useful rules

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\tan^{-1}(x)$	$\frac{1}{1+x^2}$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$

$f(x)$	$\int f(x)\,dx$	
x^n	$\frac{x^{n+1}}{n+1}$	$(n \neq -1)$
$\frac{1}{x}$	$\ln\lvert x\rvert$	
$\sin(x)$	$-\cos(x)$	
$\cos(x)$	$\sin(x)$	
e^x	e^x	

general rules

f	f'
$kf(x)$	$kf'(x)$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)\,g(x)$	$f'(x)\,g(x) + f(x)\,g'(x)$
$f(g(x))$	$f'(g(x)) \cdot g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{g(x)\,f'(x) - g'(x)\,f(x)}{g(x)^2}$

f	$\int f(x)\,dx$
$kf(x)$	$k\int f(x)\,dx$
$f(x)+g(x)$	$\int f(x)\,dx + \int g(x)\,dx$

§5.7 Integration by substitution

$$\int_{x=a}^{x=b} f(x)\,dx, \quad x(u) \quad \rightsquigarrow \quad \int_{u_1=x^{-1}(a)}^{u_2=x^{-1}(b)} f(x(u))\,\frac{dx}{du}\,du$$

recall : $\frac{dx}{du} = \frac{1}{du/dx}$ integration by substitution is "reverse chain rule".

Examples $\cdot \int x\sin(x^2)\,dx$ try $x^2 = u$ $\frac{du}{dx} = 2x$

$$\int x\sin(x^2)\,dx = \int x\sin(u)\,\frac{1}{2x}\,du = \int \frac{1}{2}\sin(u)\,du = -\frac{1}{2}\cos(u) + c = -\frac{1}{2}\cos(x^2) + c$$

check!

- $\int e^{4x}\,dx$ $\quad u = 4x \quad \frac{du}{dx} = 4 \quad \int e^u \frac{1}{4}\,du = \frac{1}{4}e^u + c = \frac{1}{4}e^{4x} + c$

check!

- $\int \tan\theta\,d\theta = \int \frac{\sin\theta}{\cos\theta}\,d\theta$ $\quad \left[\text{hint: } \frac{d}{dx}\left(\ln(f(x))\right) = \frac{1}{f(x)}\cdot f'(x) \right]$

try: $u = \cos\theta$, $\frac{du}{d\theta} = -\sin\theta$

$$\int \frac{\sin\theta}{u}\cdot\frac{-1}{\sin\theta}\,d\cancel{\theta}u = -\int \frac{1}{u}\,du = -\ln|u| + c$$

$$= -\ln|\cos x| + c$$

$$= \ln|\sec x| + c.$$

check!

- $\int x\sqrt{x+1}\,dx$ $\quad$ try $u = x+1$, $\frac{du}{dx} = 1$

$$\int x\sqrt{u}\,du = \int (u-1)\sqrt{u}\,du$$

$$= \int u^{3/2} - u^{1/2}\,du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + c = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + c.$$

Definite integrals

$\int_0^2 x^2\sqrt{x^3+1}\,dx$ $\quad u = x^3+1 \quad \frac{du}{dx} = 3x^2$

$$\int_1^9 x^2\sqrt{u}\,\frac{1}{3x^2}\,du = \int_1^9 \frac{1}{3}u^{1/2}\,du$$

$$= \left[\frac{2}{9}u^{3/2}\right]_1^9 = \frac{2}{9}\cdot 27 - \frac{2}{9}.$$

§5.8 More functions

inverse trig functions

$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$ $\quad \rightsquigarrow \quad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + c$

$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$ $\quad \int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + c$

$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{|x|\sqrt{x^2-1}}$ $\quad \int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}(x) + c$

Example $\int_0^1 \frac{1}{4+x^2}\,dx$ $\quad$ try: $x = 2u$, $x^2 = 4u^2$, $\frac{dx}{du} = 2$ $\rightsquigarrow \int_0^{1/2} \frac{1}{4+4u^2}\,2\,du = \frac{1}{2}\int_0^{1/2}\frac{1}{1+u^2}\,du$

$$= \left[\frac{1}{2}\tan^{-1}(u)\right]_0^{1/2} = \frac{1}{2}\left(\tan^{-1}(\tfrac{1}{2}) - \tan^{-1}(0)\right) = \frac{1}{2}\tan^{-1}(1/2).$$

exponential functions

$$\frac{d}{dx}(e^x) = e^x \qquad \int e^x\,dx = e^x + c$$

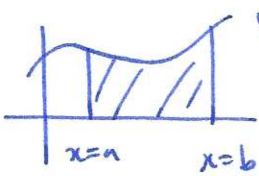
$$\frac{d}{dx}(b^x) = \frac{d}{dx}\left(e^{x\ln(b)}\right) = \ln(b)\,e^{x\ln(b)} = \ln(b)\,b^x$$

$$\int b^x\,dx = \frac{1}{\ln(b)}\,b^x + c.$$

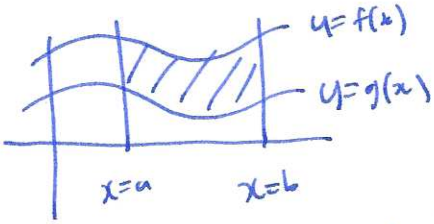
Examples $\displaystyle\int_0^1 10^x\,dx \qquad \int_0^{\pi/2} \cos\theta\;10^{\sin\theta}\,d\theta$

§ 6.1 Area between two curves

recall



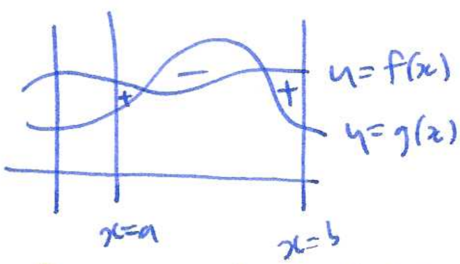
area under graph $\displaystyle\int_a^b f(x)\,dx$



area between two curves

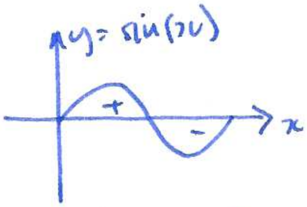
$$\int_a^b f(x) - g(x)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

warning : signs!



$$\int_a^b f(x) - g(x)\,dx$$

Q: what do you do if you want "absolute | geometric area" / unsigned area ?



$\displaystyle\int_0^{2\pi} \sin(x)\,dx = 0$ want: $\displaystyle\int_0^{2\pi} |\sin(x)|\,dx \leftarrow$ need to know when $\sin(x)$ +ve | −ve

i.e. need to find zeros of $\sin(x) = 0$ $\displaystyle\int_0^{\pi} \sin(x)\,dx + \int_{\pi}^{2\pi} -\sin x\,dx$

$$= \Big[-\cos(x)\Big]_0^{\pi} - \Big[\cos(x)\Big]_0^{2\pi} = 2 - (-2) = 4.$$