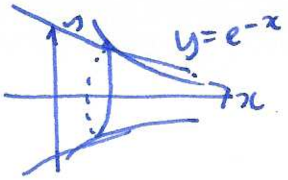
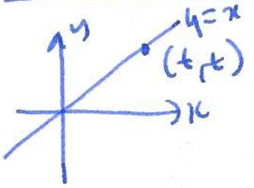
Example 

$\frac{dy}{dx} = -e^{-x}$ surface area $= \int_0^\infty 2\pi e^{-x}\sqrt{1+(-e^{-x})^2}\,dx$

sub $u = e^{-x}$, $\frac{du}{dx} = -e^{-x}$ $\quad = \int_1^0 2\pi u\sqrt{1+u^2}\,\frac{dx}{du}\,du = \int_0^1 2\pi u\sqrt{1+u^2}\,\frac{1}{u}\,du$

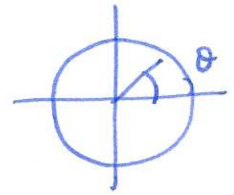
$= \int_0^1 2\pi\sqrt{1+u^2}\,du$ ← trig sub integral.

§11.1 Parametric equations



$x(t) = t$
$y(t) = t$

$c(t) = (x(t), y(t))$ parameterized curve

eg 

$c(\theta) = (\cos\theta, \sin\theta)$

problem: many different parametrization give same curve.

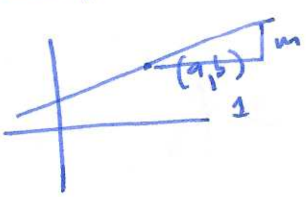
eg $c(t) = (2t, 2t)$ gives $y=x$ as does $c(t) = (t^3, t^3)$

warning: $c(t) = (t^2, t^2)$ not the same! cf $(\sin t, \sin t)$

Example describe the parameterized curve $c(t) = (t+1, t^2+1)$ in terms of $y = f(x)$:

$$\left.\begin{aligned} x &= t+1 \\ y &= t^2+1 \end{aligned}\right\} \quad \begin{aligned} t &= x-1 \\ y &= (x-1)^2+1 = x^2-2x+2 \end{aligned}$$

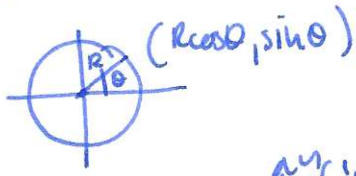
Example ① find a parametric equation for the line through (a,b) with slope m

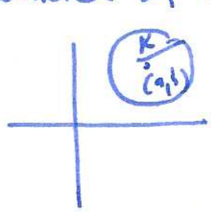


$t=0 \quad (a,b)$
$t=1 \quad (a+1, b+m)$

$c(t) = (a+t, b+tm)$

② circle of radius R centered at (a,b)





$c(\theta) = (a + R\cos\theta, b + \sin\theta)$

③ ellipse 

$c(\theta) = (2\cos\theta, \sin\theta)$

Q: how do we find the slope of the tangent line?

A: $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$ $\quad$ why: $c(t) = (x(t), y(t))$

chain rule: $\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt}$