## **Final Exam**

College Algebra and Trigonometry MTH 123, Section 6815, Spring 2012 Instructor: Abhijit Champanerkar & Jesenko Vukadinovic Version A



Version

Date: May 21st 2012

Points: 100

Time: 100 min

Name: \_\_\_\_\_

To receive full credit, you must explain your answers. Each question is worth 10 points.

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

## BEST OF LUCK

- 1. Circle the correct answer.
  - (a) The function f(x) = sin(x<sup>2</sup> + 1) is:
    (a) Odd function
    (b) Even function
    (c) Neither
  - (b) The function  $f(x) = \sin x$  is one-to-one function on the interval  $[0, 2\pi]$ .
    - (a) True (b) False (c) Neither
  - (c) For any functions f(x) and g(x),  $f \circ g(x) = g \circ f(x)$ .
    - (a) True (b) False (c) Neither
  - (d) A quadratic function always has an absolute max/min on the interval (-∞,∞).
    (a) True
    (b) False
    (c) Neither
  - (e) An exponential functions always has an absolute max/min on the interval (-∞,∞).
    (a) True
    (b) False
    (c) Neither
  - (f) The function  $f(x) = \sin 2x$  has an inverse on the interval  $[0, 2\pi]$ .
    - (a) True (b) False (c) Neither
  - (g) Simplify  $\sqrt[3]{(x^3y)^2y^4}$ . (a)  $x^4y^2$  (b)  $x^4y^4$  (c)  $x^2y^4$  (d) $x^2y^2$
  - (h) What is the average rate of change of  $f(x) = 2x^2 3$  between x = 2 and x = 3?
    - (a) 12 (b) 10 (c) 15 (d) 20

2. (a) Find the slope and equation of the line passing through points (-1, 2) and (2, -7). Write your final answer in the slope-intercept form y = mx + b.

(b) Let  $f(x) = 7 - 8x - 2x^2$ . Put f(x) in standard form  $a(x - h)^2 + k$ . Does f(x) have a maximum or minimum ? Find it.



3. (a) The graph of y = f(x) is as shown. Sketch the graphs of the following functions :

(b) The graphs of y = f(x) and y = g(x) are given below.



- (i) (f g)(2) =\_\_\_\_\_
- (ii)  $(g \circ f)(4) =$ \_\_\_\_\_
- (iii)  $(f \circ g)(-2) =$ \_\_\_\_\_
- (iv) Intervals on which f(x) increasing \_\_\_\_\_



4. (a) Sketch the line 2x - 3y = 6. Write down the x - & y-intercepts and the slope.

(b) You want to fence off a rectangular plot of land adjacent to a river (with no fence along the river). Find the area of the largest plot possible with 400 ft of fencing. Please explain your answer. (You may use a calculator for this problem.)

(ii) 
$$f(x) = e^{3x-2} - 6$$

(b) If  $f(x) = \sin 2x$  and  $g(x) = 3^{x-1}$ , find the following. (i)  $f \circ g(x)$ 

(ii)  $g \circ f(x)$ 

6. (a) Evalutate the following. (i)  $\log_3 \frac{1}{27}$ .

(ii)  $\log_5 7$ .

(b) Solve the following equations. (i)  $\log_2(3-2x) = 3$ .

(ii) 
$$e^{5x} - 9 = 0$$
.

7. (a) John deposited \$18,000 today in a bank where the interest rate is 5% compounded continuously according to the formula A(t) = P e<sup>rt</sup>.
(i) How much will he have after 6 years.

(ii) How long does it take to double his money?

(b) A certain radioactive material decays according to the formula n(t) = n<sub>0</sub> e<sup>-0.0231t</sup>
(i) If in 2010 we have 80 gram of this material, how much will remain in 2025.

(ii) Find the half life of this material?

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$					
$\cos  heta$					
an  heta					

8. (a) Fill in the table using exact values.

(b) Find the reference numbers for the angles below and the exact value of the following. (Answers in decimal will recieve a zero.)
(i) cos(120°)

(ii)  $\tan(\frac{5\pi}{4})$ 

9. (a) Let  $\cos \theta = -3/5$ , and  $\theta > 0$  is in quadrant III. Find the exact values of the remaining five trigonometric functions.

(b) Sketch one period of the graph of  $y = 3\sin(2x)$  to scale. Find the amplitude and period. Mark five equidistant points on the x-axis and the values of the function at those points on the graph.





## **10.** Sketch the graph of any five of the functions below.

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## ROUGH WORK

The Final Exam will be given on May 24, 2010 at 6:30pm in 2S-215.

1. Write down the equations of lines with the given description.

- (a) A line which passes through the points (1, 2) and (-2, 3).
- (b) A line with slope 3 and passing through the point (2, 2).
- (c) A line which passes through the origin and parallel to the line 3x + 5y = 3.
- 2. Let y = f(x) be the graph given below.



(a)	What is the domain of $f$ ?	
(b)	How many relative minima does $f$ have ? Write the $x$ and $y$ values of the relative minima.	

3. Use your calculator to find the only positive root of this equation:

$$x^3 + x^2 - 4x - 2 = 0$$

4. Solve the following equations using the quadratic formula:

(a)  

$$\frac{3}{2}x^2 + \frac{1}{2}x - 1 = 0$$
  
(b)  
 $\sqrt{x + 25} - 2x + 16 = 0$ 

- 5. Let  $f(x) = \sqrt{x+2} 5$ .
  - (a) Explain how you get this graph from the graph of  $g(x) = \sqrt{x}$ .
  - (b) Find the domain of f(x).
- 6. Solve the following inequality and write the solution in interval notation (this means round parenthesis and/or square brackets). Also, sketch your answer on the real line.

$$\left|\frac{3}{2}x - 1\right| < 4$$

7. Complete the square to find the vertex of the following parabola. Find the x-intercepts as well. Sketch the graph.

$$f(x) = x^2 - 6x + 8$$

8. Simplify the following expression

$$\frac{(8x^3)^{-2/3}y^3}{32(x\sqrt{y})^3}$$

9. Given the graph of  $f(x) = 3^{-x}$ , describe how to get the graph of  $g(x) = 3^{-(x-2)} - 1$ . Sketch both graphs on the same set of axes.

10. Evaluate

$$\log_2 \frac{1}{64}$$
,  $\ln e^{-0.2}$ ,  $\log_3 20$ 

11. Use the properties of the logarithm to write the following expression as a sum, difference, and/or constant multiple of logarithm.

$$\log_2 \frac{8x^4(y-5)^2}{z^3}, \qquad \log \sqrt[5]{rac{z^4y^5}{100 a^3}}$$

12. Solve the following equations

$$\log(x+21) + \log x = 2, \qquad e^{2x-3} = 5$$

- 13. Alan has \$1000 today deposited in a bank where the interest rate is 5% per year compounded continuously. How much will he have 3 years from now? How much if the interest is compounded monthly? How long must he wait to have \$2000 in his account if the interest is compounded continuously?
- 14. Given  $\sin(\alpha) = -3/5$  with  $\frac{3\pi}{2} < \alpha < 2\pi$ , find the exact values of  $\cos(\alpha)$  and  $\tan(\alpha)$ . Draw a picture that explains your work.
- 15. (a) Determine the reference angle for  $\theta = 225^{\circ}$ , and plot  $\theta$  on the unit circle.
  - (b) Find the exact values of  $\sin(225^\circ)$ , of  $\cos(225^\circ)$ , and of  $\tan(225^\circ)$ .
  - (c) Convert  $225^{\circ}$  to radians.
- 16. At a distance of 135 ft on the ground, a light source shines onto a cloud. The angle between the ground and the light spot is measured to be  $67.35^{\circ}$ . How high is the cloud in the air?
- 17. Sketch one period of the graph  $y = 2\sin(2x) 1$ . Label the highest and the lowest point of your graph. Find the amplitude and the period.
- 18. Simplify

$$(1 - \sin \theta)(1 + \sin \theta) - \cos^2 \theta = ?$$