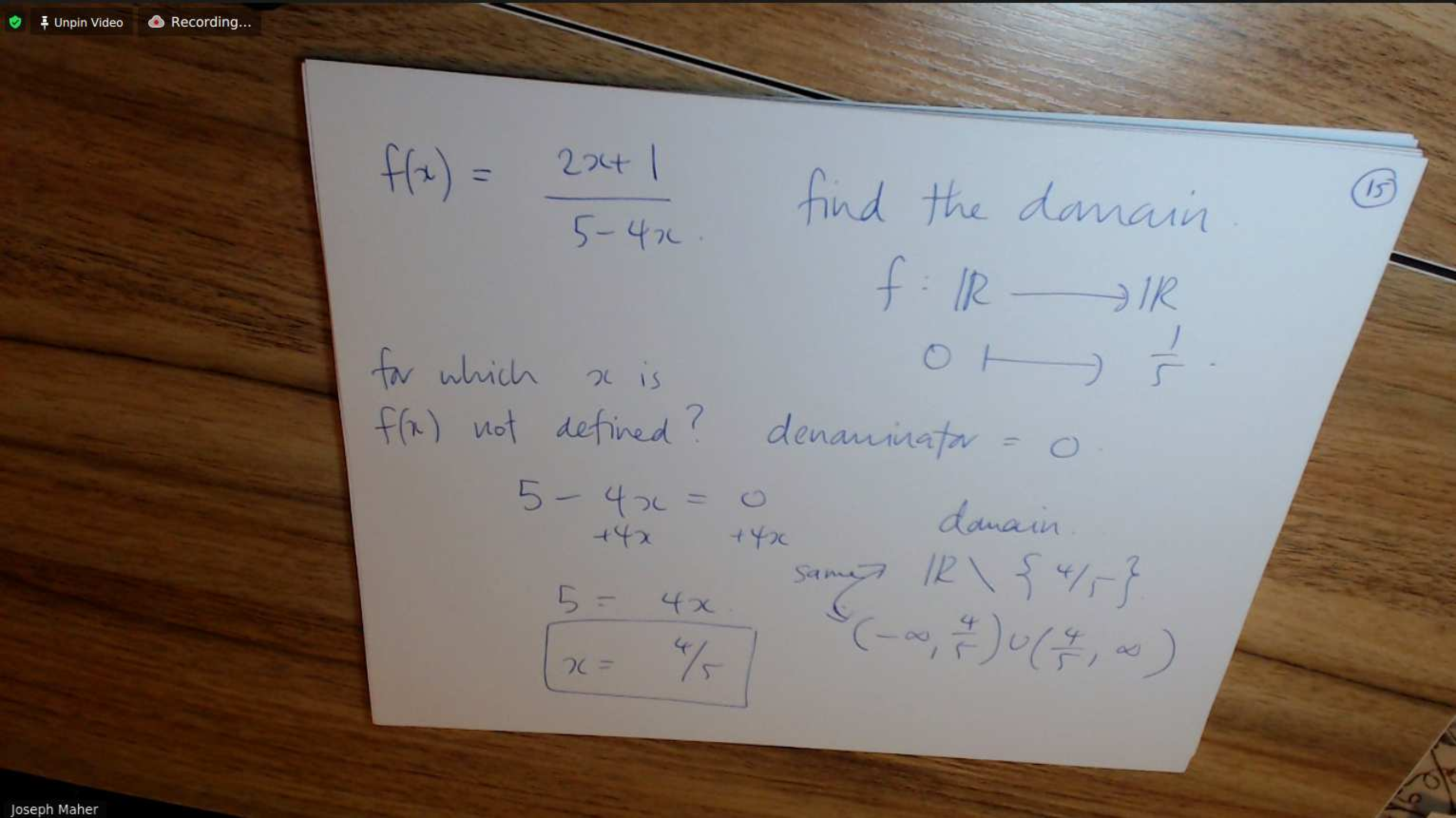
$$\frac{2x+1}{5-4x} = 0$$

$$\begin{aligned} 2x+1 &= 0 \\ -1 \quad & \quad -1 \end{aligned}$$

$$2x = -1$$

$$\boxed{x = -\frac{1}{2}}$$

$f(-\frac{1}{2}) = 0$



$f(x) = \frac{2x+1}{5-4x}$. find the domain. ⑮

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$0 \longmapsto \frac{1}{5}.$$

for which x is $f(x)$ not defined? denominator $= 0$.

$$5 - 4x = 0$$
$$+4x \qquad +4x$$

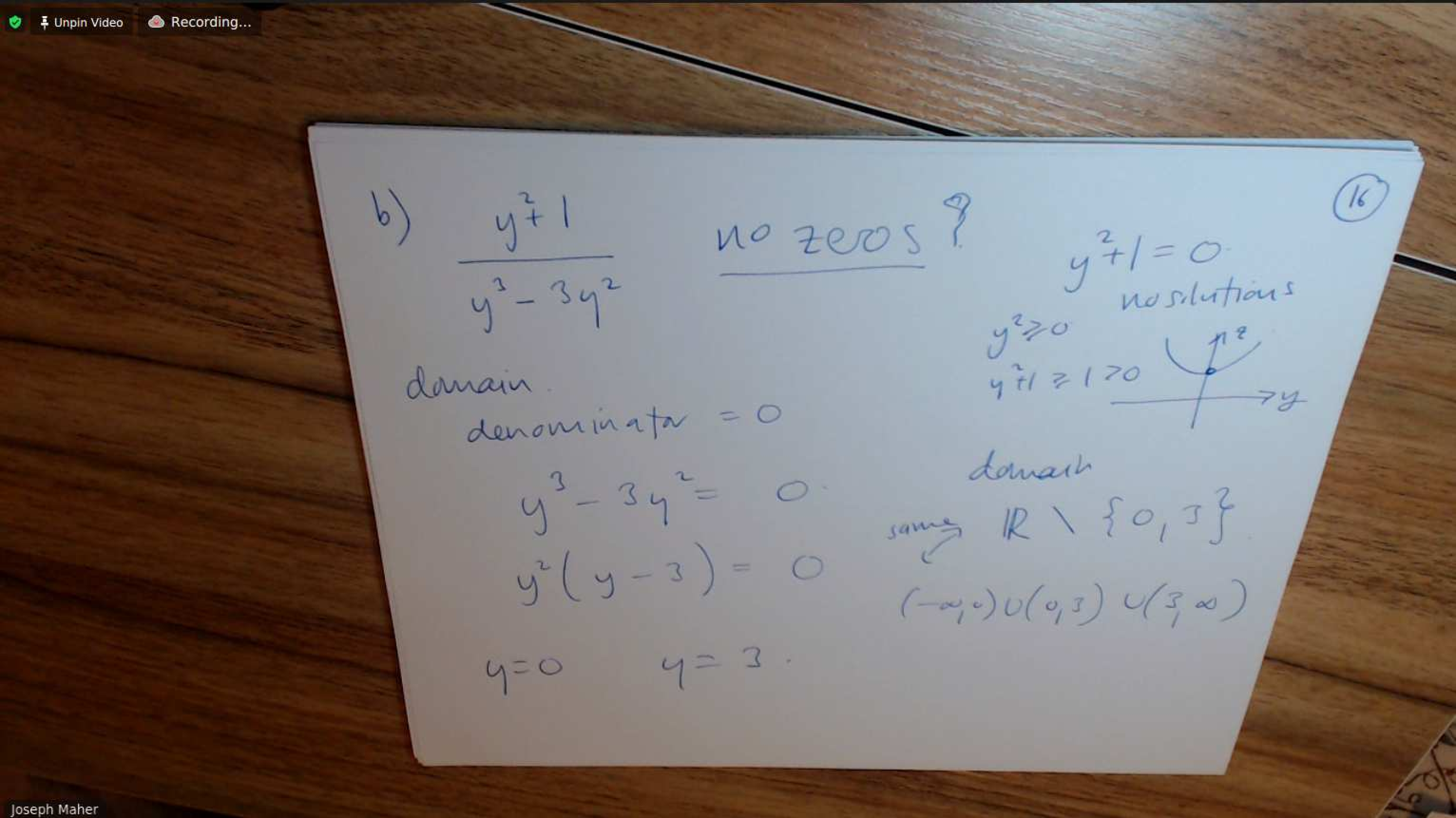
$$5 = 4x.$$

$$\boxed{x = 4/5}$$

domain.

same $\rightarrow$ $\mathbb{R} \setminus \{4/5\}$

$$(-\infty, \tfrac{4}{5}) \cup (\tfrac{4}{5}, \infty)$$



b) $\dfrac{y^2+1}{y^3-3y^2}$

no zeros?

$y^2+1=0$
no solutions

$y^2 \geq 0$
$y^2+1 \geq 1 > 0$

domain.
denominator $= 0$

$y^3-3y^2 = 0$
$y^2(y-3) = 0$

$y=0 \qquad y=3$.

domain
$\mathbb{R} \setminus \{0,3\}$

same

$(-\infty,0)\cup(0,3)\cup(3,\infty)$

Q2

⑰

$$\left(\frac{36\,s^4\,t^4}{s^3\,t^{9/2}}\right)^{-\frac{1}{2}}$$

$$\left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^{+1}$$

$$= \left(\frac{s^3\,t^{9/2}}{36\,s^4 t^4}\right)^{1/2}$$

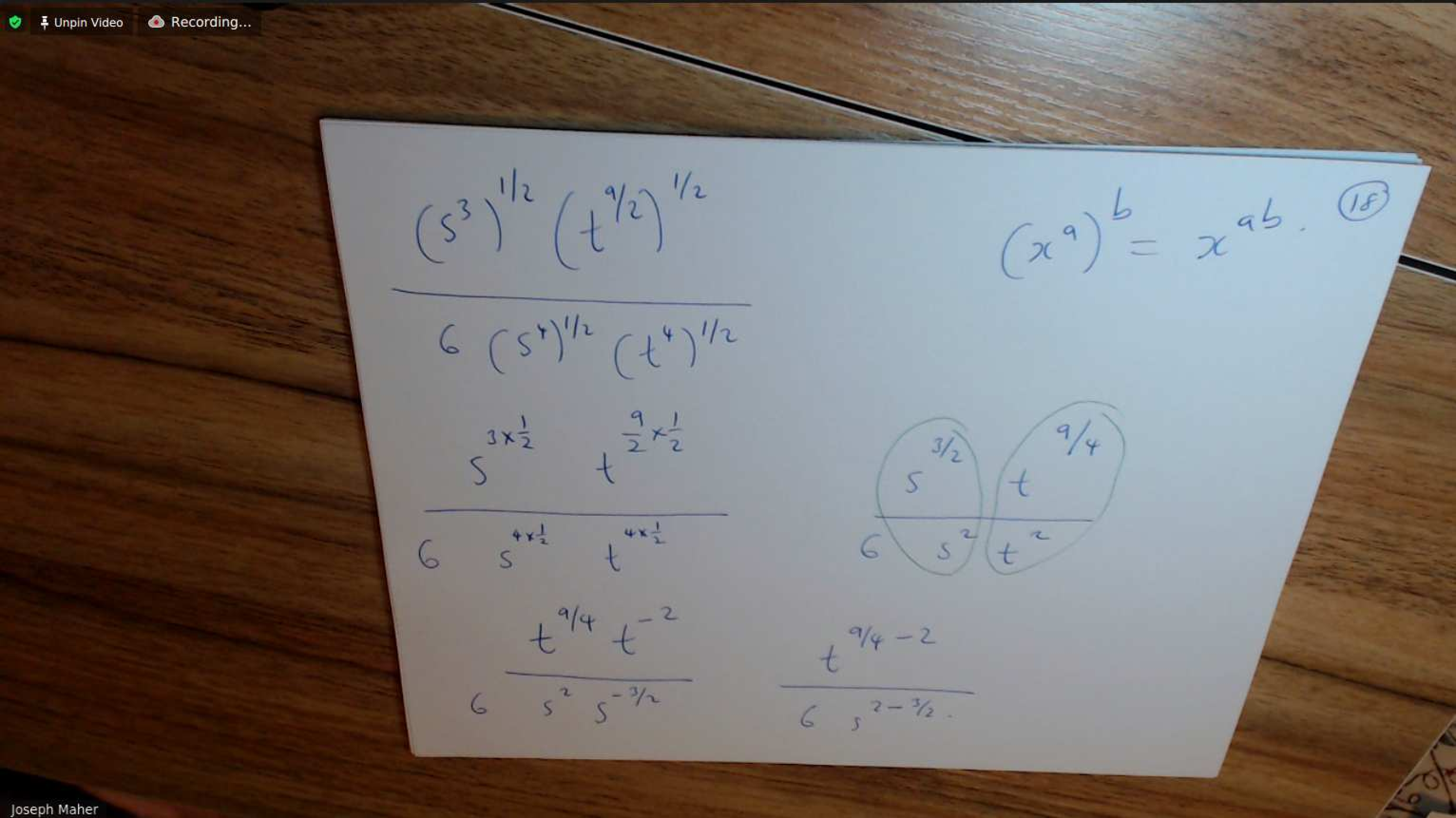
$$\sqrt[n]{x} = x^{1/n}$$

$$\sqrt{x} = x^{1/2}$$

$$= s^{3/2} \quad \frac{(s^3)^{1/2}(t^{9/2})^{1/2}}{(36)^{1/2}(s^4)^{1/2}(t^4)^{1/2}}$$

$$(36)^{1/2} = \sqrt{36}$$

$$= 6$$



$$\frac{(s^3)^{1/2}(t^{9/2})^{1/2}}{6(s^4)^{1/2}(t^4)^{1/2}}$$

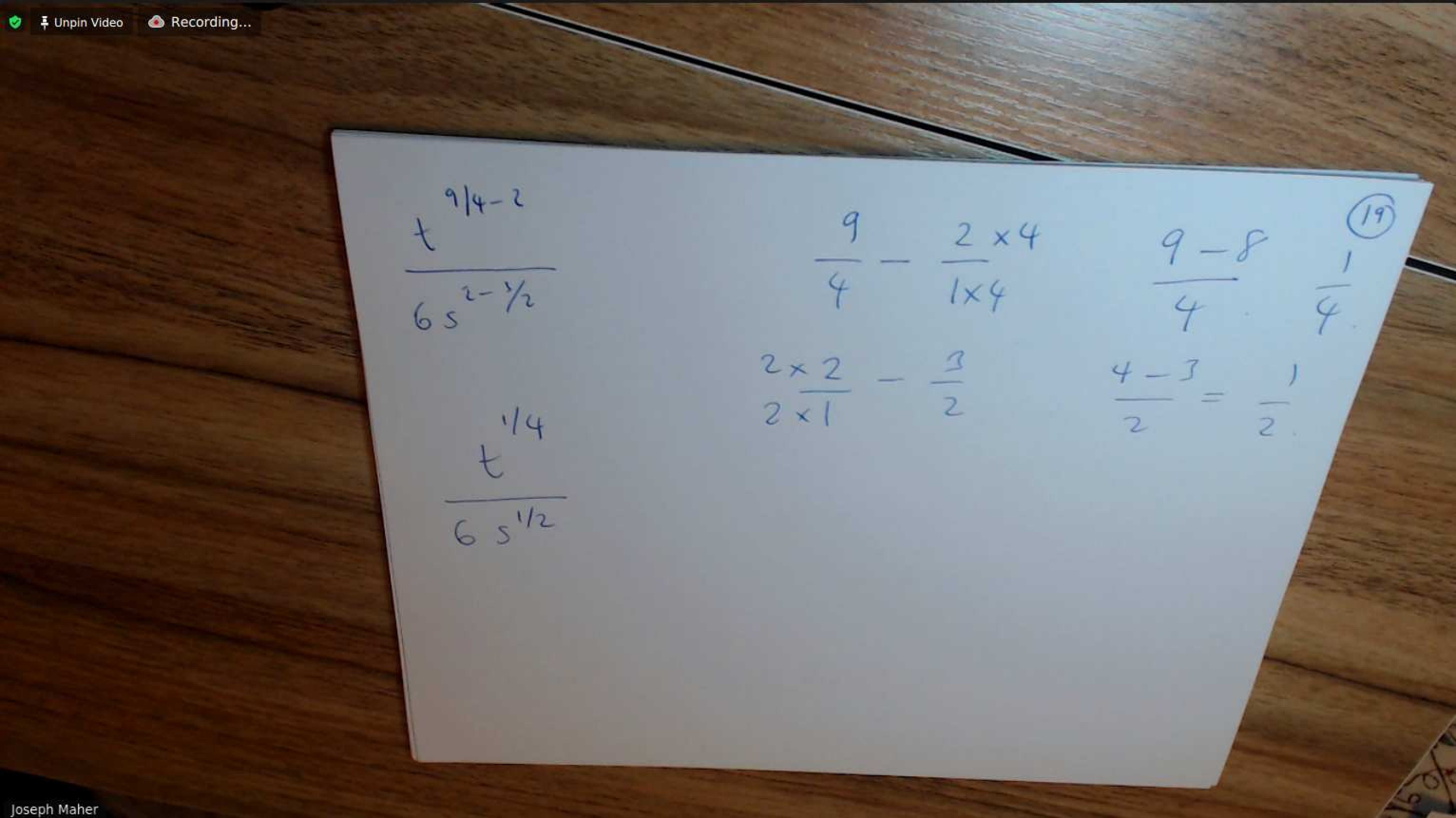
$(x^a)^b = x^{ab}$. ⑱

$$\frac{s^{3\times\frac{1}{2}}\quad t^{\frac{9}{2}\times\frac{1}{2}}}{6\quad s^{4\times\frac{1}{2}}\quad t^{4\times\frac{1}{2}}}$$

$$\frac{s^{3/2}\ t^{9/4}}{6\quad s^{2}\ t^{2}}$$

$$\frac{t^{9/4}\,t^{-2}}{6\quad s^{2}\,s^{-3/2}}$$

$$\frac{t^{9/4-2}}{6\ s^{2-3/2}}.$$



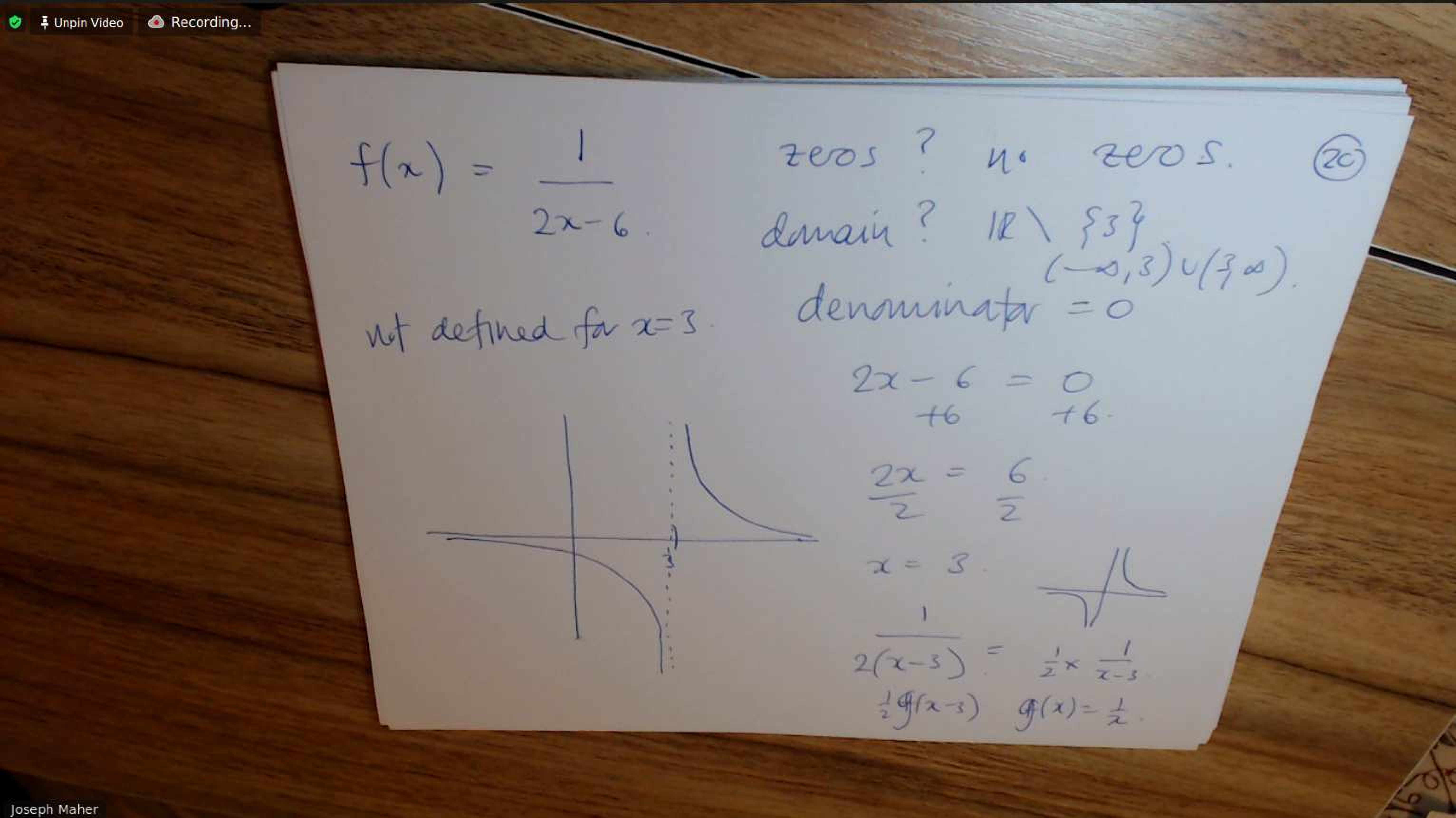
⑲

$$\frac{t^{9/4-2}}{6s^{2-3/2}}$$

$$\frac{9}{4} - \frac{2\times 4}{1\times 4} \qquad \frac{9-8}{4} \qquad \frac{1}{4}.$$

$$\frac{2\times 2}{2\times 1} - \frac{3}{2} \qquad \frac{4-3}{2} = \frac{1}{2}.$$

$$\frac{t^{1/4}}{6s^{1/2}}$$



⑳

$f(x) = \frac{1}{2x-6}$.

zeros? no zeros.

domain? $\mathbb{R} \setminus \{3\}$

$(-\infty,3)\cup(3,\infty)$.

not defined for $x=3$.

denominator $=0$

$2x-6 = 0$

$+6 \quad +6$.

$\frac{2x}{2} = \frac{6}{2}$.

$x = 3$.

$\frac{1}{2(x-3)} = \frac{1}{2}\times\frac{1}{x-3}$.

$\frac{1}{2}g(x-3) \quad g(x)=\frac{1}{x}$.

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㉑

Q5

$$x^{-\frac{1}{2}}(x+3)^{1/2} + x^{1/2}(x+3)^{-1/2}.$$

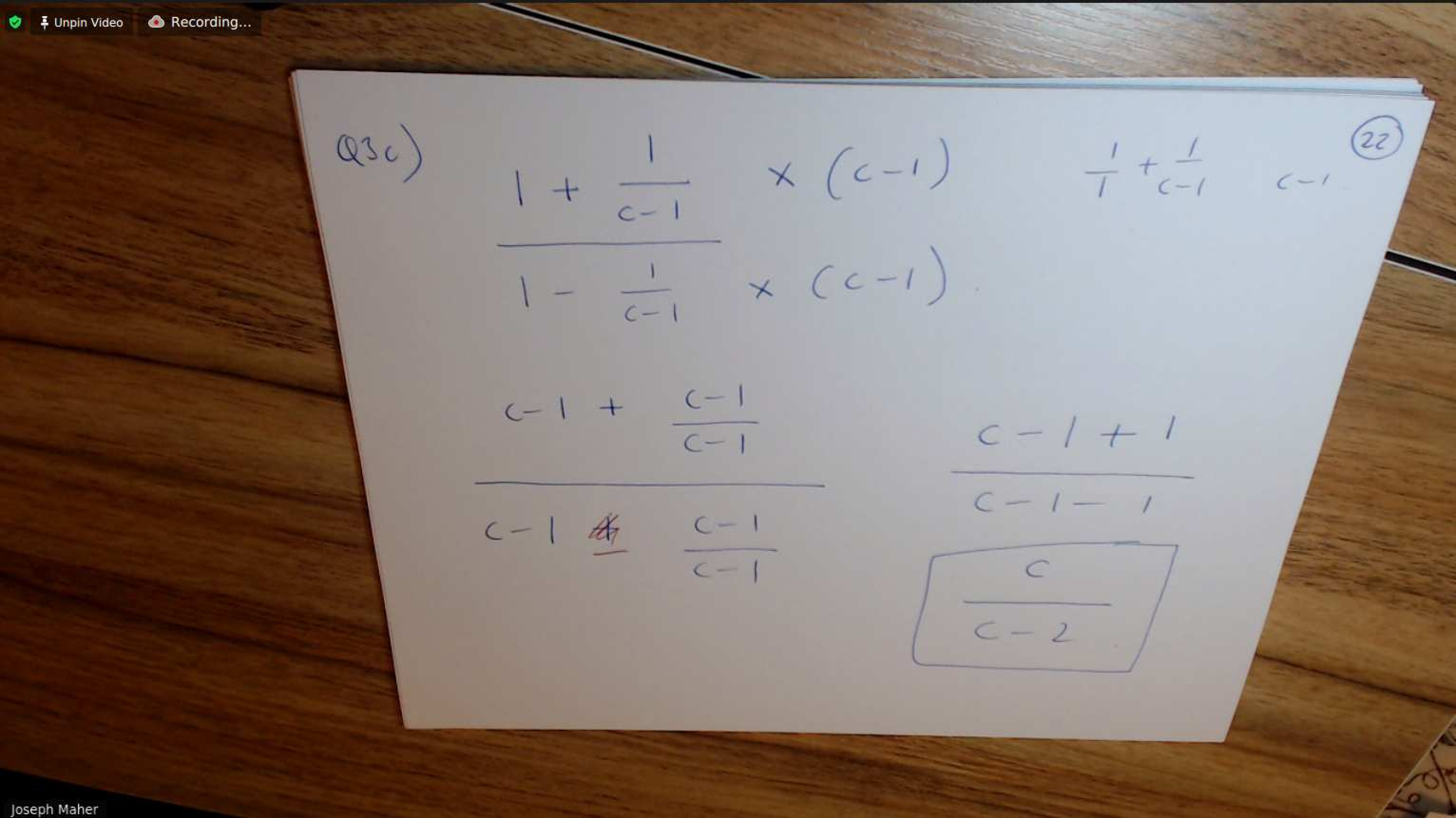
$$y^{-1} = \frac{1}{y}$$

$$\frac{(x+3)^{1/2}}{(x+3)^{1/2}} \times \frac{(x+3)^{1/2}}{x^{1/2}} + \frac{x^{1/2} \times x^{1/2}}{(x+3)^{1/2} \times x^{1/2}} \qquad x^{1/2}(x+3)^{1/2}$$

$$\frac{(x+3)^{1} + x^{1}}{x^{1/2}(x+3)^{1/2}}$$

$$\boxed{\frac{2x+3}{x^{1/2}(x+3)^{1/2}}}$$

Joseph Maher



Q3c)

㉒

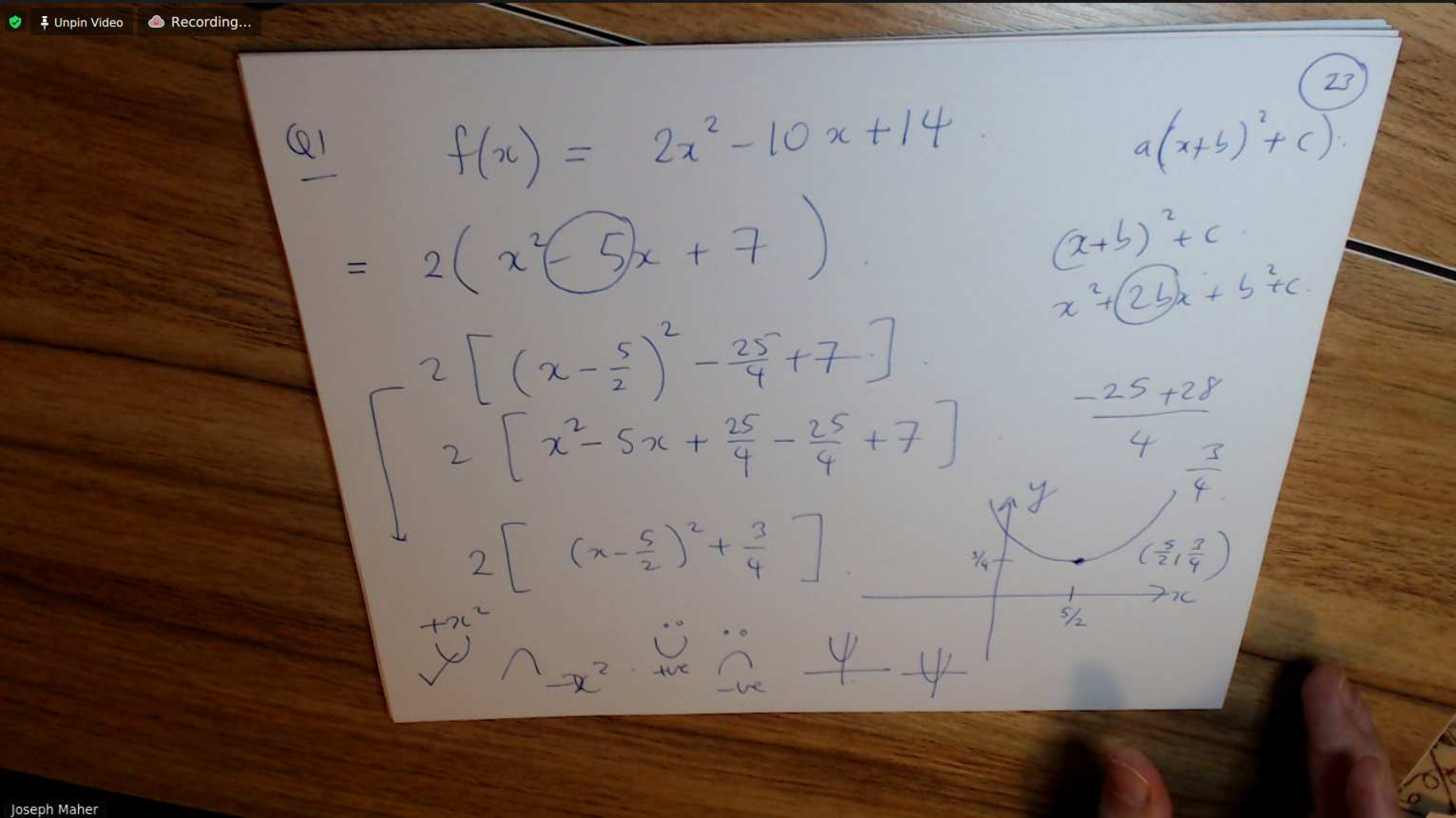
$$\frac{1+\frac{1}{c-1} \quad \times (c-1)}{1-\frac{1}{c-1} \quad \times (c-1)}$$

$\frac{1}{1}+\frac{1}{c-1} \qquad c-1$

$$\frac{c-1+\frac{c-1}{c-1}}{c-1 \; \cancel{+}\!\!- \; \frac{c-1}{c-1}}$$

$$\frac{c-1+1}{c-1-1}$$

$$\boxed{\frac{c}{c-2}}$$



Q1 $f(x) = 2x^2 - 10x + 14$.

$a(x+b)^2 + c$.

$= 2(x^2 \boxed{-5}x + 7)$.

$2\left[(x - \frac{5}{2})^2 - \frac{25}{4} + 7\right]$.

$2\left[x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 7\right]$.

$2\left[(x - \frac{5}{2})^2 + \frac{3}{4}\right]$

$(x+b)^2 + c$.

$x^2 + \boxed{2b}x + b^2 + c$.

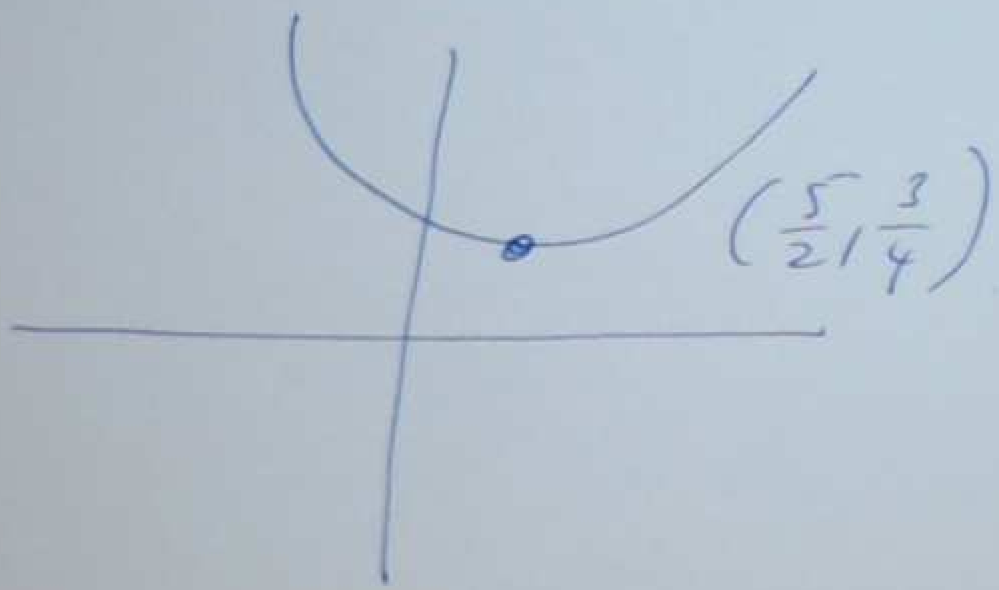
$\frac{-25+28}{4}$

$\frac{3}{4}$.

$(\frac{5}{2}, \frac{3}{4})$

$2x^2-10x+14$

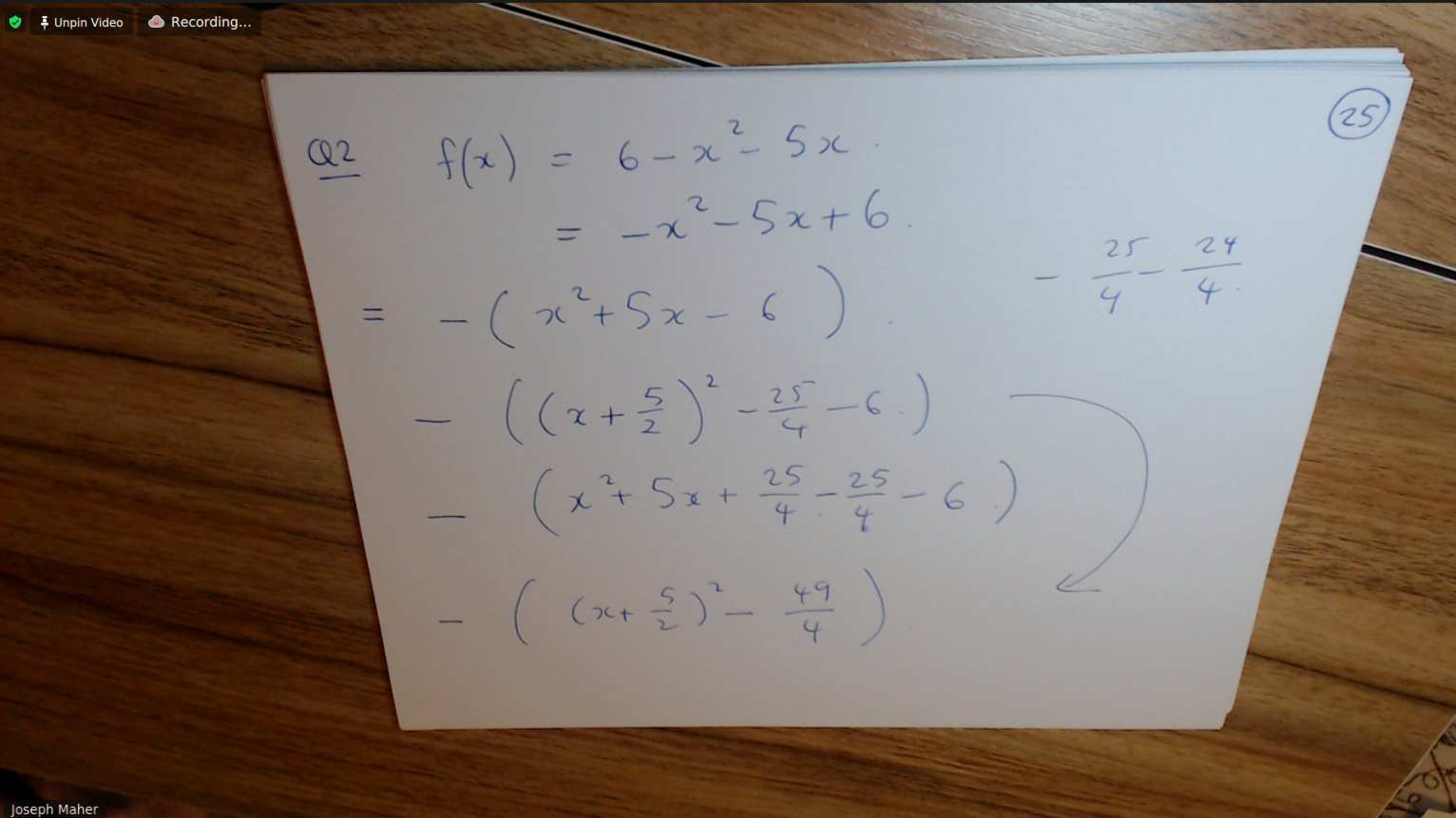
$2\left(\left(x-\frac{5}{2}\right)^2+\frac{3}{4} \right)$



minimum value is $\frac{3}{4}$

occurs at $x=\frac{5}{2}$

no zeros (doesn't hit x-axis)



Q2

$$f(x) = 6 - x^2 - 5x.$$

$$= -x^2 - 5x + 6.$$

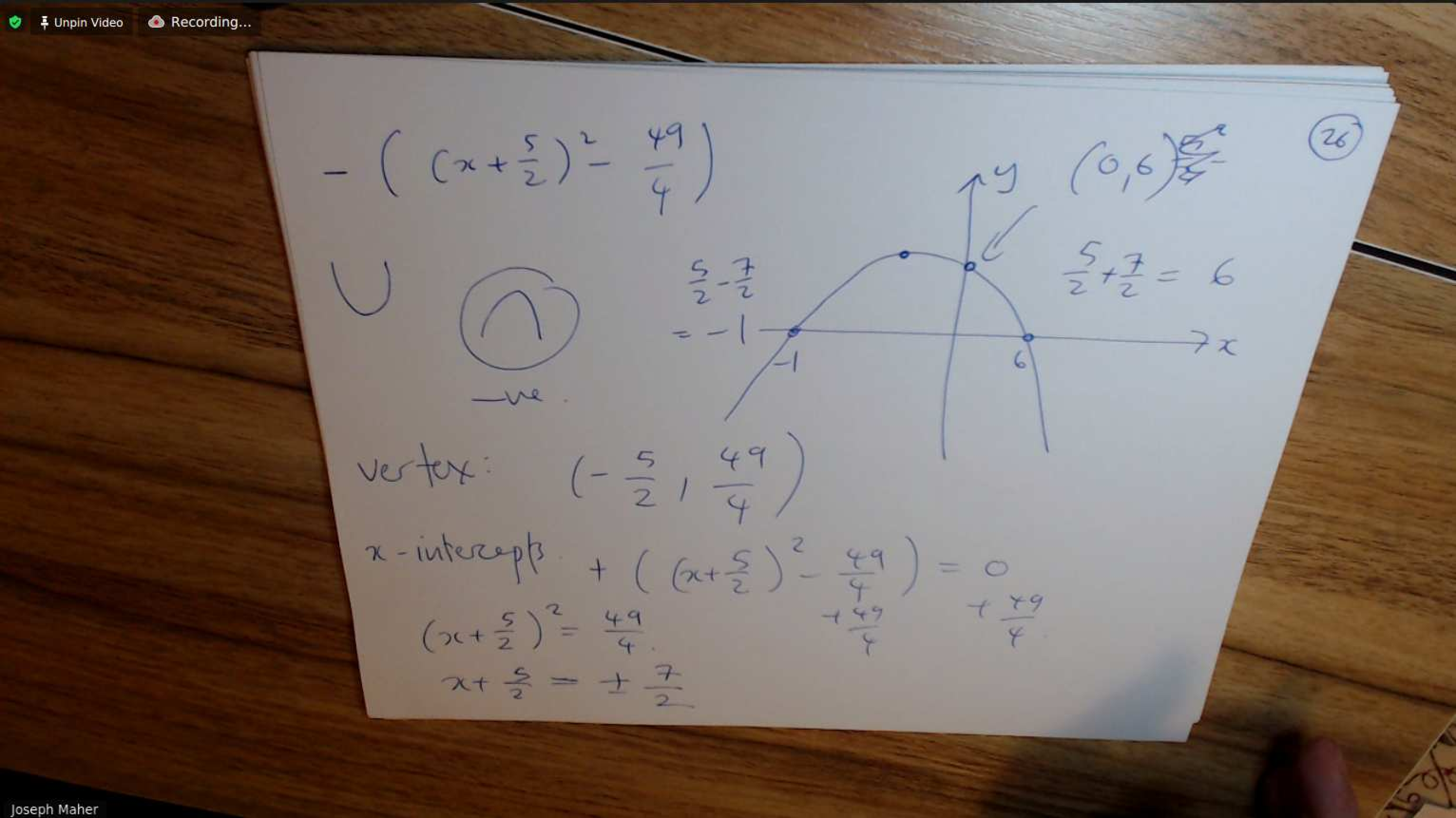
$$= -\left(x^2 + 5x - 6\right).$$

$$-\frac{25}{4} - \frac{24}{4}.$$

$$-\left(\left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - 6.\right)$$

$$-\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 6.\right)$$

$$-\left(\left(x + \frac{5}{2}\right)^2 - \frac{49}{4}\right)$$



$$-\left(\left(x+\frac{5}{2}\right)^2 - \frac{49}{4} \right)$$

—ve

(0,6)

26

$\frac{5}{2}-\frac{7}{2}$
$= -1$

$\frac{5}{2}+\frac{7}{2} = 6$

vertex: $\left(-\frac{5}{2}, \frac{49}{4}\right)$

x-intercepts $+\left(\left(x+\frac{5}{2}\right)^2 - \frac{49}{4} \right) = 0$

$+\frac{49}{4}$ $+\frac{49}{4}$

$\left(x+\frac{5}{2}\right)^2 = \frac{49}{4}$

$x+\frac{5}{2} = \pm\frac{7}{2}$

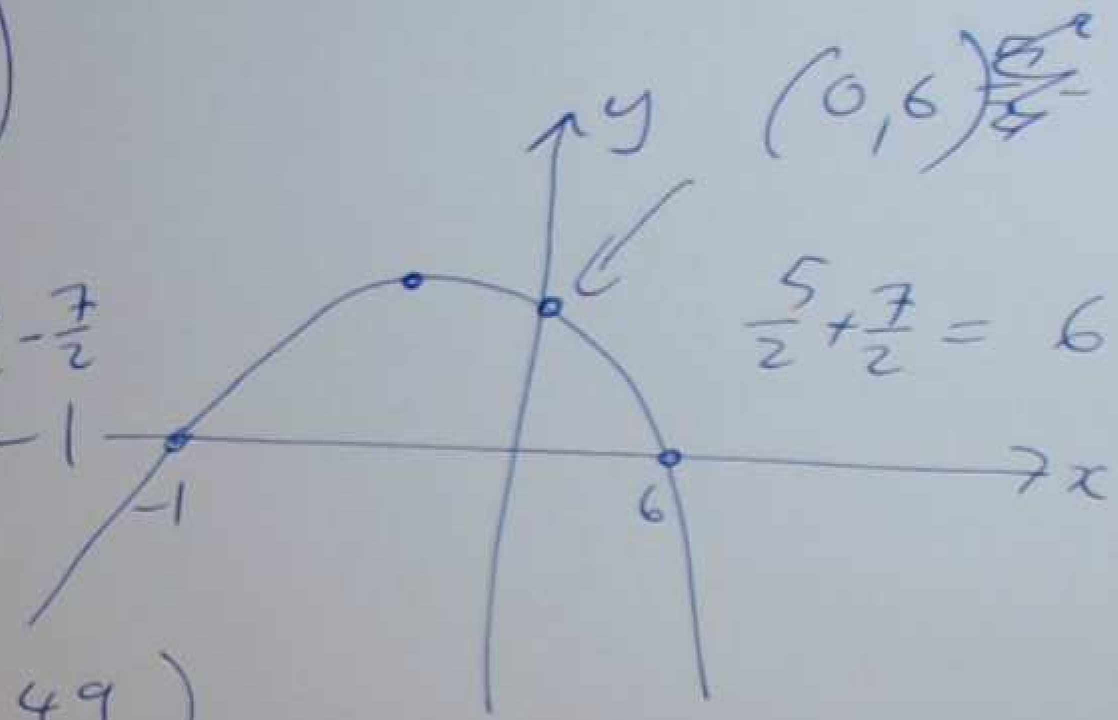
$$-\left(\left(x+\frac{5}{2}\right)^2-\frac{49}{4}\right)$$

$\cup$ $\textcircled{\cap}$

-ve

$\frac{5}{2}-\frac{7}{2}$

$=-1$



vertex: $\left(-\frac{5}{2}, \frac{49}{4}\right)$

x-intercepts $+\left(\left(x+\frac{5}{2}\right)^2-\frac{49}{4}\right)=0$

$+\frac{49}{4}$ $+\frac{49}{4}$

$\left(x+\frac{5}{2}\right)^2=\frac{49}{4}$.

$x+\frac{5}{2}=\pm\frac{7}{2}$

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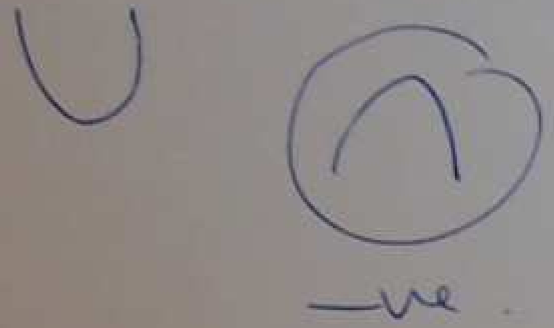
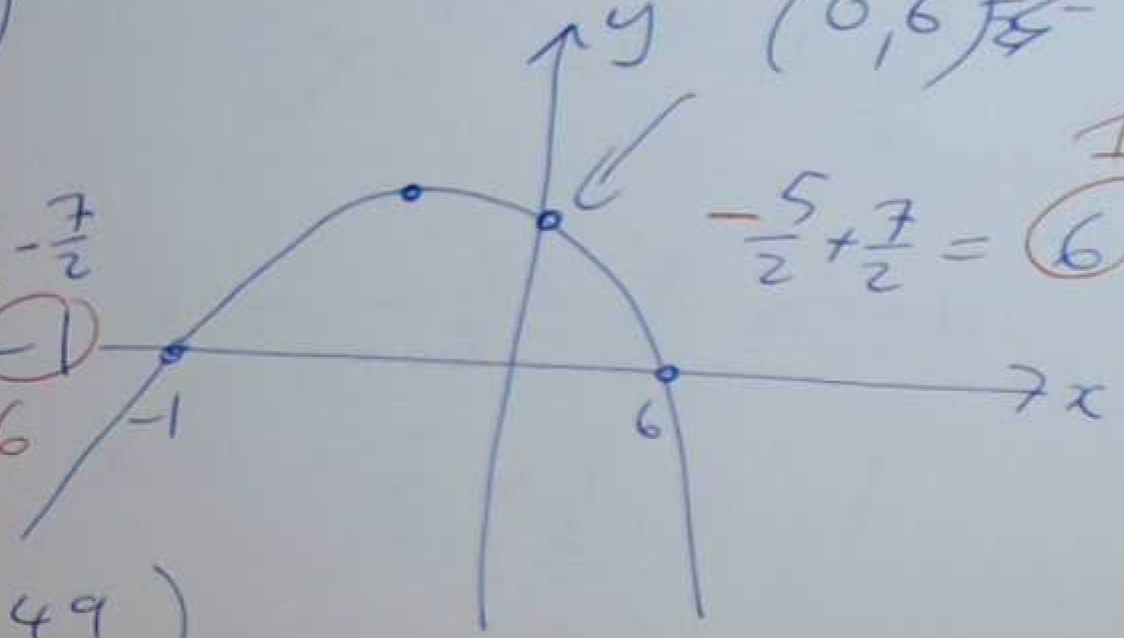
$$-\left(\left(x+\frac{5}{2}\right)^2 - \frac{49}{4} \right)$$



$-\frac{5}{2}-\frac{7}{2}$

$= \boxed{-1}$

-6



$-\frac{5}{2}+\frac{7}{2} = \boxed{6}$

1

vertex: $\left(-\frac{5}{2}, \frac{49}{4}\right)$

x-intercepts. $+\left(\left(x+\frac{5}{2}\right)^2 - \frac{49}{4} \right) = 0$

$+\frac{49}{4}$ $\quad$ $+\frac{49}{4}$

$\left(x+\frac{5}{2}\right)^2 = \frac{49}{4}$

$x + \frac{5}{2} = \pm \frac{7}{2}$

Joseph Maher

$$x + \frac{5}{2} = \pm \frac{7}{2}.$$

$$-\frac{5}{2} = -\frac{5}{2}$$

$$x = -\frac{5}{2} \pm \frac{7}{2}.$$

$$x = -\frac{5}{2} - \frac{7}{2} = -6$$

$$x = -\frac{5}{2} + \frac{7}{2} = 1$$

solutions are $-6, 1$.