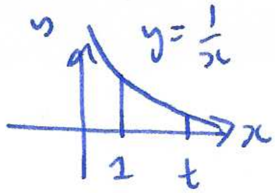
Example



between 1 and ∞
1 and t, let $t \to \infty$

volume $V = \int_1^t \frac{\pi}{x^2} dx = \pi \left[-\frac{1}{x}\right]_1^t = \pi\left(1-\frac{1}{t}\right)$

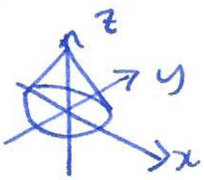
$\to \pi$ as $t \to \infty$.

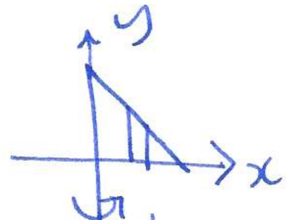
surface area $A = 2\pi \int_1^t \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\, dx > 2\pi \int_1^t \frac{1}{x}\, dx = 2\pi \left[\ln|x|\right]_1^t$

$\geqslant 2\pi \ln(t) \to \infty$ as $t \to \infty$.

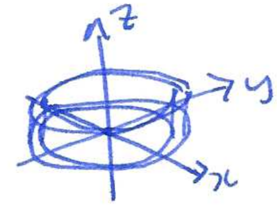
§6.4 Cylindrical shells

volume of cone :





rotate around y-axis

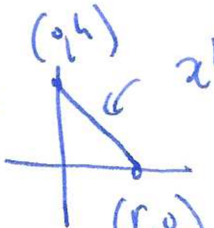


volume of shell $V_i \approx 2\pi x_i y_i \Delta x_i$

total volume $V = \sum V_i = \sum 2\pi x_i y_i \Delta x_i$

$V = 2\pi \int_a^b x f(x)\, dx$

Example



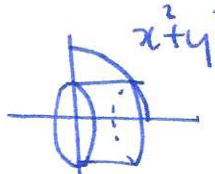
$xh + yr = hr$

$V = \int_0^r 2\pi x \left(\frac{hr - xh}{r}\right) dx$

$V = \frac{2\pi h}{r} \int_0^r x(r-x)\, dx$

$= 2\pi \frac{h}{r} \int_0^r xr - x^2\, dx = 2\pi \frac{h}{r}\left[\frac{1}{2}x^2 r - \frac{1}{3}x^3\right]_0^r = 2\pi \frac{h}{r}\left(\frac{1}{2}r^3 - \frac{1}{3}r^3\right) = \frac{1}{3}\pi r^2$

Example



$x^2+y^2=r^2$

volume of hemisphere : rotate horizontal rectangle about x-axis.

$V = 2\pi \int y f(y)\, dy.$

$V = \int_0^r 2\pi y \sqrt{r^2-y^2}\, dy = 2\pi \left[\frac{2}{3}(r^2-y^2)^{3/2} \cdot -\frac{1}{2}\right]_0^r = -\frac{2\pi}{3}(0-r^3) = \frac{2}{3}\pi r^3.$

§7.1 Integration by parts

"reverse product rule" product rule : $(uv)' = u'v + uv'$

$$\int (uv)'\,dx = \int u'v + uv'\,dx$$

$$uv = \int u'v\,dx + \int uv'\,dx$$

$$\boxed{\int uv'\,dx = uv - \int u'v\,dx}$$ ← integration by parts formula

Examples ① $\int x\sin x\,dx$ $\quad u = x \quad v' = \sin(x)$

$u' = 1 \quad v = -\cos(x)$

$$\int \underbrace{x}_{u}\,\underbrace{\sin(x)}_{v'}\,dx = \underset{u}{x}\,(\underset{v}{-\cos(x)}) - \int \underset{u'}{1}\,(\underset{v}{-\cos x})\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + c$$ check!

② $\int \underbrace{x}_{u}\underbrace{e^x}_{v'}\,dx$ $\quad u = x \quad v' = e^x$

$u' = 1 \quad v = e^x$

Q: spose we chose these the other way round?

$$= uv - \int u'v\,dx = xe^x - \int 1\,e^x\,dx = xe^x - e^x + c$$ check!

observation $\int_a^b uv'\,dx = [uv]_a^b - \int_a^b u'v\,dx$

③ $\int_1^2 \ln(x)\,dx = \int_1^2 \underbrace{1}_{v'}\cdot\underbrace{\ln(x)}_{u}\,dx$ $\quad u = \ln(x) \quad v' = 1$

$u' = \frac{1}{x} \quad v = x$

$$= [x\ln(x)]_1^2 - \int_1^2 x\cdot\frac{1}{x}\,dx$$

$$= 2\ln(2) - 1\ln(1) - [x]_1^2 = 2\ln(2) - 1$$