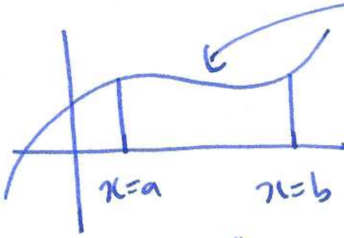
§8.1 Arc length and surface area



Q: how long? A:

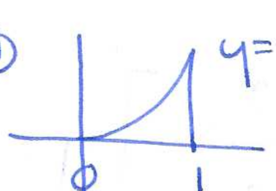
$$|\Delta L| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\text{length} = \sum_{i=1}^{n} \Delta L_i = \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} = \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\, \Delta x_i$$

take limit as $|\Delta x_i| \to 0$: arc length $= \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

$$= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Examples ①



$y = x^2$ $\quad \frac{dy}{dx} = 2x$ $\quad$ arc length: $\int_0^1 \sqrt{1+4x^2}\, dx$

(trig sub integral)

② $f(x) = \frac{1}{12}x^3 + \frac{1}{x}$ on $[1,2]$

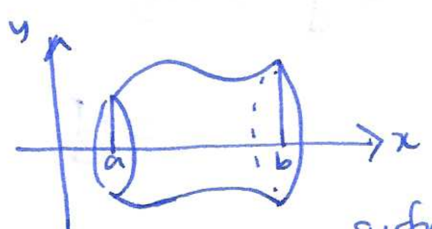
$$f'(x) = \frac{1}{4}x^2 - \frac{1}{x^2}$$

$$\text{arc length} = \int_1^2 \sqrt{1 + \left(\frac{1}{4}x^2 - \frac{1}{x^2}\right)^2}\, dx = \int_1^2 \sqrt{1 + \frac{1}{16}x^4 - \frac{1}{2} + \frac{1}{x^4}}\, dx$$

$$= \int_1^2 \sqrt{\frac{1}{16}x^4 + \frac{1}{2} + \frac{1}{x^4}}\, dx = \int_1^2 \sqrt{\left(\frac{1}{4}x^2 + \frac{1}{x^2}\right)^2}\, dx = \int_1^2 \frac{1}{4}x^2 + \frac{1}{x^2}\, dx$$

$$= \left[\frac{x^3}{12} - \frac{1}{x}\right]_1^2 = \frac{8}{12} - \frac{1}{4} - \frac{1}{12} + 1 = \frac{4}{3}$$

Area of surface of revolution



$$\text{area} \approx \sum_{i=1}^{n} \Delta L_i\, f(x_i)\, 2\pi \qquad \Delta L_i = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

take limit as $|\Delta x_i| \to 0$

surface area of volume of revolution $= \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2}\, dx$