MTH 232 Calculus 2

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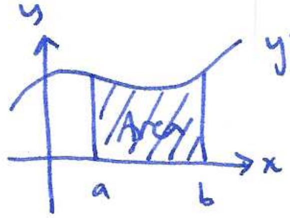
- math tutoring 1S-214?
- students w/ disabilities.

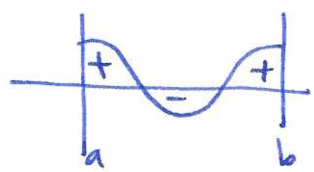
Text: Calculus, early transcendentals, Rogawski + Adams.
HW: webworks / Matlab projects / quizes.

§5.2 Definite integral



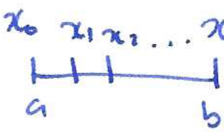
intuition: $\int_a^b f(x)\,dx =$ area under the curve $y = f(x)$ between $x=a$, $x=b$.

note: signed area



formal defn: Riemann sum $R(f, P, c) = \sum f(c_i)\Delta x_i$, $\Delta x_i = |x_i - x_{i-1}|$

f function
P partition of $[a,b]$



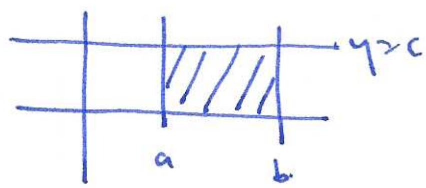
c choice of point $c_i \in [x_{i-1}, x_i]$

$$\int_a^b f(x)\,dx = \lim_{\|P\| \to 0} R(f, P, c)$$

$\|P\| = \max \Delta x_i$

useful properties

$$\int_a^b c\,dx = c(b-a)$$



$$\int_a^b f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$

reversing limits : $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

0-length intervals : $\int_a^a f(x)\,dx = 0$

adjacent intervals : $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

§5.3 Indefinite integrals

Defn A function $F(x)$ is an anti-derivative for $f(x)$ if $F'(x) = f(x)$

General antiderivative : if $F(x)$ is an anti-derivative for $f(x)$, then any other antiderivative is of the form $F(x)+c$ for some constant c.

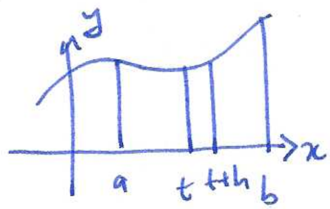
[Proof : suppose f has antiderivatives F, H, then $(F-H)' = F' - H' = f - f = 0$
$\Rightarrow$ $F - H$ is constant function]

notation : $\int f(x)\,dx = F(x) + c$ means $F(x)+c$ is the general antiderivative for $f(x)$.

§5.4 Fundamental theorem of calculus I

Thm (FTC①) suppose $f(x)$ is cts on $[a,b]$ and $F(x)$ is an antiderivative for $f(x)$, i.e. $F'(x) = f(x)$. Then $\int_a^b f(x)\,dx = F(b) - F(a)$

intuition :



consider $\int_a^t f(x)\,dx \leftarrow$ function of t!

Q: what is the rate of change wrt t?

recall $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ so $\frac{d}{dt}\left(\int_a^t f(x)\,dx \right)$

$$= \lim_{h\to 0} \frac{\int_a^{t+h} f(x)\,dx - \int_a^t f(x)\,dx}{h} = \lim_{h \to 0} \frac{\int_t^{t+h} f(x)\,dx}{h}$$

$\approx$ area of rectangle $\frac{f(t) \times h}{h} \approx f(t)$

i.e. $\int_a^t f(x)\,dx$ is an antiderivative for $f(x)$, so $\int_a^t f(x)\,dx = F(t) + c$

Q: what is the constant? $t=a$: $\int_a^a f(x)dx = 0 = F(a)+c \Rightarrow c=-F(a)$.

so $\int_a^t f(x)\,dx = F(t) - F(a)$ $\square$.

Example $\int_2^3 \sqrt{x} + \frac{1}{x} + \sin(x)\,dx = \left[\frac{2x^{3/2}}{3} + \ln|x| - \cos(x)\right]_2^3$

$= \frac{2(3)^{3/2}}{3} + \ln|3| - \cos(3) - \left(\frac{2.2^{3/2}}{3} + \ln|2| - \cos(2)\right)$.

§5.5 Fundamental theorem of calculus II

Thm (FTC ②) let $f(x)$ be a cts function on $[a,b]$, then $A(x) = \int_a^x f(t)dt$ is an antiderivative for $f(x)$, i.e. $A'(x) = f(x) = \frac{dA}{dx} \Leftrightarrow \frac{d}{dx}\int_a^x f(t)dt = f(x)$

Furthermore: $A(a) = 0$

Example: $\int_0^x e^{-t^2}dt$ $\leftarrow$ a function w/ derivative e^{-x^2}.

Example: what about $\int_0^{x^2} \sin(t)\,dt$ $\leftarrow$ function of a function!

to find $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt$ set $A(x) = \int_0^x \sin(t)\,dt$, then $A'(x) = \sin(t)$

so $\frac{d}{dx}\int_0^{x^2}\sin(t)\,dt = \frac{d}{dt}\left(A(x^2)\right) \overset{\text{chain rule}}{=} A'(x^2)\,(x^2)' = A'(x^2)\,2x = \sin(x^2).2x$.

[aside: when people say "not every formula can be integrated" what do they mean? If $f(x)$ is cts then $\int_0^x f(t)dt$ is an integral for $f(x)$, but we may not be able to write it as a formula involving basic functions.

Analogy:
- $\sqrt{2} \approx 1.414\ldots$ is a real number but not a fraction
- $x^5 - x - 1$ has 1 real root which cannot be written as an expression involving rational numbers and fractional powers. [Galois Theory]
- e^{-x^2} has an integral that can't be written as a combination of elementary functions [differential Galois theory].