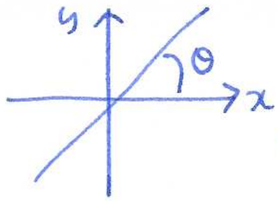
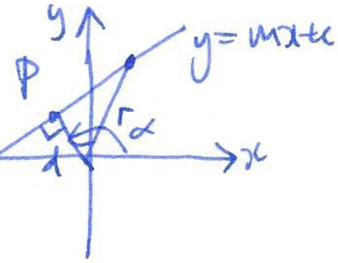


straight line through origin $\quad y = mx$

$\theta = \tan^{-1}(m) \quad , \theta = \tan^{-1}(m) + \pi$

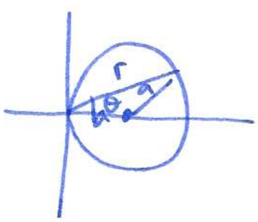
convention: $(-r, \theta) = (r, \theta + \pi)$, so can just write $\theta = \tan^{-1}(m)$

general line:



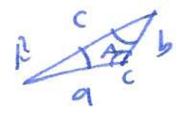
in polars: let P be the closest point to origin $O = (0,0)$, and let $d(O,P) = d$

$$\frac{d}{r} = \cos(\theta - \alpha) \qquad r = \frac{d}{\cos(\theta - \alpha)} = d\sec(\theta - \alpha)$$



cartesian: $(x-a)^2 + y^2 = a^2$

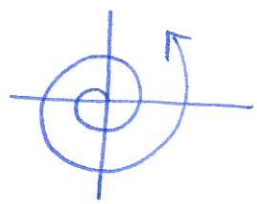
polar: $\quad r = 2a\cos\theta$



cosine rule: $\quad a^2 = b^2 + c^2 - 2bc\cos\theta$

so $\quad a^2 = r^2 + a^2 - 2ar\cos\theta \rightsquigarrow r^2 = 2ar\cos\theta \Rightarrow r = 2a\cos\theta$

<u>Examples</u> sketch $r = \theta$



$r = \sin\theta$

$r = \sin(2\theta)$ etc.

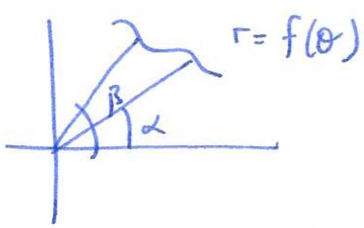
can always try: $x^2 + y^2 = r^2$, $\quad x = r\cos\theta$, $\quad y = r\sin\theta$, $\quad y/x = \tan\theta$

$y = x^2 \rightsquigarrow \quad r\sin\theta = r^2\cos^2\theta$

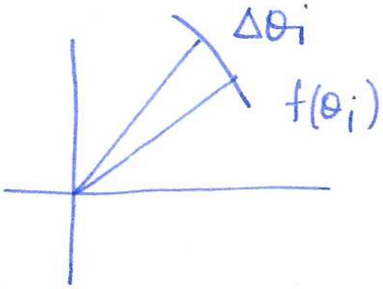
$\sin\theta = r\cos^2\theta$

$$r = \frac{\sin\theta}{\cos^2\theta} = \tan\theta\sec\theta$$

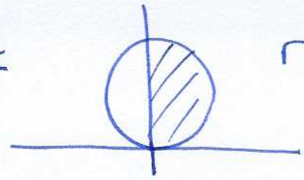
§11.4 Area and arc length in polars



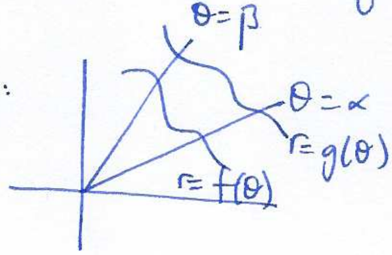
$$\text{area} = \int_\alpha^\beta \frac{1}{2}(f(\theta))^2 \, d\theta$$



$$\text{area } \Delta A \approx \frac{\pi r^2}{2\pi/\Delta\theta_i} = \frac{1}{2} r^2 \Delta\theta_i$$

Example  $r = 4\sin\theta$ $\quad$ area $= \int_0^{\pi/2} \frac{1}{2}(4\sin\theta)^2 \, d\theta = \int_0^{\pi/2} 8\sin^2\theta \, d\theta$

$$= 8\int_0^{\pi/2} \tfrac{1}{2}(1-\cos 2\theta)\,d\theta = \Big[4\theta - 2\sin 2\theta\Big]_0^{\pi/2} = 4\cdot\tfrac{\pi}{2} - 0 = 2\pi$$

area between two curves: 

area $= \frac{1}{2}\int_\alpha^\beta (g(\theta))^2 - (f(\theta))^2 \, d\theta$

<u>arc length</u> $r = f(\theta)$ is a parameterised curve with

$$x = r\cos\theta = f(\theta)\cos\theta$$
$$y = r\sin\theta = f(\theta)\sin\theta$$

so $\frac{dx}{d\theta} = f'(\theta)\cos\theta + f(\theta)(-\sin\theta)$

$\frac{dy}{d\theta} = f'(\theta)\sin\theta + f(\theta)\cos\theta$

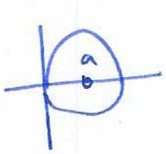
arc length $s = \displaystyle\int_\alpha^\beta \sqrt{(f'\cos\theta - f\sin\theta)^2 + (f'\sin\theta + f\cos\theta)^2}\, d\theta$

$$\left.\begin{aligned} &(f')^2\cos\theta - 2f'f\cos\theta\sin\theta + f^2\sin^2\theta \\ &(f')^2\sin\theta + 2f'f\cos\theta\sin\theta + f^2\cos\theta \end{aligned}\right\} = (f')^2 + (f)^2$$

so arc length in polars is $s = \displaystyle\int_\alpha^\beta \sqrt{(f'(\theta))^2 + (f(\theta))^2}\, d\theta$

Example circle $r = 2a\cos\theta$ $\quad f(\theta) = 2a\cos\theta$

$f'(\theta) = -2a\sin\theta$



$$\int_0^{\pi} \sqrt{4a^2\cos^2\theta + 4a^2\sin^2\theta}\, d\theta = \int_0^{\pi} 2a\, d\theta = \Big[2a\theta\Big]_0^{\pi} = 2\pi a$$